Wholesale Price Discrimination and Consumer Search*

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Abstract

We analyze the incentives behind wholesale price discrimination in a vertically related market with heterogeneous consumer search. By engaging in wholesale price discrimination, a manufacturer creates asymmetries between ex-ante symmetric retailers and indirectly screens searching consumers. Low-cost retailers sell to a disproportionate share of low search cost consumers, giving them stronger incentives to compete; high-cost retailers also lower their margins given they have a smaller customer base as low search cost consumers leave. We show that this pricing practice creates more competition in the retail market and increases manufacturer's profit, but leads to lower consumer welfare. Without commitment, it is not an equilibrium outcome, however. Legislation requiring recommended retail prices to attract positive sales serves as a commitment device enabling the manufacturer to engage in price discrimination. Without legislation aiming to protect them, consumers are better off.

JEL Classification: D40; D83; L13

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1 Introduction

Empirical studies have found that wholesale price discrimination is practiced in many important markets, including gasoline markets (Hastings (2009)) and supermarkets (Villas-Boas (2009)). This price discrimination practice used by manufacturers to charge different prices to different retailers is an important concern in many antitrust cases as well. [1] This paper provides a new theory of why manufacturers with market power may engage in wholesale price discrimination and analyzes its welfare implications. The crucial feature of our theory is that retail markets are characterized by consumer search and that consumers differ in their search cost. By setting different wholesale prices to different retailers, the manufacturer stimulates search and creates a more competitive retail market, which for given wholesale prices, boosts his profits.

Under wholesale price discrimination, a manufacturer charges a low wholesale price to some retailers and a high wholesale price to others, resulting in low and high retail prices in the downstream market. Expecting some price dispersion, without knowing which retailer charges lower prices, consumers' initially search at random, but depending on their search cost, they will follow different search paths after their first search. Wholesale price discrimination effectively is a mechanism to (indirectly) screen searching consumers: observing a high retail price at their first search low search cost consumers continue to search, while others will buy immediately. As a consequence, retailers do not face the same composition of search costs among their consumers; the demand of low cost retailers consists of a relatively larger share of low search cost consumers. As low search cost consumers are more price sensitive, they will induce more competition between low cost retailers. In addition, because of the increased competition among the low cost retailers, consumers with higher search cost may also find it attractive to continue searching, forcing the high cost retailer also to lower its margins. Thus, both low and high cost retailers may have lower margins under wholesale price discrimination. As lower retail margins, *ceteris paribus* increase manufacturer profit, the manufacturer may consider to engage in wholesale price discrimination to increase profits.

Whether or not a manufacturer actually engages in wholesale price discrimination depends on the extent to which he can commit himself to the wholesale prices he sets. In industries that have relatively stable cost and demand patterns, a manufacturer may commit by setting long-term wholesale contracts with his retailers. Under commitment,

[1] See, e.g., the claim of Games People Play, a retailer for golf equipment in the US, against Nike, ruled by the federal district court in Beaumont, Texas in February 2015 (Games People Play, Inc. v. Nike, Inc.; case number 1:14-CV-321), or, earlier cases such as the decision on the European sugar industry in 1973 where the Commission ruled that, "the granting of a rebate which does not depend on the amount bought [...] is an unjustifiable discrimination [. . .]" (Recital II-E-1 of Commission decision 73/109/EC), or the the Michelin I judgement where the European Commission in 1981 contested the alleged discriminatory nature of wholesale prics (Recital 42 of Commission decision 81/969/EEC).

we show that wholesale price discrimination increases the manufacturer profits and reduces retail profits and consumer surplus. In other industries, it may be difficult to commit to charging different retailers different prices. Without commitment, wholesale price discrimination cannot be sustained as an equilibrium outcome as the manufacturer can increase his profits by secretively deviating to an individual retailer. The reason is that without commitment wholesale price discrimination can only be an equilibrium when the manufacturer makes identical profits over all retailers (those that received a low and a high wholesale price). If not, the manufacturer is better off by charging the same wholesale price to all retailers. This equal profit condition is inconsistent with the other equilibrium conditions that the low and high wholesale price have to satisfy.

For markets where the manufacturer cannot commit to his wholesale prices by means of long-term contracts, we look at the effect of recommended retail prices and the regulation imposed by the Code of Federal Regulations used by the Federal Trade Commission. Recommended retail prices are non-binding recommendations of manufacturers at which prices retailers should sell their product. As retailers are free to deviate from the recommendation, an important question is whether these recommendations affect market behaviour and if so how. Competition authorities have a concern that recommended retail prices negatively affect competition through their impact on consumers. In the United States, for example, the Code of Federal Regulations used by the Federal Trade Commission states that "to the extent that list or suggested retail prices do not in fact correspond to prices at which a substantial number of sales of the article in question are made, the advertisement of a reduction may mislead the consumer". In this paper, we argue that despite its intention to protect consumers, the Code of Federal Regulations may actually aversely affect them and that it is not in the interest of consumers (or total surplus) to require that a substantial number of sales is sold at the recommended retail price.

The importance of consumer search is implicitly recognized by the Code of Federal Regulations where it rightfully observes that a recommended retail price may also be addressed to consumers (and not only to retailers) and may affect consumers' purchasing behaviour. What the Code does not envisage, however, is that the restrictions imposed effectively facilitate manufacturers to partially commit to wholesale price discrimination as follows. The manufacturer may announce the price the high cost retailer finds it optimal to sell her product as the recommended retail price. Given the announcement, she should make sure that at least some products are sold at this price and thus she is not free to deviate and sell to all retailers at the lower wholesale price that generates more profits for him. We show that once this possible deviation is eliminated, wholesale price discrimination can be sustained as an equilibrium outcome and that the average wholesale and retail prices increase, increasing manufacturer profits, but decreasing retailers' profits and consumer welfare.

In summary, we show that a firm (manufacturer) may price discriminate between ex ante symmetric intermediaries or retailers, because of its competitive effect on a market in which he is not active himself. In vertical markets, where the retail market is characterized by consumer search and the manufacturer can commit to wholesale prices, wholesale price discrimination increases manufacturer profits. In other markets, where long-term commitment is not feasible, wholesale price discrimination is not an equilibrium outcome. Recommended retail prices are ineffective if they are not accompanied by a restriction such as the one imposed by the Code of Federal Regulations. In conjunction with the regulation, a recommended retail price acts as a commitment device of the manufacturer that enables wholesale price discrimination. Some retailers follow the recommended retail price, as this is simply their optimal price given their individual wholesale price. Other retailers sell at a price below the recommended retail price as they receive lower wholesale prices. Thus, the observation that recommendations often do not bind in practice as most products sell at a price below the recommended retail price follows naturally from our framework.

Apart from these main results, we also make methodological contributions to the literature on consumer search in vertical markets. Where previous literature (see below for a discussion) considers a discrete search cost distribution with a fraction of shoppers and non-shoppers having identical search costs, we have results for generalized search cost distributions. Unlike the literature, this also allows us to have a general existence of equilibrium results even for uniform wholesale pricing.

There are several branches of the literature to which this paper contributes. First, there is a literature on price discrimination in intermediate goods markets. The seminal papers in this literature, Katz [9], DeGraba [4] and Yoshida [16], have built arguments in favour of banning wholesale price discrimination. The basic starting point in these papers is that downstream firms differ in their efficiency levels. An unconstrained monopolist manufacturer may then choose to charge higher wholesale prices to more efficient firms. Uniform pricing constraints the monopoly power of the manufacturer increasing total surplus. Inderst and Valleti [7] and O'Brien [12] show that a ban on discrimination may have the opposite effect if the assumption of an unconstrained manufacturer is relaxed. To distinguish our theory from this literature, we consider a setting where all retailers are ex ante symmetric and all consumers have identical demand.[2] The novelty of our paper is that in many markets consumers must engage in costly search to get to know market prices. By taking into consideration information frictions regarding retail prices, a manufacturer may purposefully create asymmetries between retailers by engaging in wholesale price discrimination.

[2]This is also the reason why we prefer to speak of wholesale markets and not of intermediate goods, or input markets. Wholesale markets stress that the only difference between retailers may be caused by manufacturers selling at different prices.

Second, there is a recent literature on vertically related industries with consumer search (Janssen and Shelegia [8], Garcia, Honda, and Janssen [6], Garcia and Janssen [5] and Asker and Bar-Isaac [1]). Janssen and Shelegia [8] show that markets can be quite inefficient if consumers search sequentially while not observing the wholesale arrangement between the manufacturer and retailers. They consider a setting a la Stahl (1989) where all non-shoppers have the same search cost. Importantly, and in contrast to our paper with its focus on wholesale price discrimination, the manufacturer always sets the same wholesale price to all retailers and retailers know this. In addition, unlike Janssen and Shelegia [8], equilibrium existence is not an issue in our paper due to the general (continuous) search cost distribution we consider. Garcia, Honda, and Janssen [6] show that the inefficiency of vertical markets with consumer search continues to hold if there are many manufacturers and retailers engage in sequential search among these manufacturers. Garcia and Janssen [5] allows for wholesale price discrimination, but mainly focuses on how a manufacturer can correlate his wholesale prices to increase profits. By contrast, we focus on the competitive impact of wholesale price discrimination by changing the search cost composition of different retailers. In contrast to Garcia and Janssen [5], low cost retailers will not have monopoly power in our context as the manufacturer offers the low wholesale price to at least two retailers, maintaning competitive pressure between them. Asker and Bar-Isaac [1] study the impact of minimum advertised prices (MAPs). They see different potential roles for MAPs with price discrimination as one of them. Their model of price discrimination is very different from ours, however. They study a market where consumers have different valuations and the consumer search cost distribution is "Varian (1980) like", where some consumers compare all prices and others always buy at the first store they visit (as their search cost is prohibitively high). The rationale for wholesale price discrimination in their paper is therefore close to the traditional role for price discrimination in extracting surplus from consumers with different valuations. In contrast, in our model consumers have identical valuations and wholesale price discrimination is a way to screen consumers with different search cost. By imposing wholesale price discrimination the manufacturer endogenously determines how many consumers search beyond the first firm. We therefore have a purely informational story of price discrimination.

Third, there is a recent literature explaining how non-binding recommended retail prices may affect market behavior.[3] Buehler and Gärtner [2] and Lubensky [10] use a framework where recommended retail prices are used by the manufacturer to signal production cost. Buehler and Gärtner [2] see recommended retail prices as communication devices between a manufacturer and its retailers and where recommended retail prices are part of a relational contract enabling the manufacturer and retailer to maximize joint surplus in a indefinitely repeated setting. Lubensky [10] is closer in spirit to our model as

[3] Two empirical papers (Faber and Janssen (2008) and De los Santos et. al. (2016)) show that recommended retail prices do affect market behavior.

he shows that a manufacturer can use recommended retail prices to signal his production cost to searching consumers. As both, consumers and the manufacturer, prefer more search when the manufacturer production cost is low and less search when it is high, the manufacturer's recommendation informs consumers via cheap talk of its cost. In contrast to these papers, uncertainty concerning manufacturer cost does not play a role in our setting. Thus, one part of our paper may explain that recommended retail prices are used even in markets where the manufacturer's cost is stable over time and uncertainty should not play a big role.

Fourth, in a market where high search cost consumers are less price sensitive than low search cost consumers, Salop [13] shows that a monopolist who directly sells to consumers may engage in price discrimination: as low search cost consumers continue to search if they first encounter a high prices, higher prices attract a disproportionally large fraction of consumers with higher search cost, who (by assumption) are also less price-sensitive. As it is optimal for the monopolist to charge higher prices to consumers who are less price sensitive, the monopolist may indeed want to screen consumers with different search cost. Unlike our purely informational theory of price discrimination, Salop [13] follows the classical idea of price discrimination as distinguishing between consumers with different valuations. In addition, his argument is based on the assumption that the monopolist is committed to charging prices according to a price distribution and that any deviation from this distribution is observed by consumers. It is difficult to see, however, how consumers may observe a price distribution, while maintaining the assumption underlying the search cost literature that the consumer does not know the prices the firm sets. By studying a vertical supply chain, our paper, in contrast, can make a distinction between a manufacturer commiting on wholesale prices to retailers, while consumers search for retail prices. We also show in this paper that a limited form of commitment towards is suficient to make our argument work.

Finally, while most papers in the search literature assume at most two different levels of search cost (see, e.g., Stahl [14]), there do exist some papers that consider more general forms of heterogeneity in consumers' search costs, such as Stahl [15], Chen and Zhang [3] and Moraga-González, Sándor, and Wildenbeest [11]. In contrast to these papers, however, we focus on vertically related industry structures and this paper is the first to consider general forms of search cost heterogeneity in vertically related industry structures. One of the results of this literature is that there exists a continuum of equilibria if the measure of consumers with zero search cost is zero. We show that this is not true if the manufacturer can commit to wholesale prices, whereas it continues to be true in the last part of of our paper where we consider partial commitment through regulations.

The remainder of this paper is organized as follows. In the next section, we present the details of the model we consider. The impact of wholesale price discrimination is discussed in Section 3. Section 4 discusses the commitment case under both uniform pricing and wholesale price discrimination. Uniform pricing and wholesale price discrimination under the non-commitment case are discussed in Section 5, where we discuss the non-existence of an equilibrium with wholesale price discrimination. When the manufacturer cannot commit to his wholesale prices, we also discuss the issue of multiplicity of equilibria. In both sections, we first provide analytic results for the case where the search costs vanish, followed by a numerical analysis for the linear demand case that allows us to show the robustness of our theoretical results. In Section 6, we analyze the implications of imposing that some sales take place at the list price, as the Code of Federal Regulations does. Finally, in Section 7 we conclude and discuss possible extensions.

2 The Model

We focus on a vertically related industry with a monopolist manufacturer in the upstream market supplying a homogeneous good to $N \geq 3$ retailers.[4] The manufacturer's production costs are normalized to zero. In principle the manufacturer can charge a different wholesale price w_i to every retailer i so that formally the manufacturer's strategy is a tuple $(w_1, w_2, ..., w_N)$. We will focus on two types of equilibria: (i) in a uniform pricing equilibrium the manufacturer chooses $w_i = w^*$, whereas in an equilibrium with price discrimination the manufacturer chooses two prices w_L^* and w_H^*, with $w_L^* < w_H^*$, and charges some retailers the low and others the high wholesale price. Retailers take their wholesale price as given and do not have other costs except for the wholesale price paid to the manufacturer for each unit they sell. Observing only their own wholesale price retailers compete in prices and choose their retail strategy $p(w)$.[5]

There is a unit mass of consumers, each demanding $D(p)$ units of the good if they buy at price p. We make standard assumptions on the demand function so that it is well-behaved. In particular, there exists a $\overline{p}$ such that $D(p) = 0$ for all $p \geq \overline{p}$ and the demand function is continuously differentiable and downward sloping whenever demand is strictly positive, *i.e.*, $D'(p) < 0$ for all $0 \leq p < \overline{p}$. For every $w \geq 0$, the retail monopoly price, denoted by $p^M(w)$ is uniquely defined by $D'(p^M(w))(p^M(w) - w) + D(p^M(w)) = 0$ and $D''(p)(p - w) + 2D'(p) < 0$. Note that for $w = 0$, this condition gives that the profit function of an integrated monpolist is concave. We denote by $p^M(w^M)$ the double marginalization retail price, which arises in case there would be a monopoly at both levels of the supply chain.

In order to observe prices consumers have to engage in costly sequential search with perfect recall. Consumers differ in their search cost s. Search costs are distributed on the interval $[0, \overline{s}]$ according to the distrbution function $G(s)$, with $G(0) = 0$. We denote

[4]To study the effects of wholesale price discrimination, it is important there are at least three retailers so that there are at least two retailers that get the lowest wholesale price and there is still some competition among these retailers.

[5]Whenever there is no confusion possible, we drop subscripts i.

by $g(s)$ the density of the search cost distrbution, with $g(s) > 0$ for all s in the interval $[0, \overline{s}]$ and $-\infty < g'(s) < \infty$. In numerical examples, we consider $G(s)$ to be uniformly distributed. For most part of the analysis, it does not matter whether or not the first search is costly, and we proceed assuming the first search is for free so that we do not have to consider the participation constraint of consumers (which for small enough $\overline{s}$ will always be satisfied). As consumers are not informed about prices before they search, an equal share of consumers visits each retailer at the first search.

The timing is as follows. First, the manufacturer sets wholesale prices to all firms. Second, given the information available to them an individual retailer i sets her retail price p_i, where $i = 1, ..., N$. Each retailer observes only her own wholesale price and does not observe the wholesale prices the manufacturer offers to other retailers. Finally, consumers sequentially search for retail prices. Consumers do not observe wholesale prices (see, e.g., Janssen and Shelegia [8]). The difference between the case where the manufacturer can and cannot commit is that in the latter case, but not in the former, the manufacturer may deviate from the wholesale prices consumers and other retailers expect. The commitment case can either be viewed as a theoretical benchmark, or as a stylized description of markets where the manufacturer is committed to long-term contracts, where retailers may learn the retail prices their competitors set.[6] Generally speaking, when analyzing the no commitment case, we look for a pure strategy Perfect Bayesian Equilibrium (PBE), where on the equilibrium path, retailers' and consumers' beliefs are updated using Bayes' rule, while we consider out-of-equilibrium beliefs that are passive. We will define the equilibrium notion in more detail in the beginning of Section 5 when we analyze this case.

3 The retail market

As explained in the Introduction, the main reason why a manufacturer may want to price discriminate between different retailers is to create a more competitive retail market. In this section, we explain in detail the mechanism by means of which this works and characterize the behaviour of consumers and retailers. Passive beliefs on the part of consumers imply that if a consumer observes an unexpected retail price, she believes that the retailer that she has visited has deviated and that all other retailers charge their equilibrium prices. In case of wholesale price discrimination when consumers expect different retail prices to prevail we consider that if she observes a price in a neighborhood of one of the prices she expected, then the consumer believes that the deviation comes from a retailer that was expected to set a price that is closest to the observed price.

[6]In a study of first-mover advantage, Bagwell (1995) has shown that a player's ability to commit is equivalent to the observability of his actions. In our world with a manufacturer, multiple reailers and many consumers, the issue of commitment is more subtle as the manufacturer may commit to an individual retailer, or to retailers in gemeral, without committing to consumers.

As a benchmark, consider first the case of uniform pricing where all retailers are expected to be charged the same wholesale price w^*.[7] Let $p^*(w^*)$ denote the equilibrium price charged by all retailers (which is the retail price consumer expect). To determine the equilibrium retail price, we need to investigate how a retailer's demand depends on his price, which in turn depends on how consumers' search behaviour depends on a price deviation. If a consumer buys at a deviation price $\widetilde{p} > p^*$, he gets a surplus of $\int_{\widetilde{p}}^{1} D(p)dp$. Using passive beliefs, a consumer with search cost s continues to search for the equilibrium price $p^*(w^*)$, if[8]

$$s < \int_{p^*(w^*)}^{1} D(p)dp - \int_{\widetilde{p}}^{1} D(p)dp.$$

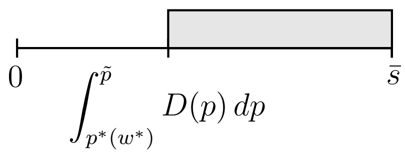

Fig 3.1 Share of consumers that buy at the deviating retailer

Thus, of all consumers who visit a retailer deviating to a price $\widetilde{p} > p^*(w^*)$ a fraction $1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)$ will continue buying from him. Therefore, the deviating retailer's profit in a uniform pricing equilibrium equals:

$$\pi_r(\widetilde{p}, p^*) = \frac{1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)}{N} D(\widetilde{p})(\widetilde{p} - w^*).$$

A retailer will never set a price larger than the retail monopoly price as he can always guarantee himself the retail monopoly profits (by lowering his price). Even if he does not attract additional consumers by doing so, he would make more profits over consumers that anyway will visit him. Thus, we must have that in equilibrium $D^{'}(p^*)(p^* - w^*) + D(p^*) \geq 0$.

Maximizing retail profits and making use of the equilibrium condition $\widetilde{p}(w^*) = p^*(w^*)$, yields that $p^* \leq p^M(w^*)$ and

$$-g(0)D^2(p^*)(p^* - w^*) + D^{'}(p^*)(p^* - w^*) + D(p^*) = 0. \tag{1}$$

[7]Here, we consider that consumers have correct beliefs concerning the wholesalesale price retailers face. In section 5, where we analyze non-commitment, we allow for the possibility that a retailer knows that the manufacturer has deviated from the wholesale price consumers expect.

[8]Note that as the first search is for free, all consumers become active and search for prices. Alternatively, if the first search is also costly, then all consumers remain active if the expected surplus of being active, given by $\int_{p^*}^{1} D(p)dp$, is larger than the maximal search cost $\bar{s}$.

Note that the equilibrium retail price is independent of the number of active retailers.[9] Note also that, in principle, *from the perspective of the retailers* the first-order condition can be satisfied with a weak inequality as retailers will never have an incentive to lower their price as long as $p^* \leq p^M(w^*)$, given that consumers search for lower prices and do not observe these prices until at the retailer in question, retailers do not attract more consumers by lowering their prices. However, in *the vertical model* taking the incentive of the manufacturer into account, it can never be the case that (1) holds with strict inequality. The reason is that in that case the manufacturer could increase profits by increasing her wholesale price. This will always be profitable as retailers will not adjust their retail price and therefore the manufacturer demand will not be affected.

To illustrate the impact of wholesale price discrimination on the retail market, consider the situation where a manufacturer is expected to set a wholesale price of w_L^* to $N-1$ retailers and w_H^* to 1 retailer. In the main part of the paper this is also the setting we consider and in subsequent sections we will explain why. A first effect on consumer search is that the low search cost consumers who happen to encounter the high cost retailer setting p_H^* will continue to search for lower retail prices. In particular, defining $\widehat{s} = \int_{p_L^*}^{p_H^*} D(p)dp$, all consumers who happen to observe p_H^* at their first search and have a search cost $s < \widehat{s}$ continue to search.[10] More generally, If a consumer observes a price p_H in the neighbourhood of p_H^*, then he will continue to search if his search cost is such that

$$s < \widehat{s} + \int_{p_H^*}^{1} D(p)dp - \int_{p_H}^{1} D(p)dp.$$

[9] This does *not* imply, however, that if retailers and consumers would expect that some retailers are foreclosed by the manufacturer (for example by receiving such a high wholesale price that they cannot effectively compete), while all the remaining retailers receive identical wholesale offers, retailers would behave in exactly the same way as in the case when all N retailers would receive the same wholesale price. The reason is that in the above analysis, it is taken for granted that if a consumer continues to search he will always find the equilibrium price on the next search. This will not be the case under foreclosure, however, as in that case the chance of finding a low retail price will be smaller and consumers will be more reluctant to search. This gives retailers more market power. Thus, a manufacturer will not want to foreclose retailers from the market. Wholesale price discrimination is, as we will see in the next Section, more subtle than foreclosure.

[10] If there would be $m^* < N-1$ retailers getting a low wholesale price, then the critical search cost value $\widehat{s}$ can be defined as:

$$\left(\frac{m^*}{N-1} + \frac{N-m^*-1}{N-1}\frac{m^*}{N-2} + .. + \frac{N-m^*-1}{N-1}\frac{N-m^*-2}{N-2} \cdot .... \cdot 1\right)\widehat{s} = \int_{p_L^*}^{p_H^*} D(p)dp$$

This expression is more complicated as there is a probability that the consumers will not immediately encounter p_L^* on her next search. The following is true, however: if a consumer continues to search after the first observation of p_H^* he will certainly continue to search after observing p_H^* each subsequent time as the chance of observing p_L^* on the next search round becomes higher. This gives the expression above.

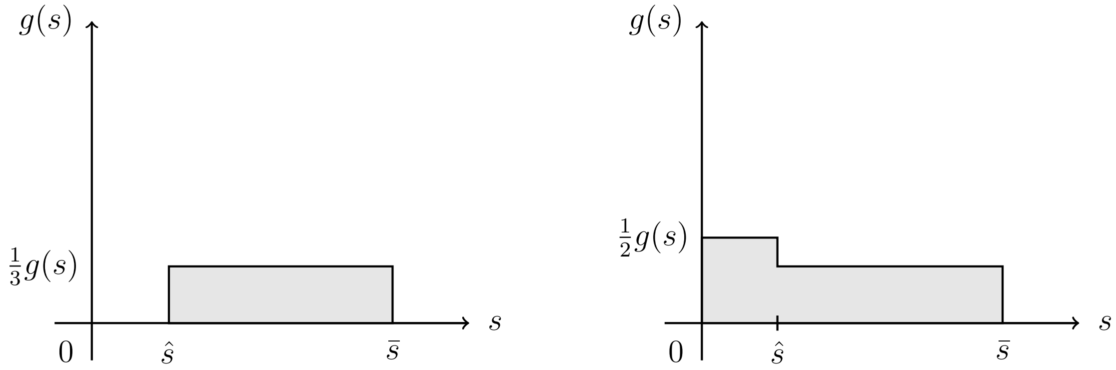

Fig 3.2 Different demand compositions of the high and low cost retailers for N=3.

Therefore, the profit of a retailer who has observed a wholesale price w_H in the neighbourhood of w_H^* and sets a price p_H in the neighbourhood of p_H^* will be:

$$\pi_r^H(p_H, p_L^*; w_H^*) = \frac{1}{N}\left(1 - G\left(\int_{p_L^*}^{p_H} D(p)dp\right)\right) D(p_H)(p_H - w_H^*). \tag{2}$$

A second effect of wholesale price discrimination on consumer search comes from considering consumers who have observed a price p_L, in a neighbourhood of p_L^*, on their first search. Believing such a deviation comes from a retailer that received a wholesale price of w_L^* consumer are less inclined to continue to search compared to when the manufacturer does not engage in wholesale price discrimination as now, there is a probability that consumers will encounter an even higher retail price if they continue to search. As low search cost consumers will continue to search until they find the lowest expected price p_L^* in the market, the benefit of search equals $\int_{p_L^*}^{p_L} D(p)dp$, whereas the expected cost of search equals $\frac{N-2}{N-1}s + \frac{1}{N-1}2s = \frac{N}{N-1}s$. Thus, these first time consumers encountering a price p_L will continue to search if their search cost is $s < \frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp$.

For a low cost retailer contemplating a deviation to a price $p_L > p_L^*$ there is, however, an important third, indirect effect of wholesale price discrimination on consumer search. Due to the fact that low search cost consumers continue to search if they observe p_H^* on their first search, low cost retailers will serve a disproportionately larger share of low search cost consumers. Therefore, they are losing relatively more consumers if they deviate and increase their prices.

Combining the second and third effect, when deviating to a price p_L with $p_L^* < p_L < p_H^*$, a low cost retailer's profit function will be:

$$\pi_r^L(p_L; p_L^*, p_H^*, w_L^*) = \frac{1}{N}\left[1 - G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right) + \frac{G\left(\int_{p_L}^{p_H^*} D(p)dp\right)}{(N-1)}\right] D(p_L)(p_L - w_L^*). \tag{3}$$

Thus, there are two important differences in this profit function relative to the uniform pricing case. First, the term $\frac{N-1}{N}$ in the first $G(s)$ function reflects the second effect described above. Second, the last term in the square brackets, reflects the third indirect effect of low cost retailers having a disproportionately large share of low search cost consumers.

The different effects of wholesale price discrimination on consumer search have important implications for competition in the retail market as can be seen from taking the first-order conditions of the profit function of the different retailers. Taking the first-order condition of (2) with respect to p_H and substituting $p_H = p_H^*$ yields

$$-\frac{g(\widehat{s})D^2(p_H^*)(p_H^* - w_H^*)}{1 - G(\widehat{s})} + \left[D^{'}(p_H^*)(p_H^* - w_H^*) + D(p_H^*)\right] = 0. \tag{4}$$

Comparing this FOC condition with that in (1) of the uniform pricing equilibrium condition from the previous Section reveals that *ceteris paribus* the only difference is that the first term is multiplied by the reverse hazard rate $\frac{g(\widehat{s})}{1-G(\widehat{s})}$ instead of by $g(0)$. As this first term is negative, ceteris paribus this implies that high cost retailers will have lower margins if, and only if $\frac{g(\widehat{s})}{1-G(\widehat{s})} > g(0)$. This is, for example, the case if the search cost distribution has an increasing reverse hazard rate (as, for instance, the uniform distribution). This is one of the important effects of wholesale price discrimination discussed in the Introduction: as (some) competitors have lower retail prices, it is more attractive for consumers to continue searching if they have visited a high cost retailer, which imposes a more severe competitive constraint on these retailers. High cost retailers have fewer buying customers (represented by $1 - G(\widehat{s})$) and an upward deviation from the equilibrium price will cause $g(\widehat{s})$ consumers to leave relative to $g(0)$ in the uniform pricing equilibrium. In relative terms, the impact on the retailer of consumers leaving is larger.

Taking the first-order condition of (3) with respect to p_L yields

$$\begin{aligned} 0 &= \left[1 - G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right) + \frac{G\left(\int_{p_L}^{p_H^*} D(p)dp\right)}{(N-1)}\right][D'(p_L)(p_L - w_L^*) + D(p_L)] \\ &\quad - \left(\frac{N-1}{N} g\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right) - \frac{g\left(\int_{p_L}^{p_H^*} D(p)dp\right)}{N-1}\right) D^2(p_L)(p_L - w_L^*), \end{aligned}$$

which evaluated at the equilibrium value yields

$$-\frac{\left(\frac{(N-1)^2}{N} g(0) + g(\widehat{s})\right) D^2(p_L^*)(p_L^* - w_L^*)}{(N-1) + G(\widehat{s})} + \left[D^{'}(p_L^*)(p_L^* - w_L^*) + D(p_L^*)\right] = 0. \tag{5}$$

Comparing this FOC with that in (1) of the uniform pricing equilibrium reveals that *ceteris paribus* the only difference is that the first term is multiplied by $\frac{\frac{(N-1)^2}{N} g(0)+g(\widehat{s})}{(N-1)+G(\widehat{s})}$ instead of $g(0)$. The easiest way to compare these to terms is for the uniform distribution where $g(s)$ is constant. In that case, the term in (5) is larger than $g(0)$ if, and only if, $G(\widehat{s}) < 1/N$. Especially, when N is small, this term may create an important difference and illustrates an important effect of wholesale price discrimination as discussed in the Introduction: even though low search cost consumers may be less inclined to continue to search (as they may not directly find another low cost retailer), the fact that low cost retailers are more frequently visited by low search cost consumers outweighs this effect.

The above discussion shows that *ceteris paribus* retail margins may be lower because of wholesale price discrimination. *Ceteris paribus* here mainly is a clause relating to the wholesale prices: the first-order conditions are similar to the first-order retail condition under uniform pricing if evaluated at the same wholesale price. The important question then is how these changes in the first-order retail price conditions impact on the incentives of the manufacturer to set his wholesale prices. It should be clear that it is not optimal for the manufacturer to induce an equilibrium where $\widehat{s} \geq \overline{s}$. If that would be an equilibrium, retailers receiving a high wholesale offer reacting with a retail price p_H^* would be effectively foreclosed from the market, giving the remaining retailers more market power as the only effect that remains would be the second effect on consumer search discussed above. Thus, in the remaining of this paper, we will consider equilibria with wholesale price discrimination where $0 < \widehat{s} \leq \overline{s}$.

We will now first analyze the manufacturer's behaviour when he can commit to the wholesale prices charged (Section 4) and then the case where he cannot (Section 5).

4 Commitment

In this section, we compare uniform pricing to wholesale price discrimination in case the manufacturer is able to commit to the wholesale prices it sets. We start with the easiest case, which is the benchmark of uniform pricing.

4.1 Uniform pricing

Under full commitment, the manufacturer chooses a wholesale price w that maximizes his profit $\pi(w) = wD(p(w))$, where $p(w)$ is implicitly defined by (1). Thus, with uniform pricing and full commitment the wholesale price w is set such that

$$\frac{\delta \pi}{\delta w} = wD'(p(w))\frac{\delta p^*}{\delta w} + D(p(w)) = 0. \tag{6}$$

To determine the optimal wholesale price we still have to evaluate $\frac{\partial p^*}{\partial w}$. From (1) it follows that:

$$-2g(0)D(p^*)(p^*-w)\frac{\partial p^*}{\partial w} - g(0)D^2(p^*)(\frac{\partial p^*}{\partial w}-1) + \left(\left(D^{''}(p^*)(p^*-w)+D'(p^*)\right)\frac{\partial p^*}{\partial w} + D'(p^*)(\frac{\partial p^*}{\partial w}-1)\right) =$$

so that

$$\frac{\partial p^*}{\partial w}=\frac{D'(p^*)-g(0)D^2(p^*)}{-2g(0)D(p^*)D'(p^*)(p^*-w)-g(0)D^2(p^*)+(D''(p^*)(p^*-w)+2D'(p^*))}.$$

If the search cost distribution becomes concentrated close to 0, in the limit $g(0)$ has to become very large. From (1) it can be seen that as $g(0)\to\infty$, $p^*(w^*)\to w^*$. This is quite intuitive: when all consumers have arbitrarily small search cost, retailers do not have any market power and their retail margins should become arbitrarily small as well. If $g(0)\to\infty$ and $p^*\to w^*$, the expression for $\frac{\partial p^*}{\partial w}$ reduces to 1 so that the wholesale price is equal to that of an integrated monopolist.

The following Proposition summarizes this result and analyzes the limiting behavior of the wholesale and retail price in a neighborhood of $\overline{s}=0$.

Proposition 1 *Consider $\overline{s}\to 0$. If the manufacturer commits to a uniform wholesale price, then the uniform retail and wholesale prices converge to $p^*=w^*$, where w^* solves $w^*D^{'}(w^*)+D(w^*)=0$. Moreover, we have that $\frac{dp^*}{d\overline{s}}=0$ and $\frac{dw^*}{d\overline{s}}=-\frac{1}{D(p^*)}$.*

From Proposition 1 it follows that in a neighbourhood of $\overline{s}=0$ we have that

$$\frac{d\Pi^M}{d\overline{s}}=\frac{dw^*}{d\overline{s}}D(p^*)+w^*D'(p^*)\frac{dp^*}{d\overline{s}}=-1.$$

For search cost distributions that are not concentrated around 0, it is difficult to obtain analytic results. The general expressions allow us to solve numerically, however, for different demand functions and search cost distributions. To be able to compare these numerical results across the different environments we analyze in the paper, we focus on the case of linear demand $D(p)=1-p$ and a uniform search cost distribution $g(s)=1/\overline{s}$.

The figure below clearly shows how retail and wholesale prices react to an increasing support of the search cost distribution: the retail price is increasing, while the wholesale price is decreasing: where retailers have more market power because of the increasing importance of search costs, the manufacturer tries to let demand not decrease to sharply by lowering the wholesale price. As a result, retail profits are increasing, the manufacturer profit is decreasing and consumers are worse off.

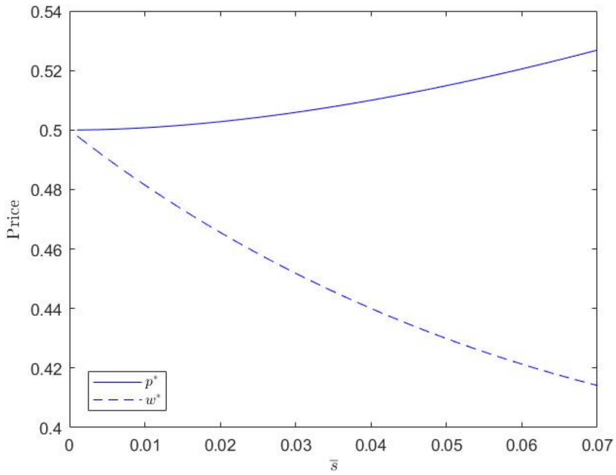

Fig 4.1. Uniform retail and wholesale prices for different values of $\overline{s}$

4.2 Wholesale Price Discrimination

When engaging in wholesale price discrimination, the manufacturer generally faces two questions: (i) how many different wholesale prices to set and (ii) how many retailers will be offered which price? Denoting the lowest wholesale price by w_L, it is clear that in equilibrium at least two retailers should get the lowest price. The reason is that if one retailer knows it is getting the lowest price, then it does not face any competition from other retailers up to the second lowest equilibrium retail price in the market. Therefore, this retailer would then set a retail price (almost) equal to the second lowest equilibrium retail price in the market, giving the manufacturer an incentive to increase the lowest wholesale price. Thus, to keep a competitive constraint on the retailers receiving the lowest wholesale price, there should be at least two retailers being offered w_L. This is why, for wholesale price discrimination to make sense we require that $N \geq 3$. If $N = 3$, wholesale price discrimination implies that two retailers should get the lower wholesale price w_L and one should get a higher wholesale price w_H. For $N > 3$, the question how the equilibria are affected by how many retailers get the lowest wholesale price is nontrivial. Thus, we simply focus on showing that wholesale price discrimination in its more simple form where the manufacturer chooses to set one low wholesale price w_L to $N - 1$ retailers and another high wholesale price w_H to 1 retailer increases manufacturer profits. Retailers will react to these prices by setting (possibly) different retail prices. When there is no confusion we use the notation $p(w_i)$, or simply $p_i, i = L, H$, to denote price reactions (or prices) of retailers who have received a low or a high wholesale price. The question whether the manufacturer can further increase profits by setting more than two different wholesale prices is left for future research.

Thus, the manufacturer will choose two different wholesale prices, w_L and w_H, to directly maximize his profit:

$$\pi(w_L, w_H) = \frac{1}{N}\left[1 - G(\widehat{s})\right] w_H D(p_H^*(w_H)) + \frac{N-1+G(\widehat{s})}{N} w_L D(p_L^*(w_L))$$

Under commitment, the manufacturer takes into account how retail prices change in reaction to changes in w_L and w_H. To derive these reactions, we have to consider that each of the retail first-order conditions (4) and (5) stipulate that a retailer's reaction to its own wholesale price depends on the other retail price they expect through its impact on $\widehat{s}$. That is, (5) describes a relationship where p_L depends on w_L and p_H and (4) describes a relationship where p_H depends on w_H and p_L. Thus, for every fixed pair of wholesale prices (w_L, w_H) we can solve for the retail reactions by simultaneously solving (4) and (5). In this way, the retail equilibrium reactions are given by $p_L^*(w_L, w_H)$ and $p_H^*(w_L, w_H)$, i.e., both retail prices depend directly on the corresponding wholesale price, but also indirectly on the other wholesale price, through its influence on the other retail price.

Using these reactions, we can solve the manufacturer's profit maximization problem and arrive at the following Proposition.

Proposition 2 *Consider $\overline{s} \to 0$. If the manufacturer commits to wholesale price discrimination selling to $N-1$ retailers at a low wholesale price w_L and to 1 retailer at a high wholesale price w_H, then optimal wholesale prices w_L^* and w_H^* and the corresponding retail prices p_L^* and p_H^* converge to w^* and p^*, with $p^* = w^*$ solving $w^* D'(w^*) + D(w^*) = 0$. Moreover, we have that*

$$\frac{d\left(p_L^* - w_L^*\right)}{d\overline{s}} = \left(\frac{N}{2\left(N^2+1\right)\left(N^2-N+1\right)} + \frac{N\left(2N-1\right)}{\left(2N^2-N+2\right)}\right)\frac{1}{D(p_L^*)},$$

$$\frac{d\left(p_H^* - w_H^*\right)}{d\overline{s}} = \left(-\frac{1}{2\left(N^2+1\right)} + \frac{2N^2-N+1}{2N^2-N+2}\right)\frac{1}{D(p_L^*)},$$

and the fraction of consumers that continue to search after visiting the high cost retailer, $G(\widehat{s}) = \int_{p_L^*}^{p_H^*} D(p)dp/\overline{s}$ is given by

$$D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}}\right) = \frac{4N^2-N+4}{2(N^2+1)(2N^2-N+2)}.$$

It follows that $\frac{d\Pi^M}{d\overline{s}} > -1$.

Thus, even though we cannot pin down the first-order approximations of the individual retail and wholesale prices, the price differences are such that we can unambiguously claim that the manufacturer makes more profit by price discriminating. In the previous Section, we have argued that if $G(\widehat{s}) < 1/N$ the low cost retailers will price more competitively under wholesale price discrimination than under uniform pricing. From the Proposition, it is easy to see that the manufacturer prices in such a way that this condition is indeed fulfilled in a neighborhood of $\overline{s} = 0$. Indeed, as $\frac{d\left(p_i^* - w_i^*\right)}{d\overline{s}} < \frac{1}{D(p_L^*)} = \frac{d(p^* - w^*)}{d\overline{s}}, i = L, H$, it is clear that both retailers make lower margins under wholesale price discrimination than under uniform pricing.

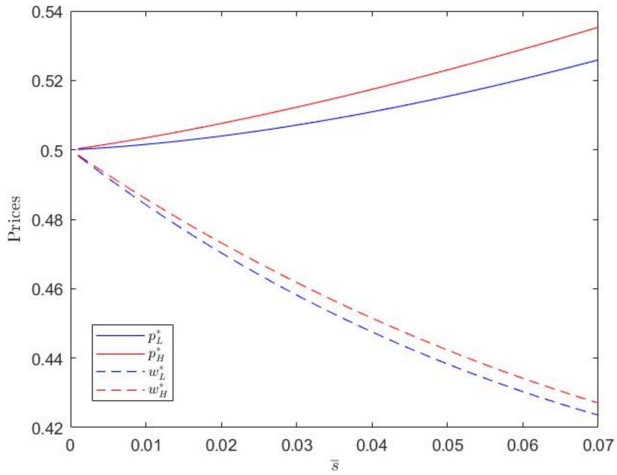

Fig 4.2. Prices under wholesale price discrimination for different values of $\overline{s}$

The above result shows that the manufacturer is better off, while retailers are worse off, because of wholesale price dsicrimination. What is not clear, however, from the above first-order approximations is whether consumers are better off. The numerical analysis below, for linear demand and a uniform search cost distribution clearly show, however, that consumers are also worse off.

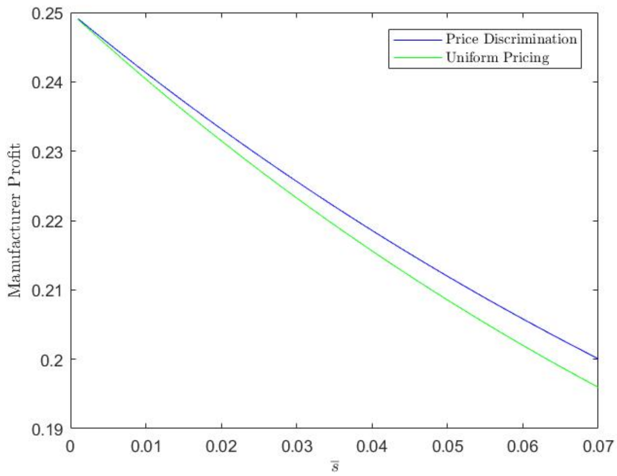

Fig 4.3. Manufacturer's Profit under full commitment for different values of $\overline{s}$

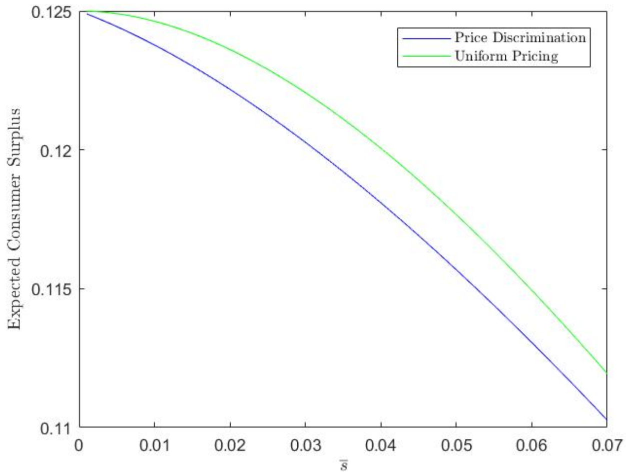

Fig 4.4. Expected Consumer Surplus under full commitment for different values of $\overline{s}$

5 No commitment

We start this Section by defining more precisely the equilibrium notion we use. For uniform wholesale pricing, we define a Perfect Bayesian Equilibrium (PBE) with passive out-of-equilibrium beliefs, denoted by $(w^*, p^*(w))$, as follows:[11]

Definition 3 *A uniform pricing equilibrium is defined by a tuple $(w^*, p^*(w))$ and an optimal sequential search strategy for all consumers such that (i) the manufacturer maximizes profits given $p^*(w)$ and consumers' optimal search strategy, (ii) retailers maximize their retail profits given the wholesale price they observe, their beliefs about the wholesale prices observed by other retailers and consumers' optimal search strategy and (iii) consumers' sequential search strategy is optimal given $(w^*, p^*(w))$ and their beliefs about retail prices not yet observed. Beliefs are updated using Bayes' rule whenever possible. Off-the-equilibrium path, beliefs are passive, i.e.,*

[11]Off-the equilibrium beliefs are important at two levels, First, consider a consumer who observes a price p different from $p^*(w^*)$. To determine how a retailer optimally reacts to the wholesale price w^* it is important to specify how a consumer reacts to a deviation from $p^*(w^*)$. This in turn depends on consumer beliefs about prices they believe they will encounter if they continue to search. For example, if consumers would have symmetric beliefs, they would believe that other retailers would set the same price if they observe a price $p \neq p^*(w^*)$ and in this case, they will decide not to continue to search. Symmetric beliefs would give full monopoly power to retailers, independent of the search cost distribution. If, on the other hand, consumers would have passive beliefs, they would believe that other retailers continue to set $p^*(w^*)$ if they observe a price $p \neq p^*(w^*)$ and in this case, the consumers with low enough search cost will continue to search if the price they observe is such that $p > p^*(w^*)$. As consumer search plays an important role in our analysis, we will adopt passive beliefs when consumers observe an out-of equilibrium price. For retailers, the issue is a little more subtle, but given the passive beliefs of consumers it seems most natural to also impose passive beliefs on retailers.

- *Retailers always believe that their competitors received a wholesale price w^* independent of the wholesale price they observed themselves;*
- *Consumers believe that retailers that are not searched yet have set a retail price $p^*(w^*)$ independent of the retail price(s) they already observed.*

To define passive beliefs for an equilibrium with wholesale price discrimination, we have to be more specific. First, the equilibrium should specify how many retailers observe the low and how many retailers observe the high wholesale price. Suppose on the equilibrium path there are $1 \leq m^* \leq N-1$ retailers that received w_L and the remaining $N - m^*$ received w_H. A consumer that observes an on-the-equilibrium path price of $p^*(w_L^*)$ believes that if he continues to search, there is a probability of $\frac{N-m^*}{N-1}$, respectively $\frac{m^*-1}{N-1}$, that he will observe a price of $p^*(w_H^*)$, respectively $p^*(w_L^*)$ on his next search. However, if the consumer observes an on-the-equilibrium path price of $p^*(w_H^*)$ he believes that if he continues to search, there is a probability of $\frac{N-m^*-1}{N-1}$, respectively $\frac{m^*}{N-1}$,he will observe a price of $p^*(w_H^*)$, respectively $p^*(w_L^*)$ on his next search. That is, even on the-equilibrium path the beliefs about retail prices on the next search depend on which retail price is observed.

Consider then a consumer who observes a price p slightly larger than $p^*(w_H^*)$. Even if he has passive beliefs, he has to have a belief whether it was a high or a low cost retailer that has deviated. We will argue that an equilibrium requires that at prices p in the neighbourhood of $p^*(w_H^*)$ the consumer believes it is a high-cost retailer that has deviated. The reason is as follows. Suppose that the consumer randomly attributes the deviation price, or that he attributes it to a low cost retailer. In that case, after observing a price $p > p^*(w_H^*)$ the consumer would become more pessimistic about finding lower prices on his next search than after observing the equilibrium price $p^*(w_H^*)$. More consumers would then decide not to continue searching if they observe a deviation price p in the neighbourhood of $p^*(w_H^*)$ than after observing $p^*(w_H^*)$, but this would make it profitable to deviate for a high cost retailer. Thus, to have an equilibrium it is necessary that consumers attribute deviation prices in the neighbourhood of $p^*(w_H^*)$ to a retailer that was supposed to have a high cost. If a consumer observes other out-of-equilibrium prices, we are more free to specify which retailer the consumer blames for such a price. Therefore, in the equilibrium definition below we do not restrict these beliefs further than necessary. Not to have our results be driven by out-of-equilibrium beliefs that favour retail competition, in the main part of the analysis we will say that consumers attribute deviations to a low cost retailer if the deviation price p in the neighbourhood of $p^*(w_L^*)$ so that beliefs are continuous in a neighbourhood of both equilibrium prices. For consistency reasons, we always invoke similar beliefs for retailers.

Thus, we define an equilibrium with wholesale price discrimination as follows.

Definition 4 *An equilibrium with wholesale price discrimination is defined by a tuple* $((w_L^*, w_H^*), p^*(w), m^*)$, *with* $w_L^* < w_H^*$, *and an optimal sequential search strategy for all consumers such that (i) the manufacturer maximizes profits given* $p^*(w)$ *and consumers' optimal search strategy, (ii) retailers maximize their retail profits given the wholesale price they observe, their beliefs about the wholesale prices observed by other retailers and consumers' optimal search strategy and (iii) consumers' sequential search strategy is optimal given* $(w^*, p^*(w))$ *and their beliefs about retail prices not yet observed. Beliefs are updated using Bayes' rule whenever possible. Off-the-equilibrium path are passive and satisfy at least the following restrictions:*

- *A retailer observing a wholesale price* w *in the neighbourhood of* w_H^* *believes that* m *competitors receive a wholesale price of* w_L^*, *while the remaining* $N - m - 1$ *competitors receive a wholesale price of* w_H^*;
- *If consumers observe a retail price* p *in the neighbourhood of* $p^*(w_H^*)$ *they believe that a high cost retailer is responsible for setting this price.*

5.1 Uniform pricing

To determine the wholesale equilibrium price under uniform pricing without commitment, we should consider that it is not optimal for the manufacturer to deviate to one retailer and offer him a wholesale price w (keeping the other retailers at w^*). If the manufacturer would deviate in this way, his profits would be:

$$\pi(w^*, w) = \frac{1}{N}\left[\left(N - 1 + G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right) w^* D(p^*(w^*)) + \left(1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right) wD(\widetilde{p}(w))\right]$$

This expression is easily understood. Of the consumers who encounter a price of $\widetilde{p}(w)$ at their first search (which is a fraction $1/N$ of them) a fraction $G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)$ continues to search for the equilibrium retail price as their search cost is low enough, while the consumers with a search cost larger than $\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp$ will buy at the deviation price $\widetilde{p}(w)$. All other consumers buy at the equilibrium price $p^*(w^*)$. A uniform pricing equilibrium requires that the first-order condition evaluated at $w = w^*$ is nonpositive:

$$N\frac{\partial \pi}{\partial w} = g(0)D(\widetilde{p})\frac{\partial \widetilde{p}}{\partial w}\left(w^* D(p^*(w^*)) - wD(\widetilde{p}(w))\right) + \left(1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right)\left(wD'(\widetilde{p})\frac{\partial \widetilde{p}}{\partial w} + D(\widetilde{p})\right)$$

$$= \left(1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right)\left(w^* D'(p^*)\frac{\partial \widetilde{p}(w^*)}{\partial w} + D(p^*)\right) \leq 0,$$

which reduces to

$$w^* D'(p^*(w^*))\frac{\partial \widetilde{p}(w^*)}{\partial w} + D(p^*(w^*)) \leq 0. \tag{7}$$

As with the retailer's maximization problem, the manufacturer does not have an incentive to lower his wholesale price as long as $p^* < \min(p^M(w^*), p^M(w^M))$ as retailers will not follow suit and keep their price at the equilibrium level if this condition is satisfied. In this case, the only requirement we have to impose is that the manufacturer does not want to increase his wholesale price and this is what (7) requires. On the other hand, nothing we have said so far precludes the possibility that the solutions to (1) and (7) result in such a high wholesale (and retail) price that $w^*D(p^*(w^*)) < w^M D(p^M(w^M))$. In this case, it would be optimal, however, for the manufacturer to deviate to w^M to all retailers and they will respond by setting $p^M(w^M)$. Thus, another necessary condition that an equilibrium needs to fulfil is that the manufacturer's equilibrium profit satisfies $w^*D(p^*(w^*)) \geq w^M D(p^M(w^M))$.

To finalize the description of an equilibrium, we still have to evaluate how $\widetilde{p}$ changes with a change in w. For this we need to to determine the best response function of retailers to non-equilibrium wholesale prices, taking into account that consumers do not observe the manufacturer deviation and blame the individual retailer for observing a non-equilibrium price. Given the retailers' profit function, an individual retailer will react to deviations in w by setting $\widetilde{p}$ such that

$$-g\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right) D^2(\widetilde{p})(\widetilde{p}-w)+\left(1-G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right)\left(D'(\widetilde{p})(\widetilde{p}-w)+D(\widetilde{p})\right)=0. \tag{8}$$

Thus, the retailer's best response to any w depends on w itself as well as on the equilibrium price $p^*(w^*)$ that is expected by consumers. Observe that in this equilibrium the retailer's reaction is smaller than the retail monopoly price due to the fact that low search cost consumers continue to search if a retailer would deviate to this price. In the proof of the next Proposition we show that if we evaluate this reaction at the equilibrium values we obtain:

$$\frac{\partial \widetilde{p}(w^*)}{\partial w} = \frac{D'(\widetilde{p}) - g\left(0\right) D^2(\widetilde{p})}{-g'\left(0\right) D^3(\widetilde{p})(\widetilde{p}-w) - 3g\left(0\right) D(\widetilde{p})D'(\widetilde{p})(\widetilde{p}-w) - 2g\left(0\right) D^2(\widetilde{p}) + 2D'(\widetilde{p}) + 2D''(\widetilde{p})(\widetilde{p}-w)}, \tag{9}$$

where we use p^* as a short-hand notation for $p^*(w^*)$.

We then have the following result.

Proposition 5 *A uniform pricing equilibrium has to satisfy (1), (7), where $\frac{\partial \widetilde{p}(w^*)}{\partial w}$ is given by (9) and $w^*D(p^*(w^*)) \geq w^M D(p^M(w^M))$.*

As in the commitment case, if $g(0) \to \infty$ we have that $p^*(w^*) \to w^*$. What is perhaps more surprising is that when $g(0) \to \infty$ and $p^* \to w^*$ we can solve (7) for w^*. It turns out that when $g(0) \to \infty$ the expression for $\frac{\partial \widetilde{p}(w^*)}{\partial w}$ reduces to $\frac{1}{2}$ so that the wholesale price is significantly larger than that of an integrated monopolist. The next Proposition states the result.

Proposition 6 *If $\overline{s}$ is small enough, an equilibrium exists. If $\overline{s} \to 0$ any uniform pricing equilibrium converges to $p^* = w^*$, where w^* solves $\frac{1}{2}w^* D^{'}(w^*) + D(w^*) \leq 0$. Moreover, If $\overline{s} \to 0$*

$$\frac{dp^*}{d\overline{s}} = -\frac{2D'(p^*)}{D(p^*)\left(wD''(p^*) + 3D'(\tilde{p})\right)} < 0$$

and

$$\frac{dw^*}{d\overline{s}} = -\frac{wD''(p^*) + 5D^{'}(\tilde{p})}{D(p^*)\left(wD''(p^*) + 3D'(\tilde{p})\right)} < 0.$$

In the context of a Stahl (1989) type model, where a fraction λ of consumers (the shoppers) has zero search cost and the remaining searchers all have the same search cost $s > 0$, Janssen and Shelegia (2015) show that if the search cost s is small an equilibrium exists if, and only if, λ is large enough.[12] The first part of the above Proposition says that if the search cost is small equilibrium existence is not an issue in our model where consumers have truly heterogeneous search cost and $g(s) > 0$ for all $s \geq 0$and Shelegia (2015) is due to the discreteness of the search cost distribution.

For general search cost distributions where the search cost does not vanish, it is difficult to establish general conditions that guarantee existence of equilibrium. The problem is that both at the retail level one should guarantee that an equilibrium exists for given w, while at the same time we should guarantee that the manufacturer does not have an incentive to further squeeze the retailers.

As the monopoly price of an integrated monopolist solves $pD^{'}(p) + D(p) = 0$ the second part of the Proposition establishes that the manufacturer sets much higher prices than an integrated monopolist. Even though retail margins are negligible, final retail prices are also larger than that of an integrated monopolist. This result is akin to Theorem 2 of Janssen and Shelegia (2015) where they show that as $s \to 0$, wholesale and retail price converge to a price w^* that solves $\lambda w^* D^{'}(w^*) + D(w^*) = 0$. The main reason why equilibrium prices are much higher than an integrated monopolist would set is that in a vertical context, the manufacturer may deviate from the equilibrium price without consumers noticing it. This makes the manufacturer's demand much more inelastic to her own price changes than the demand of an integrated monopolist. Theorem 2 of Janssen and Shelegia (2015) is obtained for a very special, binary search cost distribution and for duopoly retail markets only. The above result shows that the intuition is much more general and holds for any search cost distribution and for any number of retailers. Also, as in Janssen and Shelegia (2015) an equilibrium only exists if λ is large enough, their limit prices tend to be (much) smaller than in our model.

In terms of comparative statics, Proposition 6 shows that in a neighbourhood of $\overline{s} = 0$ both the wholesale and retail price are decreasing in $\overline{s}$. This implies that consumers are better off if search costs are not vanishing. Janssen and Shelegia (2015) have a similar result, but only for the case of linear demand. This result indicates that price comparison

[12]For linear demand, the critical value λ^* is approximately 0.47.

websites that effectively reduce search costs and are believed to help consumers getting better deals may in the end lead to higher prices.

Finally, if an equilibrium exists, there can be multiple equilibria due to the fact that the the first-order condition of the manufacturer only needs to hold with inequality. We will mainly focus on the equilibrium where the manufacturer makes most profits. This is the equilibrium where (7) holds with equality. Equilibria can be indexed by the wholesale price that retailers and consumers expect the manufacturer to choose. As the manufacturer is a monopolist, we think it is natural to think that retailers and consumers believe that the manufacturer chooses the wholesale price that maximizes profits. In addition, as all equilibrium wholesale prices are typically very high (higher than the monopoly price of an integrated monopolist), consumers and retailers themselves also benefit from lower wholesale prices.

Linear Demand

For linear demand $D(p) = 1 - p$, the equilibrium conditions can be considerably simplified. The retail equilibrium price in the welfare maximizing equilibrium should satisfy:

$$-g(0)(1-p^*)^2(p^*-w^*)+1-(2p^*-w^*)=0, \tag{10}$$

while, the manufacturer's equilibrium wholesale price should satisfy

$$(1-p^*)-w^*\frac{g(0)(1-p^*)^2+1}{g(0)(1-p^*)\left(2(1+w^*-2p^*)-(p^*-w^*)\right)+2}=0. \tag{11}$$

In the case of linear demand we can explicitly solve for w^* when $g(0) \to \infty$ and $p^* \to w^*$ as we get that in the limit (11) reduces to $1-w^*-w^*/2=0$ so that $w^*=2/3$ and expected consumer surplus converges to $\frac{1}{18}$.[13] Using Proposition 6 we have that $\frac{dp^*}{d\overline{s}}=-2$, $\frac{dw^*}{d\overline{s}}=-5$ and $\frac{dESC}{d\overline{s}}=(1-p^*)\frac{dp^*}{d\overline{s}}=\frac{2}{3}$.

For larger values of $\overline{s}$, we can solve (11) and (10) numerically for different values of $g(0)$. For general search cost distributions, these values are, however, not necessarily equilibrium values as the profit functions may not be quasi-concave. For example, if we would approximate a binary search cost distribution assumed in Janssen and Shelegia (2015) with a continuous bimodal distribution, we would get a similar non-existence result as the profit function is not quasi-concave. If the search cost distribution is uniform, we can, however, guarantee that in the relevant domain $wD(\widetilde{p}(w)) < w^*D(p^*)$ and that the retail profit function is quasi-concave so that $\widetilde{p}(w)$ represent the optimal reaction of the retailer to a deviation from the wholesale equilibrium price and p^* is the equilibrium retail price. In the appendix, we numerically report the profit functions under the uniform search cost distribution and different values of $\overline{s}$.

[13] If we want to characterize all equilibria, it is clear that $w^* \geq 2/3$, while the condition that deviation to the double marginalization solution is not optimal results when $g(0) \to \infty$ in the condition that $w^*(1-w^*) \geq 1/8$, or $w^* \leq \frac{2+\sqrt{2}}{4}$.

Figure 5.1. shows how the equilibrium retail and wholesale prices change for different values of $\overline{s}$. It also confirms that when $\overline{s} \to 0$ retail margins are converging to zero and $w^* \to 2/3$. Moreover, initially, for small values of $\overline{s}$ the figure also confirms that both p^* and w^* are decreasing in $\overline{s}$. The figure also depicts the retail and wholesale prices under commitment and confirms that without commitment prices are much higher than under commitment and that retail prices behave differently in these two cases: under commitment, uniform retail prices are increasing in the upper bound of the search cost distribution, while they are decreasing without commitment.

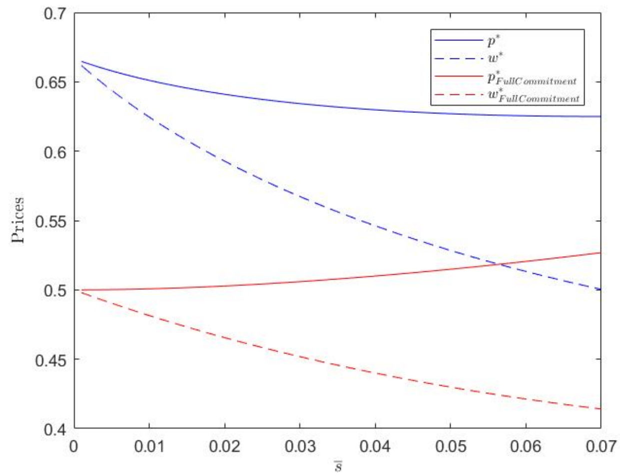

Fig 5.1. Uniform retail and wholesale prices for different values of $\overline{s}$

5.2 Wholesale Price Discrimination

We now consider whether the monopolist manufacturer wants to engage in wholesale price discrimination. We write the manufacturer's profit function when he deviates both in terms of w_H and and in terms of w_L to one low cost retailer. We obtain that:

$$\Pi_M = \frac{1}{N}\left(1 + \frac{1}{(N-1)}G\left(\int_{p_L(w_L)}^{p_H(w_H)} D(p)dp\right) - G\left(\frac{N-1}{N}\int_{p_L^*}^{\tilde{p}_L(w_L)} D(p)dp\right)\right) w_L D(\tilde{p}_L(w_L))$$

$$+\frac{N-2}{N}\left(1 + \frac{G\left(\int_{p_L^*}^{p_H(w_H)} D(p)dp\right)}{(N-1)} + \frac{G\left(\int_{p_L^*}^{\tilde{p}_L(w_L)} D(p)dp\right)}{(N-1)(N-2)} + \frac{G\left(\frac{N-1}{N}\int_{p_L^*}^{\tilde{p}_L(w_L)} D(p)dp\right)}{N-2}\right) w_L^* D(p_L^*(w_L^*))$$

$$+\frac{1}{N}\left(1 - G\left(\int_{p_L^*}^{p_H(w_H)} D(p)dp\right)\right) w_H D(p_H(w_H)).$$

This expression can be understood as follows. First, the term $\frac{1}{N}G\left(\int_{p_L^*}^{p_H(w_H)} D(p)dp\right)$ in the last line is the share of consumers that first saw $p_H(w_H)$ and continues to search as they believe that all other firms choose p_L^*. The remaining of these consumers buy at the price p_H. Each of the other retailers gets $1/(N-1)$ of the consumers that continue to search. Retailers charging p_L^* will sell to these consumers, while a retailer that charges p_L will only get a fraction of these consumers, namely those with relatively higher search cost. Since they still believe that the other retailers charge p_L^*, all consumers with a search cost smaller than $G\left(\int_{p_L^*}^{\tilde{p}_L(w_L)} D(p)dp\right)$ continue searching for the remaining retailers and buy there. Finally, there is a share of consumers that on their first search observes p_L. As the low search cost consumers know there is a probability $1/(N-1)$ that if they continue searching they may end up paying $2s$ before finding p_L^* the ones that find it worthwhile to continue searching are those that have search costs smaller than $\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp$.

The first-order condition for the manufacturer at w_H^* should be satisfied with equality. The reason is that in an equilibrium with wholesale price discrimination, a fraction $G(\widehat{s})$ of consumers continues to search if observing p_H^*. If a high-cost retailer would deviate from the equilibrium price in an upward or downward direction (in reaction to a deviation from the manufacturer) his demand and that of the manufacturer changes continuously. As equilibrium requires that such deviations are not optimal, the first-order condition of the manufacturer at w_H^* should also hold with equality. This is not the case, however, for the manufacturer at w_L^* as only upward deviations can be profitable: as consumers will only find out about the deviations once they have visited the retailer in question, downward deviations in retail price do not attract additional demand making such deviations always unprofitable.

In the proof of the Proposition below we show that the first-order conditions with respect to w_L and w_H evaluated at the equilibrium wholesale prices yield

$$w_L^* D^{'}(p_L^*(w_L))\frac{\partial p_L}{\partial w_L} + D(p_L^*) \leq 0, \tag{12}$$

and

$$(1-G(\widehat{s}))\left[w_H^* D^{'}(p_H^*)\frac{\partial p_H}{\partial w_H} + D(p_H^*)\right] + g(0)D(p_H^*)\frac{\partial p_H}{\partial w_H}\left[w_L^* D(p_L^*) - w_H^* D(p_H^*)\right] = 0, \tag{13}$$

where

$$\frac{\partial p_L}{\partial w_L} = \frac{-\frac{D(p_L^*)}{(p_L^*-w_L^*)}}{-\left[\frac{D'(p_L^*)}{D(p_L^*)}(p_L^*-w_L^*)+1\right]\left(3D'(p_L^*)+\frac{\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)}\right)+D''(p_L^*)(p_L^*-w_L^*)+-\frac{2D(p_L^*)}{p_L^*-w_L^*}} \tag{14}$$

and

$$\frac{\partial p_H}{\partial w_H} = \frac{-D(p_H^*)}{-\left(3D'(p_H^*) + \frac{g'(\hat{s})}{g(\hat{s})}\right)(p_H^* - w_H^*)\left[\frac{D'(p_H^*)(p_H^* - w_H^*)}{D(p_H^*)} + 1\right] + D''(p_H^*)(p_H^* - w_H^*)^2 - 2D(p_H^*)}. \tag{15}$$

Apart from these first-order conditions, we also need to guarantee that the manufacturer does not have an incentive to give all retailers the same wholesale price, whether it is w_L^* or w_H^*. In principle, the manufacturer could set w_L^* or w_H^* to all retailers without any retailer noticing it at their price setting stage. To make such deviations unprofitable, we have to have that the manufacturer makes equal profits over the low and high cost retailers, thus we need:

$$w_H^* D(p_H^*) = w_L^* D(p_L^*) \tag{16}$$

in any equilibrium with wholesale price discrimination. Given (16) the first-order condition with respect to w_H can be simplified to

$$w_H^* D^{'}(p_H^*) \frac{\partial p_H}{\partial w_H} + D(p_H^*) = 0. \tag{17}$$

The next Proposition shows that without commitment there does not exist an equilibrium with wholesale price discrimination.

Proposition 7 *Without commitment, an equilibrium with wholesale price discrimination requires that the equations (4), (5), (16) and (17) and the inequality (12) are satisfied. If* $\overline{s}$ *is small enough, these requirements cannot be satisfied.*

The proof of the proposition basically shows that the only way to satisfy the equal profit condition (16) and not to have an incentive to set a different high wholesale price ((17) is satisfied) is for the manufacturer to set a low wholesale price w_L^* for which it has an incentive to deviate. Alternatively, the only way to guarantee that (12) is satisfied is when the manufacturer profit per consumer is higher at the low wholesale price, w_L^* than at the high wholesale price w_H^*. However, given that retailers do not observe the wholesale prices set to their competitors, the manufacturer would then be able to profitably and secretly deviate and set w_L^* to all his retailers.

Numerical analysis, reported in the appendix, show that the conclusion of Proposition 7 also holds true when the search cost distribution is uniform and $\overline{s}$ is not small. Furthermore, we also plot the necessary equilibrium condition (12) for the demand $D(p) = (1-p)^\beta$ and different values of β, given that the other equilibrium conditions are satisfied and show that the conclusion of the proposition continues to hold even then.

6 Requiring Sales at Recommended Price

In the previous section, we have shown that wholesale price discrimination cannot be an equilibrium outcome if the manufacturer needs to make the same profits over different retailers, i.e., the equal profit condition given in (16) needs to be satisfied. In the Introduction we have argued that the Code of Federal Regulations effectively imposes restrictions on the deviations the manufacturer may contemplate once he announces a recommended price. In particular, by requiring that at least some consumers buy at the recommended retail price, the manufacturer may announce the high retail price p_H^* as a recommended retail price, and is then effectively committed to make sure that at least some consumers buy at this price. This would imply that by contemplating wholesale price discrimination and annoucing p_H^* as a recommended retail price, the manufacturer is not allowed to set to all retailers the same wholesale price w_L^*. Accordingly, the equal profit condition given in (16) would not need to hold and an equilibrium with wholesale price discrimination would only need to satisfy the conditions given in (4),(5),(12)and (17). Requiring sales at the recommended retail price, the wholesale price discriminating equilibrium that maximizes total surplus has these four equilibrium conditions holding with equality.

The next Proposition argues that when the search cost distribution is uniform, i.e., $g'(s) = 0$[14], in the most efficient equilibrium with wholesale price discrimination prices converge to the efficient equilibrium in the uniform pricing case if $\overline{s} \to 0$. One can show that $p_H^*(w_H^*) \to w_H^*$ and that this implies that $w_L^* \to w_H^*$, while $\frac{\partial \widetilde{p}_H(w^*)}{\partial w}$ reduces to $\frac{1}{2}$. Moreover, the comparative statics with respect to $\overline{s}$ is such that in a neighbourhood of $\overline{s} = 0$, the lowest wholesale and retail prices behave as in the uniform pricing equilibrium, whereas the highest wholesale and retail price charged are higher. Moreover, the difference between the lowest and highest wholesale prices, and the lowest and highest retail prices decreases in the number of retailers, and disappears if the number of retailers gets very large.

Proposition 8 *Consider $g'(s) = 0$ and regulation in place requiring sales at the recommded retail price. If $\overline{s} \to 0$ an equilibrium with effective wholesale price discrimination exists where the manufacturer announces p_H^* as the recommded retail price. The most efficient of these equilibria converges to $p_L^* = w_L^* = p_H^* = w_H^*$, where $w_L^* = w_H^* = w^*$ solves $\frac{1}{2}w^* D'(w^*) + D(w^*) = 0$. Moreover, in a neighborhood of $\overline{s} = 0$ the comparative statics with respect to $\overline{s}$ is such that*

$$\begin{aligned} \frac{dp_L^*}{d\overline{s}} &= -\frac{x}{D(p_H^*)}, \frac{dp_H^*}{d\overline{s}} = -\frac{1}{D(p_H^*)}\frac{xN-1}{N}, \\ \frac{dw_L^*}{d\overline{s}} &= -\frac{1+x}{D(p_H^*)} \text{ and } \frac{dw_H^*}{d\overline{s}} = -\frac{1}{D(p_H^*)}\frac{(1+x)N-1}{N}, \end{aligned}$$

[14]It is difficult to analyze the limit under price discrimination for other search cost distributions as we need then to take into account the relationship between $g(0)$ and $g(\widehat{s})$.

where $x = \frac{2D'(p_L^*)}{w_L^* D''(p_L^*)+3D'(p_L^*)} > 0.$

As $\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} = \frac{1}{ND(p_H^*)}$ and the comparative statics for the lowest retail price is equal to that in the uniform pricing equilibrium, consumers are worse off because of wholesale price discrimination. Interestingly, we can approximate the fraction of consumers that continue to search after visiting the high cost retailer, $\int_{p_L^*}^{p_H^*} D(p)dp/\overline{s}$, by $D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}}\right)$ which using Proposition 8 equals $\frac{1}{N}$. Compared to the analysis of Section 3, this implies indeed that the low cost retailer makes the same margin as under uniform pricing.

For larger values of $\overline{s}$ we can solve numerically for specific demand functions how the equilibrium evolves under wholesale price discrimination. For linear demand $D(p) = 1-p$, in a neighbourhood of $\overline{s} = 0$, $x = 2/3$, so that $\frac{dp_L^*}{d\overline{s}} \approx -2$, $\frac{dw_L^*}{d\overline{s}} \approx -5$, $\frac{dp_H^*}{d\overline{s}} \approx -1, \frac{dw_H^*}{d\overline{s}} \approx -3$. Figure 6.1 shows how wholesale and retail prices change for different values of $\overline{s}$. It can be seen that, under wholesale price discrimination, wholesale and retail prices are decreasing in $\overline{s}$. The figure also confirms that when $\overline{s} \to 0$, retail margins are very small and that $w_L^* \to w_L^* \to w^* \to 2/3$.

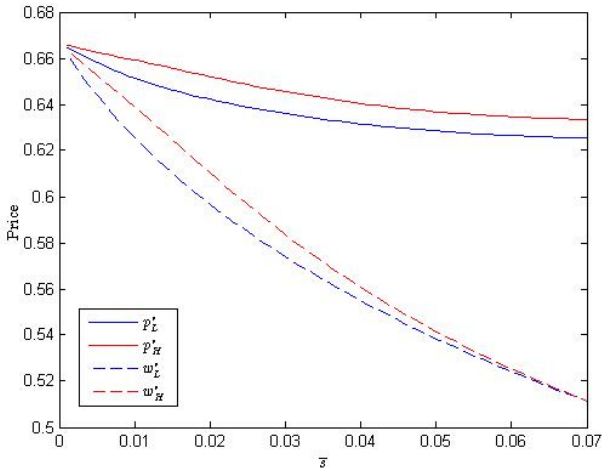

Fig 6.1 Wholesale and Retail prices for different values of $\overline{s}$

The comparison of retail prices under wholesale price discrimination and uniform pricing is depicted in Figure 6.2 for general values of $\overline{s}$. It is clear that under wholesale price discrimination, both the low and the high retail prices are larger than the retail price under uniform pricing. On the other hand, the comparison between wholesale prices is depicted in Figure 6.3. This figure reinforces Figure 6.2 that the high wholesale price that the manufacturer charges under wholesale price discrimination is larger than the wholesale price of the uniform pricing case.

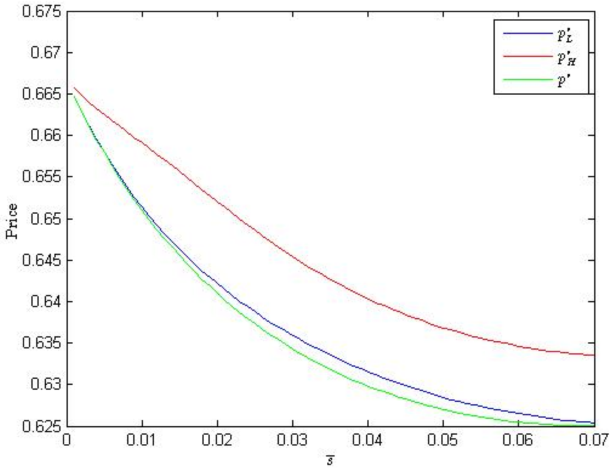

Fig 6.2 Retail prices under uniform pricing and price discrimination

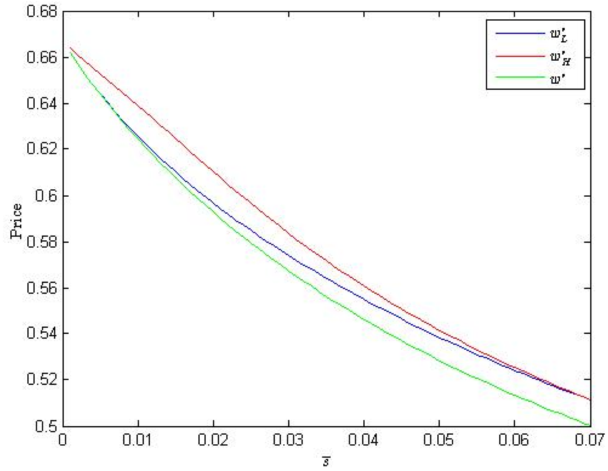

Fig 6.3 Wholesale prices under uniform pricing and price discrimination

As wholesale price discrimination acts as a mechanism that indirectly screens searching consumers, consumers with different search costs react differently to retail prices. A low search cost consumer that observes a high retail price continues to search, while others stop and buy. As a consequence, retailers do not face the same composition of search costs among their consumers. Specifically, low cost retailers' demand consists of a relatively larger share of low search cost consumers. Since low search cost consumers are more price sensitive, they will induce more competition between low cost retailers. In addition, because of the increased competition between the low cost retailers, consumers with higher search cost may also find it attractive to continue searching for lower prices forcing the high cost retailer also to lower its margins. Thus, both low and high cost retailers have lower margins under wholesale price discrimination as shown in Figure 6.4 below.

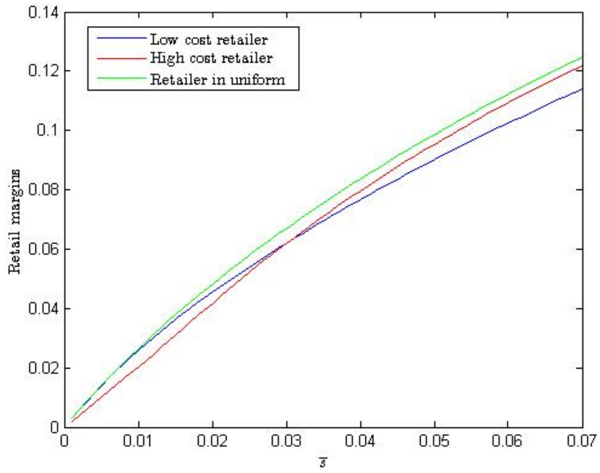

Fig 6.4 Retail margins for different values of $\overline{s}$

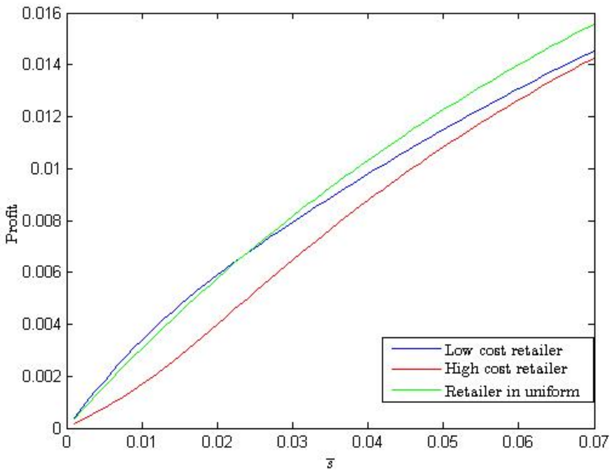

Fig 6.5 Retailers' Profit for different values of $\overline{s}$

Furthermore, Figure 6.5 shows the difference in retail profits between the uniform pricing case and the price discrimination setting. These numerical results show that despite the lower margins, the low cost retailer earns higher profits compared to a retailer under uniform pricing for smaller values of $\overline{s}$. The reason is that the difference in margins is small, while low cost retailers gain more sales due to low cost searchers that first visited the high cost retailers continuing to search for the low cost retailers. For larger values of $\overline{s}$, the numerical analysis shows that it is the lower margins that dominate the impact on the low cost retailers' profits. The profit of retailers under uniform pricing are always higher than the profit the high cost retailer makes under wholesale price discrimination.

Given the negative impact on consumer welfare, it is important to understand if the manufacturer has an incentive to engage in wholesale price discrimination. This will be the case if the manufacturer earns higher profits compared to uniform pricing. We can perform a similar analysis as with the other variables of interest. At $\overline{s} = 0$ the manufacturer makes the same profit whether or not it engages in wholesale price discrimination. When

$\overline{s}$ increases, the change in manufacturer profit under uniform pricing is given by:

$$\frac{d\Pi^M}{d\overline{s}} = \frac{dw^*}{d\overline{s}} D(p^*) + w^* D'(p^*) \frac{dp^*}{d\overline{s}}.$$

Using Proposition 6 and the fact that if $\overline{s} \to 0$ we have that $D(p^*) + \frac{1}{2} w^* D'(p^*) = 0$ it turns out that as $\overline{s} \to 0$, the first-order approximation for the change in manufacturer profits for uniform pricing is $-\frac{D'(p^*)}{\left(wD''(p^*)+3D'(\tilde{p})\right)}$. In the case of wholesale price discrimination we have that

$$\begin{aligned}
N\frac{d\pi(w_L, w_H)}{d\overline{s}} &= \frac{N-1+G(\widehat{s})}{N}\left(\frac{dw_L^*}{d\overline{s}} D(p_L^*) + w_L^* D'(p_L^*)\frac{dp_L^*}{d\overline{s}}\right) + \\
&\quad \frac{1-G(\widehat{s})}{N}\left(\frac{dw_H^*}{d\overline{s}} D(p_H^*) + w_H^* D'(p_H^*)\frac{dp_H^*}{d\overline{s}}\right) - \frac{g(\widehat{s})}{N} D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}}\right)(w_H D(p_H^*) - \\
&= D(p_L^*)\left(\frac{dw_L^*}{d\overline{s}} - 2\frac{dp_L^*}{d\overline{s}}\right) + \frac{1}{N}\left(1 - D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}}\right)\right) D(p_H^*)\left(\frac{dp_L^*}{d\overline{s}} - \frac{dp_H^*}{d\overline{s}}\right) \\
&\quad + \frac{1}{N}\left(1 - 2D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}}\right)\right) D(p_H^*)\left(\frac{dw_H^*}{d\overline{s}} - \frac{dp_H^*}{d\overline{s}} - \frac{dw_L^*}{d\overline{s}} + \frac{dp_L^*}{d\overline{s}}\right),
\end{aligned}$$

which, as $\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} = \frac{1}{ND(p_H^*)}$, $\frac{dw_L^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} = -\frac{1}{D(p_H^*)}$ and $\frac{dw_H^*}{d\overline{s}} - \frac{dp_H^*}{d\overline{s}} = -\frac{1}{ND(p_H^*)}$ reduces to

$$\begin{aligned}
\frac{d\pi(w_L, w_H)}{d\overline{s}} &= \frac{-D'(p_L^*) - w_L^* D''(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)} - \frac{N-1}{N^3} + \frac{(N-2)(N-1)}{N^3} \\
&= \frac{-D'(p_L^*) - w_L^* D''(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)} + \frac{(N-3)(N-1)}{N^3},
\end{aligned}$$

Thus, in the first-order approximation the manufacturer is strictly better off under wholesale price discrimination if $D''(p_L^*) < 0$ or $D''(p_L^*) \leq 0$ and $N > 3$.

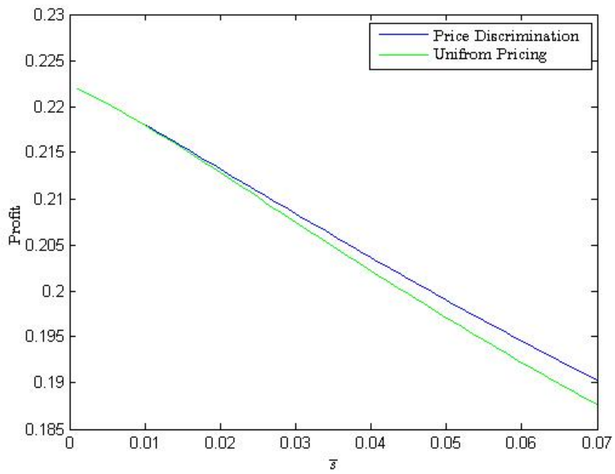

Fig 6.6 Manufacturer's Profit for different values of $\overline{s}$

For larger values of $\overline{s}$ we can show numerically that the manufacturer earns higher profit under wholesale price discrimination, but that the difference is small for smaller values of $\overline{s}$. This is confirmed in Figure 6.6 above, where the manufacturer's profits under both pricing practices is depicted.

Finally, since wholesale price discrimination leads to increased retail prices downstream this implies that consumer surplus will suffer. Under such a pricing practice, both low and high search cost consumers end up paying higher retail prices. Furthermore, a fraction of consumers with low search costs has to incur a search cost to find the low retail price p_L^*, while under uniform pricing consumer pay lower retail prices and do not have to incur a search cost.

Results regarding expected consumer surplus from propositions (5) and (8), state that while expected consumer surplus is increasing in $\overline{s}$, both under uniform and wholesale price discrimination, it increases twice as fast when the manufacturer sets uniform prices to his retailers. Figure 6.7, shows the difference in consumer surplus under these two different practices for larger values of $\overline{s}$. From the figure we can see that the impact of wholesale price discrimination on consumer surplus can be quite large. For instance, for a search cost of 0.04, consumer surplus under wholesale price discrimination decreases by approximately 6%.

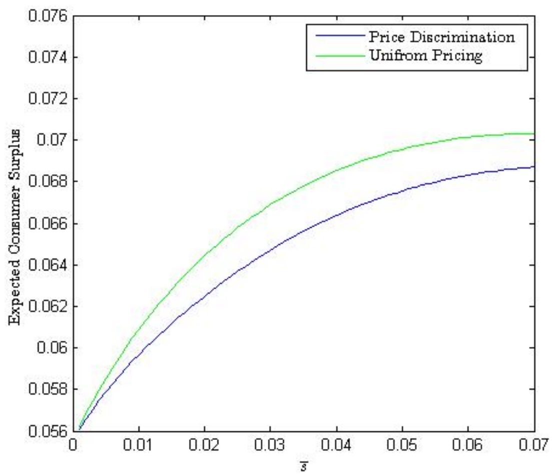

Fig 6.7 Expected Consumer Surplus for different values of $\overline{s}$

7 Discussion and Conclusion

In this paper, we have focused on a vertically related industry where consumers in the retail market have heterogeneous search cost. We showed that the manufacturer has an incentive to set different wholesale prices to different retailers in order to stimulate consumers to search for lower prices, inducing more competition between retailers and lower retail margins. If the manufacturer could commit to wholesale prices, we show that she will make more profit by discriminating between retailers, making both retailers and consumers worse off. However, in most markets firms cannot commit to prices (especially not if they are not observed by retailers that are not directly affected by them and also not by consumers). We have shown that without commitment an equilibrium with wholesale price discrimination does not exist for many market constellations. We have also shown that legislation requiring that a substantial number of sales are made at recommended retail prices gives manufacturers a possibility to partially commit to wholesale prices discrimination and that this is enough to engage in wholesale price discrimination and to announce the retail price of the high cost retailer(s) as the recommended retail price. Despite the fact that competition authorities impose such restrictions with the aim of protecting consumers, we have shown that they actually have the opposite effect. The legal requirement serves as a commitment device and eliminates the possibility of the manufacturer to deviate and charge all retailers the low wholesale price.

We have shown that once the monopolist manufacturer has the possibility to price discriminate among his retailers he will charge low prices to some retailers and high prices to others. As retailers optimally react to such wholesale prices, the downstream market will consist of low and high retail prices. Given that consumers differ in their search costs, some of them will stop and buy at the their first search, while consumers that have lower search costs can afford to continue searching and only buy at a low retail price. Therefore, the demand of high cost retailers will consist of only high search cost consumers, while the low cost retailers' demand will consist of a relatively larger share of low search cost consumers. In contrast, under uniform pricing retailers would each face the same demand composition. Thus, wholesale price discrimination acts as a mechanism that indirectly screens consumers according to their search costs.

The low search cost consumers, who are more price sensitive, increase competition between the low cost retailers, which makes even high search cost consumers more inclined to search. Also as high cost retailers have relatively fewer consumers, they also will be inclined to settle for lower retail margins in order to induce more consumers to buy. As a result, under wholesale price discrimination both types of retailers have lower margins. Finally, as wholesale price discrimination increases average wholesale and retail prices, consumers are worse off since, no matter their search cost, consumers end up buying at higher retail prices and some of them have to search twice.

Typically, price discrimination is used to differentiate between consumers with different valuations. In this paper we have focussed on a very different function of price discrimination. In our story it is essential (i) that consumers do not know which retailer has which wholesale (or retail) price and that their first search is random and (ii) that consumers *believe* that some retailers have a lower prices because they contract at a lower wholesale price. This is enough to induce more retail competition, lower retail margins and higher manufacturer profits. This mechanism may also affect other aspects of the vertical relationship between manufacturers and retailers and we think that it is worthwhile in future research to see in which type of contractual arrangements, manufacturers may induce asymmetries between retailers to induce more retail competition and when this may benefit consumers. In the Appendix, we also present an extension where we allow the manufacturer to make use of two-part tariffs and we show that, both under wholesale price commitment and under no commitment, our main results continue to hold and are thus robust to such contractual agreements.

8 Appendix

Proposition 1. Consider $\overline{s} \to 0$. If the manufacturer commits to a uniform wholesale price, then the uniform retail and wholesale prices converge to $p^* = w^*$, where w^* solves $w^* D^{'}(w^*) + D(w^*) = 0$. Moreover, we have that $\frac{dp^*}{d\overline{s}} = 0$ and $\frac{dw^*}{d\overline{s}} = -\frac{1}{D(p^*)}$.

Proof: The first part of the Proposition is proved in the main text. Here, we only discuss the comparative static result. To determine the optimal wholesale price we first evaluate $\frac{\delta p^*}{\delta w}$. From (1) it follows that

$$\frac{\partial p^*}{\partial w} = \frac{D'(p^*) - g(0)D^2(p^*)}{-2g(0)D(p^*)D'(p^*)(p^* - w) - g(0)D^2(p^*) + D''(p^*)(p^* - w) + 2D'(p^*)}$$

so that the first-order condition for the manufacturer can be written as

$$\begin{aligned} 0 &= wD'(p^*)\left(\frac{D'(p^*)}{g(0)} - D^2(p^*)\right) - 2D^2(p^*)D^{'}(p^*)(p^* - w) \\ &\quad -D^3(p^*) + \frac{D(p^*)D^{''}(p^*)(p^* - w)}{g(0)} + \frac{2D(p^*)D'(p^*)}{g(0)}. \end{aligned}$$

Taking the total differential evaluated in a neighborhood of $\overline{s} = 0$ gives

$$0 = \left(2D(p^*)D'(p^*) + wD'^2(p^*)\right) d\frac{1}{g(0)} + D'(p^*)D^2(p^*)dw,$$

which, using $D(p^*) + wD'(p^*) = 0$, gives

$$dw = -\frac{1}{D(p^*)} d\frac{1}{g(0)},$$

Taking the total differential of the first-order condition (1) of the retailer and evaluating it in a neighborhood of $\overline{s}=0$ where $g(0)\to\infty$ gives

$$d\frac{1}{g(0)}+D(p^*)dw^*-D(p^*)dp^*=0,$$

Substituting $dw=-\frac{1}{D(p^*)}d\frac{1}{g(0)}$ yields $dp^*=0$.

Proposition 2. Consider $g'=0$ and $\overline{s}\to 0$. If the manufacturer commits to wholesale price discrimination selling to $N-1$ retailers at a low wholesale price w_L and to 1 retailer at a high wholesale price w_H, then optimal wholesale prices w_L^* and w_H^* and the corresponding retail prices p_L^* and p_H^* converge to w^* and p^*, with $p^*=w^*$ solving $w^*D'(w^*)+D(w^*)=0$. Moreover, we have that

Proof: As $g'=0$ we have that $g(\widehat{s})=g(0)$ and $G(\widehat{s})=g(0)\int_{p_L^*}^{p_H^*}D(p)dp$. Maximizing

$$\pi(w_L,w_H)=\frac{1}{N}\left[1-G(\widehat{s})\right]w_HD(p_H^*)+\frac{N-1+G(\widehat{s})}{N}w_LD(p_L^*)$$

yields the following two first-order conditions

$$\begin{aligned}0 &= (w_LD(p_L^*)-w_HD(p_H^*))\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}-D(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right)\\ &\quad+\frac{[1-G(\widehat{s})]}{g(\widehat{s})}\left[D(p_H^*)+w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right]+\frac{N-1+G(\widehat{s})}{g(\widehat{s})}w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*},\end{aligned}$$

and

$$\begin{aligned}0 &= (w_LD(p_L^*)-w_HD(p_H^*))\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}-D(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right)\\ &\quad+\frac{N-1+G(\widehat{s})}{g(\widehat{s})}\left[D(p_L^*)+w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right]+\frac{[1-G(\widehat{s})]}{g(\widehat{s})}w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}.\end{aligned}$$

The total differential of the first first-order condition in the neighborhood of $\overline{s}=\frac{1}{g(0)}=0$ where $wD'(p)=-D(p)$ and $D(p_L^*)=D(p_H^*)$ can be written as

$$\begin{aligned}&\left(D(p_L^*)+w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}-w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right)\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}-D(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right)dw_L\\ &-\left(D(p_H^*)-w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}+w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right)\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}-D(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right)dw_H\\ &-\left(D(p_H^*)+w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}-w_HD'(p_H^*)\frac{\partial p_L^*}{\partial w_H^*}\right)D(p_L^*)\left(\left(\frac{\partial p_H^*}{\partial w_H^*}-\frac{\partial p_L^*}{\partial w_H^*}\right)dw_H+\left(\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*}\right)dw\right.\\ &+\left(\left[D(p_H^*)+w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right]+[N-1]\,w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right)d\frac{1}{g(\widehat{s})}=0,\end{aligned}$$

or

$$\left(\frac{\partial p_H^*}{\partial w_H^*}-\frac{\partial p_L^*}{\partial w_H^*}\right)\left(\left(1+\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*}\right)dw_L-\left(1-\frac{\partial p_H^*}{\partial w_H^*}+\frac{\partial p_L^*}{\partial w_H^*}\right)dw_H\right) \tag{18}$$
$$-\left(1-\left(\frac{\partial p_H^*}{\partial w_H^*}-\frac{\partial p_L^*}{\partial w_H^*}\right)\right)\left(\left(\frac{\partial p_H^*}{\partial w_H^*}-\frac{\partial p_L^*}{\partial w_H^*}\right)dw_H+\left(\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*}\right)dw_L\right)$$
$$+\left(\left[1-\frac{\partial p_H^*}{\partial w_H^*}\right]-[N-1]\frac{\partial p_L^*}{\partial w_H^*}\right)d\frac{1}{D(p_H^*)g(\widehat{s})}=0.$$

The total differential of the second first-order condition in the neighborhood of $\overline{s}=\frac{1}{g(0)}=0$ where $wD'(p)=-D(p)$ can be written as

$$\left(D(p_L^*)+w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}-w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right)\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}-D(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right)dw_L$$
$$-\left(D(p_H^*)-w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}+w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right)\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}-D(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right)dw_H$$
$$+\left(D(p_L^*)+w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}-w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right)D(p_L^*)\left(\left(\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*}\right)dw_L+\left(\frac{\partial p_H^*}{\partial w_H^*}-\frac{\partial p_L^*}{\partial w_H^*}\right)dw_H\right.$$
$$+\left((N-1)\left[D(p_L^*)+w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right]+w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right)d\frac{1}{g(\widehat{s})}=0,$$

or

$$\left(\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*}\right)\left(\left(1+\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*}\right)dw_L-\left(1+\frac{\partial p_H^*}{\partial w_H^*}+\frac{\partial p_L^*}{\partial w_H^*}\right)dw_H\right) \tag{19}$$
$$+\left(1-\left(\frac{\partial p_L^*}{\partial w_L^*}-\frac{\partial p_H^*}{\partial w_L^*}\right)\right)\left(\left(\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*}\right)dw_L+\left(\frac{\partial p_H^*}{\partial w_H^*}-\frac{\partial p_L^*}{\partial w_H^*}\right)dw_H\right)$$
$$+\left(N\left[1-\frac{\partial p_L^*}{\partial w_L^*}\right]-\left(1-\left(\frac{\partial p_L^*}{\partial w_L^*}-\frac{\partial p_H^*}{\partial w_L^*}\right)\right)\right)d\frac{1}{D(p_L^*)g(\widehat{s})}=0.$$

We next evaluate the different partial derivatives in the neighborhood of $\overline{s}=\frac{1}{g(0)}=0$ where $p_H^*=w_H$. To derive these partial derivatives, we rewrite the retail first-order conditions (4) and (5) as

$$f_1(p_H^*,p_L^*,w_H^*)=-D^2(p_H^*)(p_H^*-w_H)+\frac{(1-G(\widehat{s}))}{g(\widehat{s})}\left[D'(p_H^*)(p_H^*-w_H)+D(p_H^*)\right]=0. \tag{20}$$

$$f_2(p_H^*,p_L^*,w_L^*)=-\left(\frac{(N-1)^2}{N}+1\right)D^2(p_L^*)(p_L^*-w_L^*)+\frac{((N-1)+G(\widehat{s}))}{g(\widehat{s})}\left[D'(p_L^*)(p_L^*-w_L^*)+D(p_L^*)\right]= \tag{21}$$

Taking the total differential of f_1 we obtain:
$-D^2(p_H^*)\left(dp_H^*-dw_H^*\right)-D(p_H^*)\left(D(p_H^*)dp_H^*-D(p_L^*)dp_L^*\right)+D(p_H^*)d\frac{1}{g(\widehat{s})}=0$, or

$$-2dp_H^*+dw_H^*+dp_L^*+d\frac{1}{D(p_H^*)g(\widehat{s})}=0.$$

Similarly, taking the total differential of f_2 and leaving out "irrelevant" terms we obtain:

$-\left(\frac{(N-1)^2}{N}+1\right)D^2(p_L^*)\left(dp_L^*-dw_L^*\right)+D(p_L^*)\left(D(p_H^*)dp_H^*-D(p_L^*)dp_L^*\right)+(N-1)D(p_L^*)d\frac{1}{g(\widehat{s})}=0$, or

$$-\frac{N^2+1}{N}dp_L^*+\frac{N^2-N+1}{N}dw_L^*+dp_H^*+d\frac{N-1}{D(p_L^*)g(\widehat{s})}=0$$

Thus, the total effects of $w_L$and w_H on retail prices can be calculated by substituting these two equations into each other:

$$\frac{2N^2-N+2}{N}dp_L^*=2\frac{N^2-N+1}{N}dw_L^*+dw_H^*+(2N-1)\,d\frac{1}{D(p_L^*)g(\widehat{s})},$$

or

$$\left(2N^2-N+2\right)dp_L^*=2\left(N^2-N+1\right)dw_L^*+Ndw_H^*+d\frac{N\,(2N-1)}{D(p_L^*)g(\widehat{s})}, \tag{22}$$

and

$$-2\frac{N^2+1}{N}dp_H^*+\frac{N^2+1}{N}dw_H^*+\frac{N^2+1}{N}d\frac{1}{D(p_H^*)g(\widehat{s})}+\frac{N^2-N+1}{N}dw_L^*+dp_H^*+d\frac{N-1}{D(p_L^*)g(\widehat{s})}=0$$

or

$$\left(2N^2-N+2\right)dp_H^*=\left(N^2+1\right)dw_H^*+\left(N^2-N+1\right)dw_L^*+d\frac{2N^2-N+1}{D(p_L^*)g(\widehat{s})} \tag{23}$$

Thus, it follows from (22) and (23) that

$$\begin{aligned}\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*} &= \frac{(N^2-N+1)-2\,(N^2-N+1)}{(2N^2-N+2)}=\frac{-N^2+N-1}{(2N^2-N+2)}\\ \frac{\partial p_H^*}{\partial w_H^*}-\frac{\partial p_L^*}{\partial w_H^*} &= \frac{(N^2-N+1)}{(2N^2-N+2)}\end{aligned}$$

and therefore (18) can be simplified as

$$\begin{aligned}&\frac{N^2-N+1}{(2N^2-N+2)}\left(\left(1+\frac{-N^2+N-1}{(2N^2-N+2)}\right)dw_L-\left(1-\frac{(N^2-N+1)}{(2N^2-N+2)}\right)dw_H\right)\\ &-\left(1-\frac{(N^2-N+1)}{(2N^2-N+2)}\right)\left(\frac{(N^2-N+1)}{(2N^2-N+2)}dw_H+\frac{-N^2+N-1}{(2N^2-N+2)}dw_L\right)\\ &+\left(\left[1-\frac{N^2+1}{2N^2-N+2}\right]-[N-1]\frac{N}{2N^2-N+2}\right)d\frac{1}{D(p_H^*)g(\widehat{s})}=0,\end{aligned}$$

or

$$\begin{aligned}&\frac{N^2-N+1}{2N^2-N+2}\left(\frac{N^2+1}{(2N^2-N+2)}dw_L-\frac{N^2+1}{2N^2-N+2}dw_H\right)\\ &-\frac{N^2+1}{2N^2-N+2}\left(\frac{(N^2-N+1)}{(2N^2-N+2)}dw_H+\frac{-N^2+N-1}{(2N^2-N+2)}dw_L\right)+\frac{1}{2N^2-N+2}d\frac{1}{D(p_H^*)g(\widehat{s})}=0,\end{aligned}$$

or

$$2\frac{N^2-N+1}{2N^2-N+2}\left(N^2+1\right)(dw_L-dw_H)+d\frac{1}{D(p_H^*)g(\widehat{s})}=0,$$

In addition, (19) can be simplified to the same expression.

Thus, we have that

Substituting into (22) and (23) yields

$$dp_L^*-dw_L^*=\left(\frac{N}{2\left(N^2+1\right)\left(N^2-N+1\right)}+\frac{N\left(2N-1\right)}{\left(2N^2-N+2\right)}\right)d\frac{1}{D(p_L^*)g(\widehat{s})}, \tag{24}$$

and

$$dp_H^*-dw_H^*=\left(-\frac{1}{2\left(N^2+1\right)}+\frac{2N^2-N+1}{2N^2-N+2}\right)d\frac{1}{D(p_L^*)g(\widehat{s})} \tag{25}$$

Also, we can approximate the fraction of consumers that continue to search after visiting the high cost retailer, $\int_{p_L^*}^{p_H^*}D(p)dp/\overline{s}$, by

$$D(p_H^*)\left(dp_H^*-dp_L^*\right)=-\frac{N^2-N+1}{2N^2-N+2}D(p_H^*)\left(dw_L^*-dw_H^*\right)+d\frac{1}{g(\widehat{s})}\frac{1}{\left(2N^2-N+2\right)}.$$

$$D(p_H^*)\left(dp_H^*-dp_L^*\right)=\frac{N^2-N+1}{2N^2-N+2}\frac{2N^2-N+2}{2(N^2+1)(N^2-N+1)}d\frac{1}{g(\widehat{s})}+d\frac{1}{g(\widehat{s})}\frac{1}{\left(2N^2-N+2\right)}.$$

$$D(p_H^*)\left(dp_H^*-dp_L^*\right)=\left[\frac{1}{2(N^2+1)}+\frac{1}{(2N^2-N+2)}\right]d\frac{1}{g(\widehat{s})}.$$

$$D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)=\frac{4N^2-N+4}{2(N^2+1)(2N^2-N+2)}.$$

Under wholesale price discrimination and commitment the change in the optimal manufacturer profits $w_LD(p_L^*)+\frac{1-G(\widehat{s})}{N}\left(w_HD(p_H^*)-w_LD(p_L^*)\right)$ equals

$$\begin{aligned}
&\frac{N-1+G(\widehat{s})}{N}\left(\frac{dw_L^*}{d\overline{s}}D(p_L^*)+w_L^*D'(p_L^*)\frac{dp_L^*}{d\overline{s}}\right)+\\
&\frac{1-G(\widehat{s})}{N}\left(\frac{dw_H^*}{d\overline{s}}D(p_H^*)+w_H^*D'(p_H^*)\frac{dp_H^*}{d\overline{s}}\right)-\frac{g(\widehat{s})}{N}D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)\left(w_HD(p_H^*)-w_LD(p_L^*)\right)\\
=\;&D(p_L^*)\left(\frac{dw_L^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)+\frac{1-G(\widehat{s})}{N}D(p_L^*)\left(\frac{dp_L^*}{d\overline{s}}-\frac{dw_L^*}{d\overline{s}}+\frac{dw_H^*}{d\overline{s}}-\frac{dp_H^*}{d\overline{s}}\right)\\
&-\frac{D^2(p_H^*)}{N}\left(\frac{dp_H^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)\left(\frac{dw_H^*}{d\overline{s}}-\frac{dp_H^*}{d\overline{s}}-\frac{dw_L^*}{d\overline{s}}+\frac{dp_L^*}{d\overline{s}}\right)\\
=\;&D(p_L^*)\left(\frac{dw_L^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)+\frac{D(p_H^*)}{N}\left(1-G(\widehat{s})-D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)\right)\Bigg(\frac{dp_L^*}{d\overline{s}}-\frac{dw_L^*}{d\overline{s}}+\frac{dw_H^*}{d\overline{s}}-\frac{dp_H^*}{d\overline{s}}\\
=\;&D(p_L^*)\left(\frac{dw_L^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)+\frac{1}{N}\left(1-2D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)\right)\Bigg(D(p_H^*)\left(\frac{dw_H^*}{d\overline{s}}-\frac{dw_L^*}{d\overline{s}}\right)-D(p_H^*)\Bigg(\frac{dp_H^*}{d\overline{s}}\\
=\;&-\left(\frac{N}{2\left(N^2+1\right)\left(N^2-N+1\right)}+\frac{N\left(2N-1\right)}{\left(2N^2-N+2\right)}\right)+\left[\frac{N^2}{N^2+1}-\frac{2}{2N^2-N+2}\right]\Bigg[\frac{1}{2(N^2-N+1)(2\ldots}
\end{aligned}$$

This expression is larger than -1, the change in manufcturer profit under uniform pricing, if and only if,

$$-\frac{N}{2\left(N^2+1\right)\left(N^2-N+1\right)}+\left[\frac{N^2}{N^2+1}-\frac{2}{2N^2-N+2}\right]\left[\frac{1}{2(N^2-N+1)(2N^2-N+2)}\right]>\frac{-2}{(2N^2-N+}$$

or

$$\frac{1}{2\left(N^2+1\right)\left(N^2-N+1\right)}\frac{-2N^3+2N^2-2N}{2N^2-N+2}>\frac{-2}{(2N^2-N+2)}\left[1-\frac{1}{2(N^2-N+1)(2N^2-N+2)}\right]$$

or

$$\frac{N}{2\left(N^2+1\right)}<1-\frac{1}{2(N^2-N+1)(2N^2-N+2)},$$

which is true as the LHS is decreasing in N, while the RHS is increasing in N and the inequality certainly holds for N is 3.

Proposition 5. A uniform pricing equilibrium exists and has to satisfy (1), (7), where $\frac{\partial\widetilde{p}(w^*)}{\partial w}$ is given by (9) and $w^*D(p^*(w^*))\geq w^MD(p^M(w^M))$.

Proof: Apart from the expression for $\frac{\partial\widetilde{p}(w^*)}{\partial w}$ all the equilibrium conditions are explained in the main text. From (8) it follows that:

$$\begin{aligned}
&-g^{'}\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)D^3(\widetilde{p})(\widetilde{p}-w)\frac{d\widetilde{p}}{dw}-2g\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)D(\widetilde{p})D^{'}(\widetilde{p})(\widetilde{p}-w)\frac{d\widetilde{p}}{dw}\\
&-g\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)D(\widetilde{p})\left(D^{'}(\widetilde{p})(\widetilde{p}-w)+D(\widetilde{p})\right)\frac{d\widetilde{p}}{dw}-g\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)D^2(\widetilde{p})(\frac{d\widetilde{p}}{dw}-1)\\
&+\left(1-G\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)\right)\left(\left(D^{''}(\widetilde{p})(\widetilde{p}-w)+D'(\widetilde{p})\right)\frac{d\widetilde{p}}{dw}+D'(\widetilde{p})(\frac{d\widetilde{p}}{dw}-1)\right)=0,
\end{aligned}$$

or,

$$-g^{'}\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)D^3(\widetilde{p})(\widetilde{p}-w)\frac{d\widetilde{p}}{dw}-3g\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)D(\widetilde{p})D^{'}(\widetilde{p})(\widetilde{p}-w)\frac{\partial\widetilde{p}}{\partial w}-g\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)I$$

$$+\left(1-G\left(\int_{p^*(w^*)}^{\widetilde{p}}D(p)dp\right)\right)\left(D^{''}(\widetilde{p})(\widetilde{p}-w)\frac{\partial\widetilde{p}}{\partial w}+D'(\widetilde{p})(2\frac{\partial\widetilde{p}}{\partial w}-1)\right)=0$$

using the fact that we want to evaluate $\frac{d\widetilde{p}}{dw}$ at $w=w^*$ and that in that case $\widetilde{p}(w^*)=p^*(w^*)$ we can use (1) to get

$$\begin{aligned}
&-g^{'}(0)\,D^3(\widetilde{p})(\widetilde{p}-w)\frac{d\widetilde{p}}{dw}-3g(0)D(p^*)D^{'}(p^*)(p^*-w)\frac{d\widetilde{p}}{dw}-g(0)D^2(p^*)(2\frac{d\widetilde{p}}{dw}-1)\\
&+D^{''}(p^*)(p^*-w)\frac{d\widetilde{p}}{dw}+D'(p^*)(2\frac{d\widetilde{p}}{dw}-1)\\
=&\ 0,
\end{aligned}$$

which gives the expression in (9).

Proposition 6. If $\overline{s}$ is small enough, an equilibrium exists. If $\overline{s} \to 0$ any uniform pricing equilibrium converges to $p^* = w^*$, where w^* solves $\frac{1}{2}w^* D^{'}(w^*) + D(w^*) \leq 0$. Moreover, If $\overline{s} \to 0$

$$\frac{dp^*}{d\frac{1}{g(0)}} = -\frac{2D'(p^*)}{D(p^*)\left(wD''(p^*) + 3D'(\tilde{p})\right)} < 0$$

and

$$\frac{dw^*}{d\frac{1}{g(0)}} = -\frac{wD''(p^*) + 5D^{'}(\tilde{p})}{D(p^*)\left(wD''(p^*) + 3D'(\tilde{p})\right)} < 0.$$

Proof: The first part of the Proposition easily follows as the expression for $\frac{\partial \widetilde{p}(w^*)}{\partial w}$ reduces to $\frac{1}{2}$ if $g(0) \to \infty$. To show existence we first show that the manufacturer does not want to *increase* her wholesale price. In particular, we show that

$$D(\widetilde{p}) + wD'(\widetilde{p})\frac{\partial \widetilde{p}}{\partial w} \leq 0 \quad \text{ for all } w > w^*.$$

First, note that if the manufacturer deviates and sets a w to one or multiple retailers such that all consumers who visit these retailers continue to search, she cannot make more profit than in equilibrium. In the best case, if the manufacturer sticks to the wholesale equilibrium price for one retailer, she will make the same profit as in equilibrium, while if she deviates to all retailers, she will make les profit as the retailers will react by setting $\widetilde{p} = w$ and $wD(w)$ is decreasing in w for all $w > w^*$ (because $2D'(w) + wD''(w) < 0$ and the equilibrium wholesale price is such that $\frac{1}{2}w^* D^{'}(w^*) + D(w^*) \leq 0$ and thus larger than the optimal price of an integrated monpolist).

Thus, consider deviations such that some consumers still buy from the retailer where the manufacturer has deviated. In this case, the above inequality holds certainly true if the derivative of the LHS with respect to w

$$2D'(\widetilde{p})\frac{\partial \widetilde{p}}{\partial w} + wD''(\widetilde{p})\left(\frac{\partial \widetilde{p}}{\partial w}\right)^2 + wD'(\widetilde{p})\frac{\partial^2 \widetilde{p}}{\partial w^2} < 0 \quad \text{ for all } w > w^*. \tag{26}$$

From the proof of proposition 3 it follows that in a neighborhood of $\overline{s} = 0$ where $g(s) \to \infty$ $\frac{\partial \widetilde{p}}{\partial w}$ can be approximated by

$$\frac{\partial \widetilde{p}(w^*)}{\partial w} = \frac{D'(\widetilde{p}) - g\left(0\right) D^2(\widetilde{p})}{-g^{'}\left(0\right) D^3(\widetilde{p})(\widetilde{p} - w) - 3g\left(0\right) D(\widetilde{p})D^{'}(\widetilde{p})(\widetilde{p} - w) - 2g\left(0\right) D^2(\widetilde{p}) + 2D'(\widetilde{p}) + 2D''(\widetilde{p})(\widetilde{p} - w)}, \tag{27}$$

$$\frac{d\widetilde{p}}{dw} = \frac{1}{2} + \frac{3D(\widetilde{p})D^{'}(\widetilde{p})(\widetilde{p} - w)}{-2D^2(\widetilde{p})} > \frac{1}{2}$$

As $\lim_{\overline{s}\to\infty} \frac{\partial \widetilde{p}}{\partial w} = \frac{1}{2}$, it must be the case that $\frac{\partial^2 \widetilde{p}}{\partial w^2} > 0$ for small enough values of $\overline{s}$. Thus, (26) holds true if $(2D'(\widetilde{p}) + wD''(\widetilde{p}))\frac{\partial \widetilde{p}}{\partial w} < 0$. This is certainly the case as small enough values of $\overline{s}$ $2D'(\widetilde{p}) + wD''(\widetilde{p}) \approx 2D'(pw) + wD''(w) < 0$ and $\frac{\partial \widetilde{p}}{\partial w} > 0$.

We next show that the manufacturer does not want to decrease her wholesale price either. The only candidate deviation is to deviate to w^M. So, we have to compare the equilibrium profit $w^*D(p^*)$ to $w^M D(p^M(w^M))$. As for small enough values of $\overline{s}$ $w^*D(p^*)$ is close to $w^*D(w^*)$ where w^* solves $\frac{1}{2}w^*D^{'}(w^*)+D(w^*)=0$, it follows that w^* is larger than the integrated monopolist price. We will now show that w^* must be smaller than $p^M(w^M)$. Suppose to the contrary that $w^* \geq p^M(w^M)$. In that case $\frac{1}{2}p^M(w^M)D^{'}(p^M(w^M))+D(p^M(w^M)) \geq 0$.[15] From the FOC of the monopoy price $p^M(w)$ it follows that

$$\frac{dp^M(w)}{dw}=\frac{D'(p^M(w))}{2D'(p^M(w))+(p^M(w)-w)D''(p^M(w))} \leq \frac{1}{2}.$$

Hence, $p^M(w^M)D^{'}(p^M(w^M))\frac{dp^M(w)}{dw}+D(p^M(w^M)) \geq w^M D^{'}(p^M(w^M))\frac{dp^M(w)}{dw}+D(p^M(w^M)) > 0$. But this contradicts the manufacturer's optimal condition of the double marginalization price so that $w^* < p^M(w^M)$. It then follows that $w^*D(w^*) > p^M(w^M)D(p^M(w^M)) > w^M D(p^M(w^M))$ and that it is not optimal to deviate downwards either.

To establish that an equilibrium exists for small enough values of $\overline{s}$, we finally consider the retailer's decision problem. From the retailer's profit function, it follows that for all $\widetilde{p} \geq p^*$ the first-order derivative equals

$$-g\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)D^2(\widetilde{p})(\widetilde{p}-w)+D^{'}(\widetilde{p})(\widetilde{p}-w)+D(\widetilde{p}),$$

while the second-order derivative equals

$$-g'\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)D^3(\widetilde{p})(\widetilde{p}-w)-g\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)D(\widetilde{p})\left(2D^{'}(\widetilde{p})(\widetilde{p}-w)+D(\widetilde{p})\right)+D^{''}(\widetilde{p})(\widetilde{p}-w)+2$$

As $(\widetilde{p}-w)$ is close to 0 if $\overline{s}$ becomes small and as $g'(s) > -\infty$ this expression is smaller than 0 if $\overline{s}$ becomes small.

To prove the comparative statics results, we first rewrite the equilibrium condition for the manufacturer in a neighborhood of $\overline{s}=0$ as

$$\begin{aligned} 0 &= wD'(p^*)\left(\frac{D'(p^*)}{g(0)}-D^2(p^*)\right)-3D^2(p^*)D^{'}(p^*)(p^*-w)-2D^3(p^*) \\ &\quad +\frac{2D'(p^*)D(p^*)+2D^{''}(p^*)D(p^*)(p^*-w)-g^{'}(0)\,D^4(p^*)(p^*-w)}{g(0)}. \end{aligned}$$

Taking the total differential and taking into account that in a neighborhood of $\overline{s}=0, g \to \infty$ this approximately yields

$$\begin{aligned} 0 &\approx D'(p^*)\left(w^*D'(p^*)+2D(p^*)\right)d\frac{1}{g(0)}+2D'(p^*)D^2(p^*)dw \\ &\quad +\left(-w^*D''(p^*)D^2(p^*)-2w^*D'^2(p^*)D(p^*)-9D^2(p^*)D^{'}(p^*)\right)dp^*. \end{aligned}$$

[15] As $pD''(p)+2D'(p)<0$ it follows that the derivative of $\frac{1}{2}pD^{'}(p)+D(p)<0$.

As $\frac{1}{2}w^*D'(p^*) + D(p^*) = 0$ the first term is approximately equal to 0 so that we have

$$dw^* = \frac{w^*D''(p^*) + 5D^{'}(p^*)}{2D'(p^*)}dp^*.$$

From the proof of Proposition 3, we know that the total differential of the first-order condition (1) of the retailer evaluated in a neighborhood of $\overline{s} = 0$ is

$$d\frac{1}{g(0)} + D(p^*)dw^* - D(p^*)dp^* = 0.$$

Combining these two equations gives

$$d\frac{1}{g(0)} + D(p^*)\frac{w^*D''(p^*) + 3D^{'}(p^*)}{2D'(p^*)}dp^* = 0$$

or

$$\frac{dp^*}{d\frac{1}{g(0)}} = -\frac{2D'(p^*)}{D(p^*)\left(w^*D''(p^*) + 3D^{'}(p^*)\right)}.$$

so that

$$\frac{dw^*}{d\frac{1}{g(0)}} = -\frac{w^*D''(p^*) + 5D^{'}(p^*)}{D(p^*)\left(w^*D''(p^*) + 3D^{'}(p^*)\right)}.$$

As the demand function satisfies $w^*D''(p^*) + 2D^{'}(p^*) < 0$ it follows that both expressions are negative.

Proposition 7. Without commitment, an equilibrium with wholesale price discrimination requires that the equations (4),(16), (5) and (17) and the inequality (12) are satisfied. If $\overline{s}$ is small enough these requirements cannot be satisfied.

Proof. The first-order condition for profit maximization of the high-cost retailer (4) can be written as

$$-g\left(\widehat{s} + \int_{p_H^*}^{p_H} D(p)dp\right)D^2(p_H)(p_H - w_H) + \left(1 - G\left(\widehat{s} + \int_{p_H^*}^{p_H} D(p)dp\right)\right)\left[D^{'}(p_H)(p_H - w_H) + D(p_H)\right]. \tag{28}$$

Taking the total differential gives

$$\begin{aligned}
&-g'\left(\widehat{s} + \int_{p_H^*}^{p_H} D(p)dp\right)D^3(p_H)(p_H - w_H)\frac{\partial p_H}{\partial w_H} - 3g\left(\widehat{s} + \int_{p_H^*}^{p_H} D(p)dp\right)D(p_H)D'(p_H)(p_H - w_H)\frac{\partial}{\partial} \\
&-g\left(\widehat{s} + \int_{p_H^*}^{p_H} D(p)dp\right)D^2(p_H)(2\frac{\partial p_H}{\partial w_H} - 1) + \\
&\left(1 - G\left(\widehat{s} + \int_{p_H^*}^{p_H} D(p)dp\right)\right)\left[D^{''}(p_H)(p_H - w_H)\frac{\partial p_H}{\partial w_H} + D'(p_H)(2\frac{\partial p_H}{\partial w_H} - 1)\right] = 0,
\end{aligned}$$

which evaluated at the equilibrium values yields

$$
\begin{aligned}
& -g'\left(\widehat{s}\right) D^3(p_H^*)(p_H^* - w_H^*)\frac{\partial p_H}{\partial w_H} - 3g\left(\widehat{s}\right) D(p_H^*)D'(p_H^*)(p_H^* - w_H^*)\frac{\partial p_H}{\partial w_H} - g\left(\widehat{s}\right) D^2(p_H^*)(2\frac{\partial p_H}{\partial w_H} - 1) \\
+ & \quad (1 - G\left(\widehat{s}\right)) \left[D^{''}(p_H^*)(p_H^* - w_H^*)\frac{\partial p_H}{\partial w_H} + D'(p_H^*)(2\frac{\partial p_H}{\partial w_H} - 1)\right] = 0.
\end{aligned}
$$

Thus,

$$
\frac{\partial p_H}{\partial w_H} = \frac{(1 - G\left(\widehat{s}\right)) D'(p_H^*) - g\left(\widehat{s}\right) D^2(p_H^*)}{-g'\left(\widehat{s}\right) D^3(p_H^*)(p_H^* - w_H^*) - 3g\left(\widehat{s}\right) D(p_H^*)D'(p_H^*)(p_H^* - w_H^*) + (1 - G\left(\widehat{s}\right)) [D''(p_H^*)(p_H^* - w_H^*) + 2 \ldots}
$$

Using the first-order condition (4) evaluated at equilibrium values,

$$
g\left(\widehat{s}\right) D^2(p_H^*)(p_H^* - w_H^*) = (1 - G\left(\widehat{s}\right)) \left[D^{'}(p_H^*)(p_H^* - w_H^*) + D(p_H^*)\right],
$$

we can rewrite $\frac{\partial p_H}{\partial w_H}$ as

$$
\begin{aligned}
= & \quad \frac{D'(p_H^*) - \left[D^{'}(p_H^*) + \frac{D(p_H^*)}{(p_H^* - w_H^*)}\right]}{-\left(3D'(p_H^*) + \frac{g'(\widehat{s})}{g(\widehat{s})}\right)\left[\frac{D^{'}(p_H^*)(p_H^* - w_H^*)}{D(p_H^*)} + 1\right] + [D''(p_H^*)(p_H^* - w_H^*) + 2D'(p_H^*)] - 2\left[D^{'}(p_H^*) + \frac{D(p_H^*)}{(p_H^* - w_H^*)} \ldots\right.} \\
= & \quad \frac{-\frac{D(p_H^*)}{(p_H^* - w_H^*)}}{-\left(3D'(p_H^*) + \frac{g'(\widehat{s})}{g(\widehat{s})}\right)\left[\frac{D^{'}(p_H^*)(p_H^* - w_H^*)}{D(p_H^*)} + 1\right] + D''(p_H^*)(p_H^* - w_H^*) - \frac{2D(p_H^*)}{(p_H^* - w_H^*)}}.
\end{aligned}
$$

For the low-cost retailer we can perform a similar analysis to evaluate $\frac{\partial p_L}{\partial w_L}$. Taking the first-order condition of (3) with respect to p_L yields

$$
\begin{aligned}
0 \quad = \quad & \left[1 - G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right) + \frac{G\left(\int_{p_L}^{p_H^*} D(p)dp\right)}{(N-1)}\right][D'(p_L)(p_L - w_L) + D(p_L)] \\
& - \left(\frac{N-1}{N} g\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right) + \frac{g\left(\int_{p_L}^{p_H^*} D(p)dp\right)}{N-1}\right) D^2(p_L)(p_L - w_L).
\end{aligned}
$$

Taking the total differential and inserting equilibrium values gives

$$
\begin{aligned}
0 \quad = \quad & -\left[\frac{N-1}{N} g\left(0\right) + \frac{g\left(\widehat{s}\right)}{N-1}\right] D(p_L)\left[D'(p_L)(p_L - w_L) + D(p_L)\right]\frac{\partial p_L}{\partial w_L} + \\
& \left[1 + \frac{G\left(\widehat{s}\right)}{(N-1)}\right]\left[D''(p_L)(p_L - w_L)\frac{\partial p_L}{\partial w_L} + D'(p_L)(2\frac{\partial p_L}{\partial w_L} - 1)\right] \\
& - \left(\left(\frac{N-1}{N}\right)^2 g'\left(0\right) - \frac{g'\left(\widehat{s}\right)}{N-1}\right) D^3(p_L)(p_L - w_L)\frac{\partial p_L}{\partial w_L} \\
& - \left(\frac{N-1}{N} g\left(0\right) + \frac{g\left(\widehat{s}\right)}{N-1}\right) D(p_L)\left(2D'(p_L)(p_L - w_L)\frac{\partial p_L}{\partial w_L} + D(p_L)(\frac{\partial p_L}{\partial w_L} - 1)\right),
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
0 &= -3\left[\frac{N-1}{N}g\left(0\right)+\frac{g\left(\widehat{s}\right)}{N-1}\right]D(p_L)D'(p_L)(p_L-w_L)\frac{\partial p_L}{\partial w_L}+ \\
&\quad \left[1+\frac{G\left(\widehat{s}\right)}{(N-1)}\right]\left[D''(p_L)(p_L-w_L)\frac{\partial p_L}{\partial w_L}+D'(p_L)(2\frac{\partial p_L}{\partial w_L}-1)\right] \\
&\quad -\left(\left(\frac{N-1}{N}\right)^2 g'\left(0\right)-\frac{g'\left(\widehat{s}\right)}{N-1}\right)D^3(p_L)(p_L-w_L)\frac{\partial p_L}{\partial w_L} \\
&\quad -\left(\frac{N-1}{N}g\left(0\right)+\frac{g\left(\widehat{s}\right)}{N-1}\right)D^2(p_L)(2\frac{\partial p_L}{\partial w_L}-1),
\end{aligned}
$$

or

$$\frac{\partial p_L}{\partial w_L}=\frac{\left[1+\frac{G(\widehat{s})}{(N-1)}\right]D'(p_L^*)-\left(\frac{N-1}{N}g\left(0\right)+\frac{g(\widehat{s})}{N-1}\right)D^2(p_L^*}{-\left(\left(\frac{N-1}{N}\right)^2 g'\left(0\right)-\frac{g'(\widehat{s})}{N-1}\right)D^3(p_L^*)(p_L^*-w_L^*)-\left[\frac{N-1}{N}g\left(0\right)+\frac{g(\widehat{s})}{N-1}\right](3D(p_L^*)D'(p_L^*)(p_L^*-w_L^*)+2D^2(}$$

Using the first-order condition (5) evaluated at equilibrium values,

$$\left(\frac{N-1}{N}g\left(0\right)+\frac{g\left(\widehat{s}\right)}{N-1}\right)D^2(p_L^*)(p_L^*-w_L^*)=\left[1+\frac{G\left(\widehat{s}\right)}{(N-1)}\right]\left[D'(p_L^*)(p_L^*-w_L^*)+D(p_L^*)\right]$$

we can rewrite $\frac{\partial p_L}{\partial w_L}$

$$
\begin{aligned}
&= \frac{\left(D'(p_L^*)-\frac{D'(p_L)(p_L^*-w_L^*)+D(p_L)}{(p_L^*-w_L^*)}\right)}{-\frac{\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)}\left[\frac{D'(p_L^*)}{D(p_L^*)}(p_L^*-w_L^*)+1\right]-\left[\frac{D'(p_L^*)(p_L^*-w_L^*)+D(p_L^*)}{(p_L^*-w_L^*)}\right]\left(3\frac{D'(p_L^*)}{D(p_L^*)}(p_L^*-w_L^*)+2\right)+\left[D''(p_L^*)\right.} \\
&= \frac{-\frac{D(p_L^*)}{(p_L^*-w_L^*)}}{-\left[\frac{D'(p_L^*)}{D(p_L^*)}(p_L^*-w_L^*)+1\right]\left(3D'(p_L^*)+\frac{\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)}\right)+D''(p_L^*)(p_L^*-w_L^*)-\frac{2D(p_L^*)}{p_L^*-w_L^*}}.
\end{aligned}
$$

From the expressions for $\frac{\partial p_H}{\partial w_H}$ and $\frac{\partial p_L}{\partial w_L}$ it follows that in a neighborhood of $\overline{s}=0$ where $p_i^*\approx w_i^*, i=L,H$, is approximately equal to

$$\frac{\partial p_L}{\partial w_L}-\frac{\partial p_H}{\partial w_H}=-\frac{\left(3D'(p_L^*)+\frac{\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)}\right)(p_H^*-w_H^*)}{4D(p_L^*)}+\frac{\left(3D'(p_H^*)+\frac{g'(\widehat{s})}{g(\widehat{s})}\right)(p_L^*-w_L^*)}{4D(p_H^*)}.$$

We now prove that in a neighborhood of $\overline{s}=0$ if $g'(s)=0$ we have that if

$$w_H^*D'(p_H^*)\frac{\partial p_H}{\partial w_H}+D(p_H^*)=0,$$

then: $w_L^*D'(p_L^*)\frac{\partial p_L}{\partial w_L}+D(p_L^*)$

$$\approx \quad w_L^* D'(p_L^*) \left(\frac{\partial p_H}{\partial w_H} - \frac{\left(3D'(p_L^*) + \frac{\left(\frac{N-1}{N}\right)^2 g'(0) - \frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N} g(0) + \frac{g(\widehat{s})}{N-1}\right)}\right)(p_H^* - w_H^*)}{4D(p_L^*)} + \frac{\left(3D'(p_H^*) + \frac{g'(\widehat{s})}{g(\widehat{s})}\right)(p_L^* - w_L^*)}{4D(p_H^*)} \right) + D(p_L^*) > 0.$$

Our claim is true if

$$\begin{aligned} 0 \quad > \quad & (w_H^* D'(p_H^*) - w_L^* D'(p_L^*)) \frac{\partial p_H}{\partial w_H} + D(p_H^*) - D(p_L^*) + \\ & w_L^* D'(p_L^*) \left(\frac{\left(3D'(p_L^*) + \frac{\left(\frac{N-1}{N}\right)^2 g'(0) - \frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N} g(0) + \frac{g(\widehat{s})}{N-1}\right)}\right)(p_H^* - w_H^*)}{4D(p_L^*)} - \frac{\left(3D'(p_H^*) + \frac{g'(\widehat{s})}{g(\widehat{s})}\right)(p_L^* - w_L^*)}{4D(p_H^*)} \right). \end{aligned}$$

In a neighborhood of $\overline{s} = 0$ we can write $w_i^* = w^* + dw_i, D(p_i^*) = D(p^*) + D'(p_i^*)dp_i^*$ and $D'(p_i^*) = D'(p^*) + D''(p_i^*)dp_i^*$, $i = L, H$. Thus, the first-order approximation of the right-hand side is

$$\begin{aligned} 0 \quad > \quad & (D'(p^*)(dw_H - dw_L) + w^* D''(p^*)(dp_H - dp_L)) \frac{\partial p_H}{\partial w_H} + D'(p^*)(dp_H - dp_L) \\ & -w^* \frac{D'(p^*)}{4D(p^*)} \left(3D'(p^*)\left(dw_H - dw_L - (dp_H - dp_L)\right) - \frac{\left(\frac{N-1}{N}\right)^2 g'(0) - \frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N} g(0) + \frac{g(\widehat{s})}{N-1}\right)} (dp_H - dw_H) + \frac{g'(\widehat{s})}{g(\widehat{s})} (d \right. \end{aligned}$$

From the equal profit condition $w_L^* D(p_L^*) = w_H^* D(p_H^*)$ it follows that $D(p^*)dw_L + w^* D'(p^*)dp_L = D(p^*)dw_H + w^* D'(p^*)dp_H$ or

$$-w^* D'(p^*)\left(dp_H - dp_L\right) = D(p^*)\left(dw_H - dw_L\right)$$

so that using $w^* D'(p^*) \frac{\partial p_H}{\partial w_H} + D(p^*) = 0$ we have

$$dw_H - dw_L = \frac{dp_H - dp_L}{\frac{\partial p_H}{\partial w_H}}.$$

As when $\overline{s} \to 0$ $g(0) \to \infty$ and because of the assumption that $-\infty < g'(s) < \infty$ $\frac{g'(\widehat{s})}{g(\widehat{s})}$ also approaches 0 if $\overline{s} \to 0$ we can rewrite (29) as

$$\left(w^* D''(p^*) \frac{\partial p_H}{\partial w_H} + 2D'(p^*) - \frac{3}{4} \frac{w^* D'^2(p^*)}{D(p^*)} \right) (dp_H - dp_L) < 0.$$

This is clearly needs to be the case as in an equilibrium with wholesale price discrimination $dp_H - dp_L > 0$, whereas $w^* D''(p^*) \frac{\partial p_H}{\partial w_H} + 2D'(p^*) < 0$ because of the second-order condition for profit maximization.

Proposition 8. Consider $g'(s) = 0$ and regulation is in place requiring sales at the recommded retail price. If $\overline{s} \to 0$ an equilibrium with effective wholesale price discrimination exists where the manufacturer announces p_H^* as the recommded retail price. The most efficient of these equilibria converges to $p_L^* = w_L^* = p_H^* = w_H^*$, where $w_L^* = w_H^* = w^*$ solves $\frac{1}{2}w^* D^{'}(w^*) + D(w^*) = 0$. Moreover, in a neighborhood of $\overline{s} = 0$ the comparative statics with respect to $\overline{s}$ is such that

$$\frac{dp_L^*}{d\overline{s}} = -\frac{x}{D(p_H^*)}, \frac{dw_L^*}{d\overline{s}} = -\frac{1+x}{D(p_H^*)} d\frac{1}{g(0)}, \frac{dp_H^*}{d\overline{s}} = -\frac{1}{D(p_H^*)}\frac{xN-1}{N}, \frac{dw_H^*}{d\overline{s}} = -\frac{1}{D(p_H^*)}\frac{(1+x)N-1}{N},$$

where $x = \frac{2D'(p_L^*)}{w_L^* D''(p_L^*)+3D'(p_L^*)} > 0$.

Proof. We first show that if an equilibrium exists, it must be that $\frac{1}{2}w^* D^{'}(w^*) + D(w^*) = 0$ in the limit where $\overline{s} \to 0$. From (4) it is clear that in any equilibrium with wholesale price discrimination $p_H^* \to w_H^*$. As $0 < \widehat{s} < \overline{s}$, where $\widehat{s} = \int_{p_L^*}^{p_H^*} D(p)dp$, it must be the case that $p_H^* \to p_L^*$ if $\overline{s} \to 0$. Next, consider (5) if $\overline{s} \to 0$. Since also $\hat{s} \to 0$, and $D_L'^*) < 0$ while $D(p_L^*) > 0$ it must be that in any equilibrium with wholesale price discrimination $p_L^* \to w_L^*$. Thus, if $\overline{s} \to 0$ then it follows that $p_H^* \approx p_L^* \approx w_H^* \approx w_L^*$. It remains to be seen to which values the wholesale and retail prices converge. To this end, consider (10) in a neighbourhood of $\overline{s} = 0$ where $p_L^* - w_L^* = 0$. It is easy to see that

$$\frac{\partial p_L}{\partial w_L} \approx \frac{-(N-1+\frac{1}{N})D^2(p_L^*)}{-2(N-1+\frac{1}{N})D^2(p_L^*)} \approx \frac{1}{2}.$$

Thus, in a neighbourhood of $\overline{s} = 0$ the first-order condition determining w_L^* can be simplified to

$$\frac{1}{2}w_L^* D^{'}(w_L^*) + D(w_L^*) \approx 0.$$

We now prove the comparative statics results assuming an equilibrium exists and come back to the existence issue at the end of the proof. Substituting (5) into (12) and taking into account that $g'(s) = 0$ we have that (12) can be written as

$$\begin{aligned} 0 &= -w_L^* D^{'}(p_L^*)D(p_L^*) - \left[D'(p_L^*)(p_L^* - w_L^*)^2 + D(p_L^*)(p_L^* - w_L^*)\right] 3D'(p_L^*) \\ &\quad + D''(p_L^*)2D(p_L^*)(p_L^* - w_L^*)^2 - 2D^2(p_L^*) \end{aligned}$$

Taking the total differential in a neighborhood of $\overline{s} = 0$ gives

$$-D(p_L^*)D'(p_L^*)dw_L^* - w_L^* \left(D(p_L^*)D''(p_L^*) + D'^2(p_L^*)\right) dp_L^* - 4D(p_L^*)D'(p_L^*)dp_L^* - 3D^{'}(p_L^*)D(p_L^*)\left(dp_L^* - dw_L^*\right) =$$

which can be rewritten as

$$2D'(p_L^*)dw_L^* - \left(w_L^* D''(p_L^*) + w_L^* \frac{D'^2(p_L^*)}{D(p_L^*)} + 7D'(p_L^*)\right) dp_L^* = 0.$$

Thus, we have

$$dw_L^*{=}\left(\frac{w_L^* D''(p_L^*) + 5D'(p_L^*)}{2D'(p_L^*)}\right) dp_L^*. \tag{30}$$

As $g'(s) = 0$ we can write $G(\widehat{s}) = g(0)\int_{p_L^*}^{p_H^*} D(p)dp$. Using this, we can rewrite the first-order condition of the low-cost retailer as

$$\begin{aligned} 0 \;=\; & -\left(\frac{(N-1)^2}{N}+1\right) D^2(p_L^*)(p_L^*-w_L^*) + \int_{p_L^*}^{p_H^*} D(p)dp\left[D^{'}(p_L^*)(p_L^*-w_L^*)+D(p_L^*)\right] + \\ & \frac{(N-1)\left[D^{'}(p_L^*)(p_L^*-w_L^*)+D(p_L^*)\right]}{g\left(0\right)}. \end{aligned}$$

Taking the total differential in a neighborhood of $\overline{s} = 0$ gives

$$0 = -\left(\frac{(N-1)^2}{N}+1\right) D(p_L^*)(dp_L^*-dw_L^*)+(N-1)d\frac{1}{g\left(0\right)}+D(p_L^*)dp_H^*-D(p_L^*)dp_L^*. \tag{31}$$

Similarly, we can rewrite the first-order condition of the low-cost retailer as

$$-D^2(p_H^*)(p_H^*-w_H)-\left[D^{'}(p_H^*)(p_H^*-w_H)+D(p_H^*)\right]\int_{p_L^*}^{p_H^*} D(p)dp+\frac{\left[D^{'}(p_H^*)(p_H^*-w_H)+D(p_H^*)\right]}{g(0)}=0.$$

Taking the total differential in a neighborhood of $\overline{s} = 0$ gives

$$0 = -D^2(p_H^*)(dp_H^*-dw_H) + D(p_H^*)d\frac{1}{g\left(0\right)} - D^2(p_H^*)dp_H^* + D(p_H^*)D(p_L^*)dp_L^*,$$

or

$$0 = -D(p_H^*)(2dp_H^*-dw_H) + d\frac{1}{g\left(0\right)} + D(p_H^*)dp_L^*, \tag{32}$$

Finally, we consider the first-order condition of the manufacturer for the high-cost wholesale price

$$(1-G(\widehat{s}))\left[w_H^* D^{'}(p_H^*)\frac{\partial p_H}{\partial w_H}+D(p_H^*)\right]+g(0)D(p_H^*)\frac{\partial p_H}{\partial w_H}\left[w_L^* D(p_L^*)-w_H^* D(p_H^*)\right]=0.$$

This can be rewritten as

$$\left(\frac{1}{g(0)}-\int_{p_L^*}^{p_H^*} D(p)dp\right)\left[w_H^* D^{'}(p_H^*)\frac{\partial p_H}{\partial w_H}+D(p_H^*)\right]+D(p_H^*)\frac{\partial p_H}{\partial w_H}\left[w_L^* D(p_L^*)-w_H^* D(p_H^*)\right]=0,$$

so that the total differential in a neighborhood of $\overline{s} = 0$ yields

$$w_L^* D'(p_L^*)dp_L^* + D(p_L^*)dw_L^* = w_H^* D'(p_H^*)dp_H^* + D(p_H^*)dw_H^*,$$

or, using $w_L^* D'(p_L^*)\frac{1}{2} + D(p_L^*) = 0$,

$$-2dp_L^* + dw_L^* = -2dp_H^* + dw_H^*, \tag{33}$$

Thus, we should solve the four equations (30), (31), (32) and (33) to solve for the respective derivatifves. Combining (32) and (33) gives

$$D(p_H^*)(dp_L^* - dw_L^*) = d\frac{1}{g(0)}. \tag{34}$$

Combined with (30) gives

$$-D(p_H^*)\left(\frac{w_L^* D''(p_L^*) + 3D'(p_L^*)}{2D'(p_L^*)}\right) dp_L^* = d\frac{1}{g(0)},$$

or

$$dp_L^* = -\frac{1}{D(p_H^*)}\frac{2D'(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)} d\frac{1}{g(0)},$$

and

$$dw_L^* = -\frac{1}{D(p_H^*)}\frac{w_L^* D''(p_L^*) + 5D'(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)} d\frac{1}{g(0)}$$

Substitute (34) into (31) gives

$$\begin{aligned} 0 &= \frac{(N-1)}{N}(dp_L^* - dw_L^*) + dp_H^* - 2dp_L^* + dw_L^* \\ &= -\frac{1}{N}(dp_L^* - dw_L^*) + dp_H^* - dp_L^* \end{aligned}$$

Combined with the expressions for dp_L^* and dw_L^* gives

$$dp_H^* = -\frac{1}{D(p_H^*)}\left(-\frac{1}{N} + \frac{2D'(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)}\right) d\frac{1}{g(0)}.$$

Substituting all expressions into (33) yields

$$\begin{aligned} dw_H^* &= 2\left(dp_H^* - dp_L^*\right) + dw_L^* \\ &= \frac{2}{N}\left(dp_L^* - dw_L^*\right) + dw_L^* \\ &= -\frac{1}{D(p_H^*)}\left(-\frac{2}{N} + \frac{w_L^* D''(p_L^*) + 5D'(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)}\right) d\frac{1}{g(0)}. \end{aligned}$$

This proves the comparative statics results.

Finally, we prove an equilibrium with wholesale price discrimination exists if $\bar{s}$ is small enough. The first part to notice is that the comparative statics results indeed show that $p_H^* > p_L^*$ and $w_H^* > w_L^*$ in a neighborhood of $\bar{s} = 0$. Next, we will follow similar steps as in the proof of Proposition 3 but for both w_H^* and w_L^* separately to show that the manufacturer does not want to increase these respective wholesale prices beyond their equilibrium values. The parts of the proof showing that the manufacturer does not want to decrease her wholesale prices and that the retail profit functions are well-behaved are similar to the proof of Proposition 3 and will not be repeated here.

Like in the proof of Proposition 3 it is clear that the manufacturer does not want to increase its prices such that all consumers visiting that retailer will continue to search. In addition, in the range of prices where some consumers continue to buy from a retailer we need that the second-order derivative of the manufacturer's profit function with respect to $w_i, i = L, H$, is negative

$$2D'(\widetilde{p}_i)\frac{\partial \widetilde{p}_i}{\partial w_i} + w_i D''(\widetilde{p}_i)\left(\frac{\partial \widetilde{p}_i}{\partial w_i}\right)^2 + w_i D'(\widetilde{p}_i)\frac{\partial^2 \widetilde{p}_i}{\partial w_i^2} < 0 \quad \text{for } i = L, H \text{ and all } w > w^*.$$

From the proof of proposition 6 and the expression for $\frac{\partial \widetilde{p}_H}{\partial w_H}$ it follows that in a neighborhood of $\overline{s} = 0$ where $g(s) \to \infty$ $\frac{\partial \widetilde{p}_H}{\partial w_H}$ can be approximated by

$$\frac{\partial \widetilde{p}_H}{\partial w_H} \approx \frac{1}{2} + \frac{3D'(p_H^*)(p_H - w_H)}{-2D(p_H)} > \frac{1}{2}.$$

Similarly, in a neighborhood of $\overline{s} = 0$ and the expression for $\frac{\partial \widetilde{p}_L}{\partial w_L}$ can be approximated by

$$\frac{\partial \widetilde{p}_L}{\partial w_L} \approx \frac{1}{2} + \frac{3D'(p_L^*)(p_L - w_L)}{-2D(p_L)} > \frac{1}{2}.$$

Thus, we can argue that in a neighborhood of $\overline{s} = 0$ $\frac{\partial^2 \widetilde{p}_i}{\partial w_i^2} > 0$. Therefore, the second-order condition is satisfied and the manufacturer does not want to increase her wholesale prices beyond their equilibrium values.

9 Extensions

In this section we consider two extensions. First, we take up the question, raised in Section 4, of what is the optimal fraction of retailers to give a high wholesale price. Second, we take up the issue of two-part-tariffs.

9.1 The optimal fraction of retailers getting a high wholesale price

Let there be a unit mass of retailers and a γ mass of consumers. We denote with α the share of retailers to whom the manufacturer set a high wholesale price w_H. In this case, a consumer who on his first search encounters a high cost retailer selling at price p_H^*, believes that the probability that on his next search he will find a low cost retailer is $(1 - \alpha)$. Thus, a consumer will continue to search for a price of p_L^* if and only if:

$$\int_{p_H^*}^1 D(p)dp < (1-\alpha)\left[\int_{p_L^*}^1 D(p)dp - s\right] + \alpha\left[(1-\alpha)\left[\int_{p_L^*}^1 D(p)dp - 2s\right] + \alpha\left[(1-\alpha)\left[\int_{p_L^*}^1 D(p)dp - 3\right.\right.\right.$$

which given that $\alpha < 1$, yields:

$$\hat{s} = (1-\alpha)\int_{p_L^*}^{p_H^*} D(p)dp$$

Similarly, a consumer who on his first search encounters a price p_L, in the neighbourhood of p_L^*, will continue to search for a price of p_L^* if and only if:

$$\int_{p_L}^{1} D(p)dp < (1-\alpha)\left[\int_{p_L^*}^{1} D(p)dp - s\right] + \alpha\left[(1-\alpha)\left[\int_{p_L^*}^{1} D(p)dp - 2s\right] + \alpha\left[(1-\alpha)\left[\int_{p_L^*}^{1} D(p)dp - 3s\right.\right.\right.$$

which given that $\alpha < 1$, yields:

$$s < (1-\alpha)\int_{p_L^*}^{p_L} D(p)dp$$

If a high cost retailer deviates to a price $p_H > p_H^*$, then his profit will be:

$$\pi_r^H(p_H, p_L^*; w) = \gamma\left(1 - \frac{(1-\alpha)}{\overline{s}}\int_{p_L^*}^{p_H} D(p)dp\right) D(p_H)(p_H - w_H).$$

Taking FOC wrt p_H and substituting $p_H = p_H^*$, yields:

$$-\frac{(1-\alpha)D^2(p_H^*)(p_H^* - w_H)}{\overline{s} - (1-\alpha)\int_{p_L^*}^{p_H} D(p)dp} + \left[D^{'}(p_H^*)(p_H^* - w_H) + D(p_H^*)\right] = 0. \qquad (35)$$

Therefore, as $\alpha \to 0$, the equilibrium condition for p_H^* becomes the same as under the case of wholesale price discrimination with a finite number of retailer, where $N - m^* = 1$.

$$-\frac{D^2(p_H^*)(p_H^* - w_H)}{\overline{s} - \int_{p_L^*}^{p_H} D(p)dp} + \left[D^{'}(p_H^*)(p_H^* - w_H) + D(p_H^*)\right] = 0.$$

If a low cost retailer deviates to a price p_L with $p_L^* < p_L < p_H^*$, then his profit will be:

$$\pi_r^L(p_L, p_L^*, p_H) = \gamma\left[1 - \frac{(1-\alpha)}{\overline{s}}\int_{p_L^*}^{p_L} D(p)dp + \frac{(1-\alpha)\alpha}{(1-\alpha)\overline{s}}\int_{p_L}^{p_H^*} D(p)dp\right] D(p_L)(p_L - w_L).$$

which when simplified becomes:

$$\pi_r^L(p_L, p_L^*, p_H) = \gamma \left[1 - \frac{(1-\alpha)}{\overline{s}} \int_{p_L^*}^{p_L} D(p)dp + \frac{\alpha}{\overline{s}} \int_{p_L}^{p_H^*} D(p)dp\right] D(p_L)(p_L - w_L).$$

Taking the FOC wrt p_L and substituting $p_L = p_L^*$, yields:

$$-\frac{D^2(p_L^*)(p_L^* - w_L^*)}{\overline{s} + \alpha \int_{p_L}^{p_H^*} D(p)dp} + \left[D^{'}(p_L^*)(p_L^* - w_L^*) + D(p_L^*)\right] = 0. \tag{36}$$

As $\alpha \to 0$, the equilibrium condition for p_L^* becomes the same as for the uniform retail price p^*.

$$-\frac{D^2(p_L^*)(p_L^* - w_L^*)}{\overline{s}} + \left[D^{'}(p_L^*)(p_L^* - w_L^*) + D(p_L^*)\right] = 0.$$

The manufacturer's profit in equilibrium will thus be:

$$\pi(w_L^*, w_H^*) = \gamma \Bigg[(1-\alpha)\left(1 + \frac{\alpha}{\overline{s}} \int_{p_L^*}^{p_H^*} D(p)dp\right) w_L^* D(p_L^*(w_L^*)) + \alpha \left(1 - \frac{(1-\alpha)}{\overline{s}} \int_{p_L^*}^{p_H^*} D(p)dp\right) w_H^* D(p_H^*($$

If the manufacturer deviates to one low cost retailer or if he deviates to one high cost retailer, his profit will be:

$$\pi(w_L, w_H) = \gamma \Bigg[(1-\alpha)\left(1 + \frac{\alpha}{\overline{s}} \int_{p_L}^{p_H} D(p)dp\right) w_L D(p_L(w_L)) + \alpha \left(1 - \frac{(1-\alpha)}{\overline{s}} \int_{p_L}^{p_H} D(p)dp\right) w_H D(p_H($$

Taking the FOC wrt w_L and substituting $w_L = w_L^*$ and $w_H = w_H^*$, yields:

$$\left(1 + \frac{\alpha}{\overline{s}} \int_{p_L^*}^{p_H^*} D(p)dp\right) \left[D(p_L^*(w_L^*)) w_L^* \frac{\delta p_L}{\delta w_L} + D(p_L^*(w_L^*))\right] + \frac{\alpha}{\overline{s}} D(p_L^*(w_L^*)) \frac{\delta p_L}{\delta w_L} \left[w_H^* D(p_H^*(w_H^*)) - w_L^* D(p_L^* \right. \tag{37}$$

As $\alpha \to 0$ condition (3) becomes the same as the equilibrium condition for w_L^* in the finite retailers case.

Taking the FOC wrt w_H and substituting $w_H = w_H^*$ and $w_L = w_L^*$, yields:

$$\left(1 - \frac{(1-\alpha)}{\overline{s}} \int_{p_L^*}^{p_H^*} D(p)dp\right) \left[D(p_H^*(w_H^*))w_H^* \frac{\delta p_H}{\delta w_H} + D(p_H^*(w_H^*))\right] + \frac{(1-\alpha)}{\overline{s}} D(p_H^*(w_H^*)) \frac{\delta p_H}{\delta w_H} [w_L^* D(p_L^*(w \tag{38}$$

As $\alpha \to 0$ condition (4) becomes the same as the equilibrium condition for w_H^* in the finite retailers case.

Taking the FOC wrt α, yields:

$$\frac{1}{\overline{s}} \int_{p_L^*}^{p_H^*} D(p)dp\Big((1-\alpha)w_L^* D(p_L^*(w_L^*)) + \alpha w_H^* D(p_H^*(w_H^*))\Big) - w_L^* D(p_L^*(w_L^*)) \left(1 + \frac{\alpha}{\overline{s}} \int_{p_L^*}^{p_H^*} D(p)dp\right)$$
$$+ w_H^* D(p_H^*(w_H^*)) \left(1 - \frac{(1-\alpha)}{\overline{s}} \int_{p_L^*}^{p_H^*} D(p)dp\right) = 0 \tag{39}$$

Making use of equations: (1), (2), (3), (4) and (5), we obtain the following outcomes:

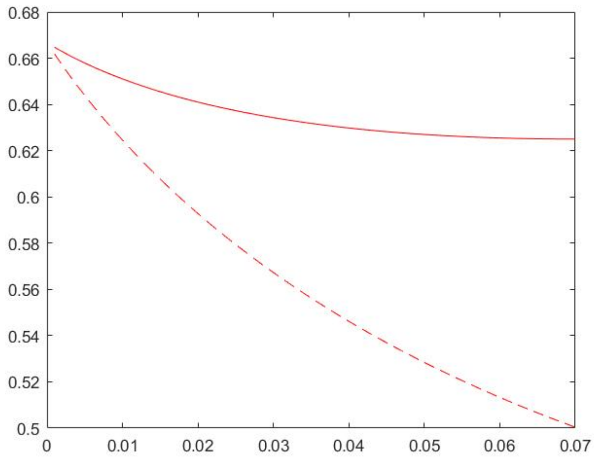

Fig 1. Retail and wholesale prices under price discrimination for different values of $\overline{s}$

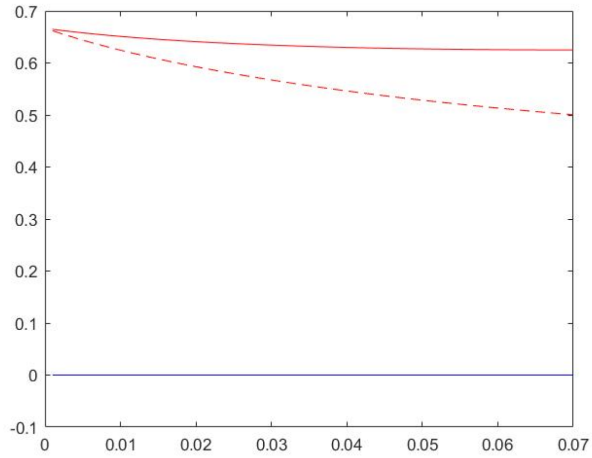

Fig 2. Optimal share of high cost retailers under price discrimination for different values of $\overline{s}$

9.2 Two-part-tariffs

We now investigate how allowing the monopolist manufacturer to have the possibility of choosing two-part tariffs affects our results. Clearly, if the manufacturer has all the bargaining power, then he will set a wholesale price that induces the retailers to choose the integrated monopolist price and set a fixed fee equal to the retail profit. Wholesale price discrimination does not add to the manufacturer's profit in this case. In most markets, however, the bargaining power is not exclusively with the manufacturer, however. In this subsection, we exogenously fix the relative bargaining power and denote by α the bargaining power of a given retailer, where α measures the share of the retail profit, the retailer can keep for himself. We, first consider the case of wholesale price commitment with two-part tariffs and later on also the case of non-commitment.

9.2.1 Full Commitment

In this setting, in an equilibrium under the uniform pricing scheme, an individual retailer's profit will be:

$$\pi_r^*(p^*) = \frac{\alpha}{N} D(p^*(w^*))(p^* - w^*).$$

Whereas, the monopolist manufacturer's profit in equilibrium is given by:

$$\pi(w^*) = w^* D(p^*(w^*)) + (1 - \alpha)(p^* - w^*) D(p^*(w^*))$$

Thus, if $\alpha = 0$, the manufacturer extracts all profits from its retailers and if $\alpha = 1$, then the profits will be the same as in the previous sections. It is clear that with this formulation, the retailer's problem is identical to the one analyzed in the previous Sections and thus the equilibrium condition for the retail price remains the same as before. On the other hand, the equilibrium condition for the uniform wholesale price changes since the manufacturer now directly maximizes:

$$\pi(w) = wD(p(w)) + (1-\alpha)(p-w)D(p(w))$$

Thus, with uniform pricing under full commitment and two-part tariffs the wholesale price w is set such that:

$$wD'(p(w))\frac{\delta p^*}{\delta w} + D(p(w)) + (1-\alpha)\left[(p-w)D'(p(w))\frac{\delta p^*}{\delta w} + (\frac{\delta p^*}{\delta w} - 1)D(p(w))\right] = 0. \quad (40)$$

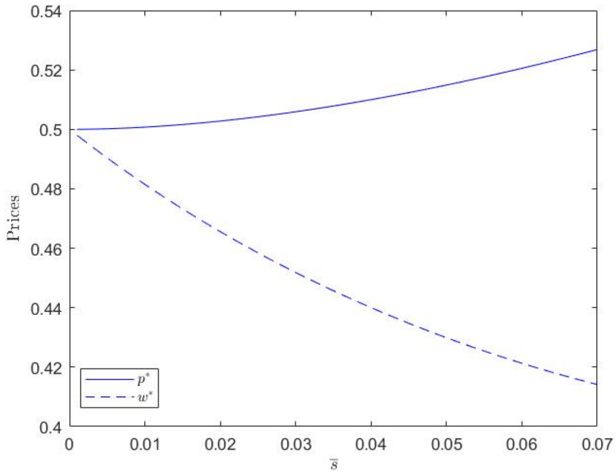

Fig 9.1 Uniform retail and wholesale prices for different values of $\overline{s}$, when $\alpha = 1$

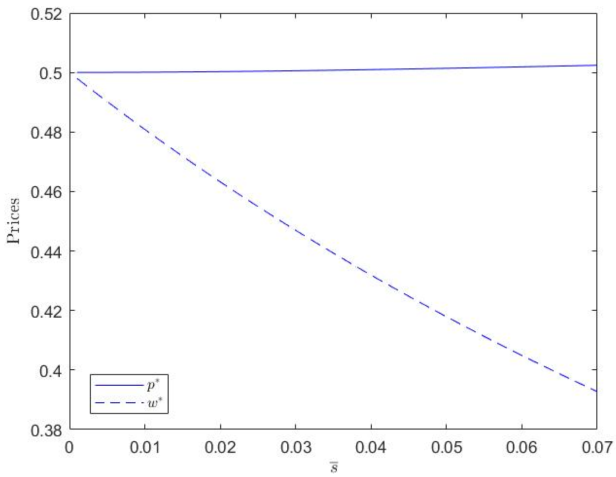

Fig 9.2 Uniform retail and wholesale prices for different values of $\overline{s}$, when $\alpha = 0.1$

Under wholesale price discrimination the manufacturer with two-part tariffs, the manufacturer will chose two different wholesale prices, w_L and w_H, to directly maximize:

$$
\begin{aligned}
\pi(w_L, w_H) \quad = \quad & \frac{1}{N}\left[1 - G(\widehat{s})\right]\left[w_H D(p_H^*(w_H)) + (1-\alpha)(p_H^* - w_H) D(p_H^*(w_H))\right] \\
& + \frac{N - 1 + G(\widehat{s})}{N}\left[w_L D(p_L^*(w_L)) + (1-\alpha)(p_L^* - w_L) D(p_L^*(w_L))\right]
\end{aligned}
$$

which yields the two following first-order conditions:

$$
\begin{aligned}
0 \quad = \quad & \left[w_H D(p_H^*) + (1-\alpha)(p_H^* - w_H) D(p_H^*) - w_L D(p_L^*) - (1-\alpha)(p_L^* - w_L) D(p_L^*)\right]\left(D(p_L^*)\frac{\partial p_L^*}{\partial w_H^*} - D(p \right. \\
& + \frac{N - 1 + G(\widehat{s})}{g(\widehat{s})}\left[w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*} + (1-\alpha)\left(\frac{\partial p_L^*}{\partial w_H^*} D(p_L^*) + (p_L^* - w_L) D'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right)\right] \\
& + \frac{[1 - G(\widehat{s})]}{g(\widehat{s})}\left[D(p_H^*) + w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*} + (1-\alpha)\left((\frac{\partial p_H^*}{\partial w_H^*} - 1) D(p_H^*) + (p_H^* - w_H) D'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
0 \quad = \quad & \left[w_H D(p_H^*) + (1-\alpha)(p_H^* - w_H) D(p_H^*) - w_L D(p_L^*) - (1-\alpha)(p_L^* - w_L) D(p_L^*)\right]\left(D(p_L^*)\frac{\partial p_L^*}{\partial w_L^*} - D(p \right. \\
& + \frac{N - 1 + G(\widehat{s})}{g(\widehat{s})}\left[D(p_L^*) + w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*} + (1-\alpha)\left((\frac{\partial p_L^*}{\partial w_L^*} - 1) D(p_L^*) + (p_L^* - w_L) D'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right)\right] \\
& + \frac{[1 - G(\widehat{s})]}{g(\widehat{s})}\left[w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*} + (1-\alpha)\left(\frac{\partial p_H^*}{\partial w_L^*} D(p_H^*) + (p_H^* - w_H) D'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right)\right]
\end{aligned}
$$

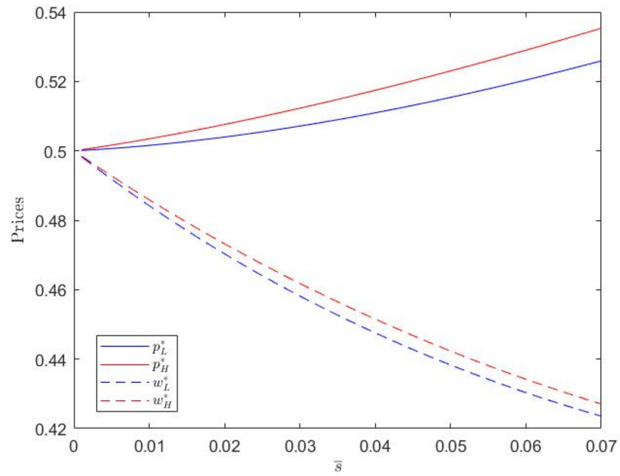

Fig 9.3 Retail and wholesale prices under wholesale price discrimination for different values of $\overline{s}$ and $\alpha = 1$

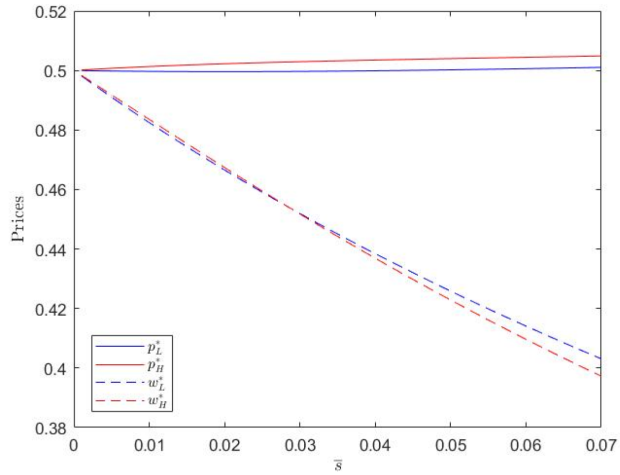

Fig 9.4 Retail and wholesale prices under wholesale price discrimination for different values of $\overline{s}$ and $\alpha = 0.1$

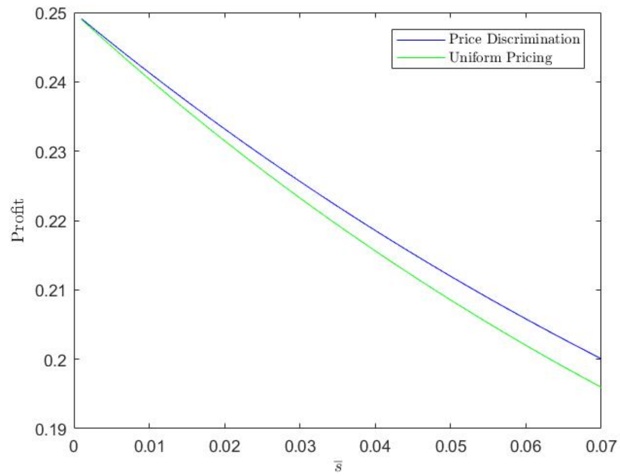

Fig 9.5 Manufacturer's Profit for different values of $\overline{s}$ and $\alpha = 1$

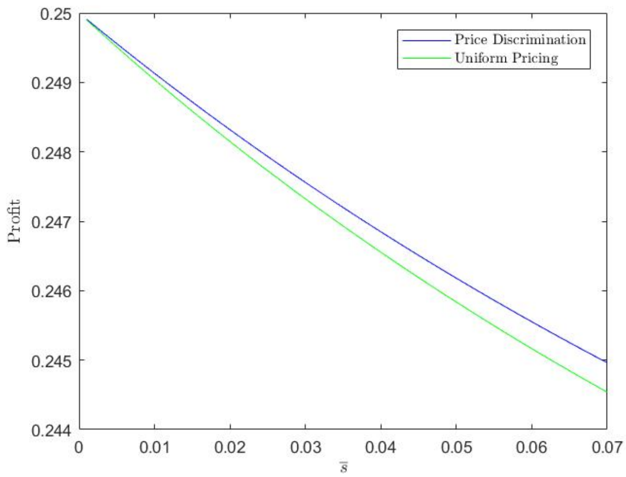

Fig 9.6 Manufacturer's Profit for different values of $\overline{s}$ and $\alpha = 0.1$

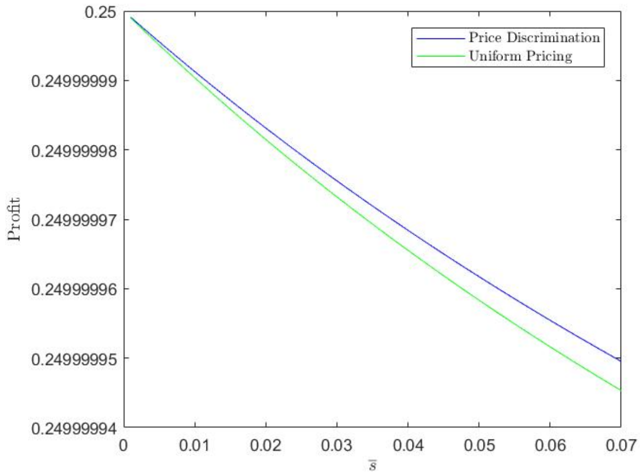

Fig 9.7 Manufacturer's Profit for different values of $\overline{s}$ and $\alpha = 0.000001$

9.2.2 No Commitment

On the other hand, under no commitment, the manufacturer's profit in case of deviation will be:

$$\begin{aligned}\pi(w^*, w') \;=\; & \left(\frac{N-1}{N} + \frac{1}{N\overline{s}}\int_{p^*(w^*)}^{\widetilde{p}(w)} D(p)dp\right)\left[w^* D(p^*(w^*) + (1-\alpha)(p^* - w^*)D(p^*(w^*))\right] + \\ & \frac{1}{N}\left(1 - \frac{1}{\overline{s}}\int_{p^*(w^*)}^{\widetilde{p}(w)} D(p)dp\right)\left[wD(\widetilde{p}(w)) + (1-\alpha)(\widetilde{p} - w)D(\widetilde{p}(w))\right],\end{aligned}$$

so that the equilibrium condition on the wholesale price becomes:

$$w^* D'(\widetilde{p}(w^*))\frac{\delta\widetilde{p}}{\delta w} + D(\widetilde{p}(w^*)) + (1-\alpha)(p^* - w^*)D'(\widetilde{p}(w^*))\frac{\delta\widetilde{p}}{\delta w} + (1-\alpha)D(\widetilde{p}(w^*))\left(\frac{\delta\widetilde{p}}{\delta w} - 1\right) = 0.$$

Under wholesale price discrimination, the retailers' analysis remains gain the same as in Section 4 and the manufacturer's profit in case of deviation will be:

$$\frac{1}{N}\left(1 + \frac{\int_{p_L^*(w_L)}^{p_H(w_H)} D(p)dp}{(N-1)\overline{s}} - \frac{N-1}{N\overline{s}}\int_{p_L^*}^{\tilde{p}_L(w_L)} D(p)dp - \frac{\int_{p_L^*}^{\tilde{p}_L(w_L)} D(p)dp}{(N-1)\overline{s}}\right)\left[w_L D(\tilde{p}_L(w_L)) + (1-\alpha)(\widetilde{p} - w_L\right.$$

$$+\frac{N-2}{N}\left(1 + \frac{\int_{p_L^*}^{p_H(w_H)} D(p)dp}{(N-1)\overline{s}} + \frac{\int_{p_L^*}^{\tilde{p}_L(w_L)} D(p)dp}{(N-1)(N-2)\overline{s}} + \frac{N-1}{N\overline{s}}\int_{p_L^*}^{\tilde{p}_L(w_L)} D(p)dp\right)\left[w_L^* D(p_L^*(w_L^*)) + (1-\alpha)\right.$$

$$+\frac{1}{N}\left(1 - \frac{1}{\overline{s}}\int_{p_L^*}^{p_H(w_H)} D(p)dp\right)\left[w_H D(p_H(w_H)) + (1-\alpha)(p_H - w_H)D(p_H(w_H))\right]$$

The equilibrium wholesale price condition for w_L^* thus becomes:

$$w_L^* D'(\widetilde{p}(w_L^*))\frac{\delta\widetilde{p_L}}{\delta w_L} + D(\widetilde{p_L}(w_L^*)) + (1-\alpha)(p_L^* - w_L^*)D'(\widetilde{p_L}(w_L^*))\frac{\delta\widetilde{p_L}}{\delta w_L} + (1-\alpha)D(\widetilde{p_L}(w_L^*))\left(\frac{\delta\widetilde{p_L}}{\delta w_L} - 1\right) = 0$$

while for w_H^* the condition becomes

$$\frac{1}{\overline{s}}D(p_H^*(w_H^*))\frac{\delta p_H}{\delta w_H}\left[w_L^* D(p_L^*(w_L^*)) + (1-\alpha)(p_L^* - w_L^*)D(p_L^*(w_L^*)) - w_H^* D(p_H^*(w_H^*)) - (1-\alpha)(p_H^* - w_H^*)\right.$$

$$+\left(1 - \frac{1}{\overline{s}}\int_{p_L^*}^{p_H^*} D(p)dp\right)\left[w_H^* D'(p_H^*(w_H^*))\frac{\delta p_H}{\delta w_H} + D(p_H^*(w_H^*)) + (1-\alpha)(\frac{\delta p_H}{\delta w_H} - 1)D(p_H^*(w_H^*)) + (1-\alpha)\right.$$

The following figures illustrates that for any $\alpha > 0$, the qualitative results of our analysis for linear pricing continue to hold and that wholesale price discrimination results in higher profit for the manufacturer and lower retail profit and consumer surplus.

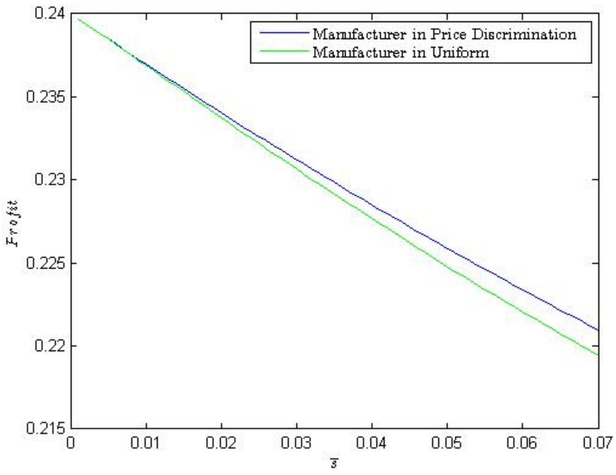

Figure 6.1 Manufacturer's Profit under the two-part tariff case: $\alpha = 0.5$

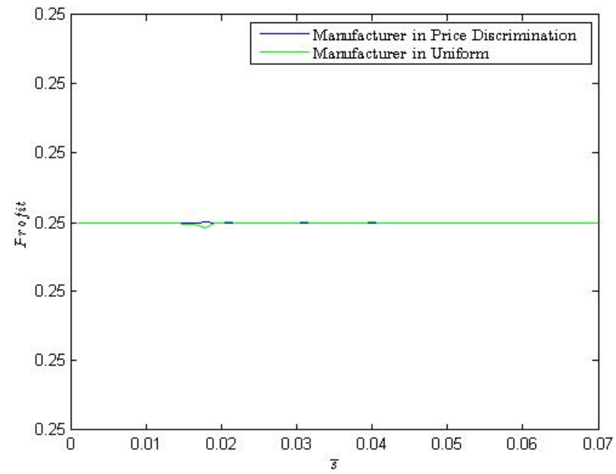

Figure 6.2 Manufacturer's Profit under the two-part tariff case: $\alpha = 0.1$

9.3 Profit Functions

9.3.1 Uniform pricing, no commitment, linear demand, uniform distribution

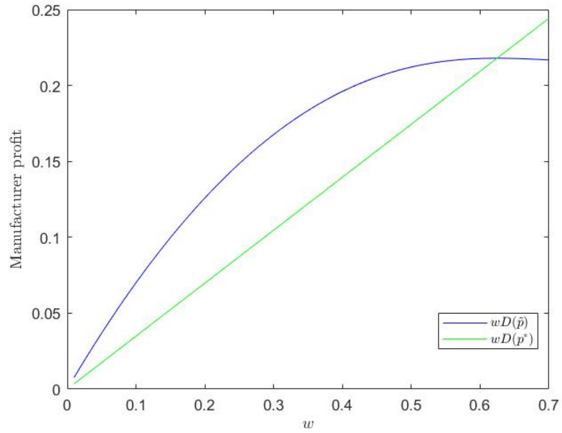

Figure 8.1. Manufacturer's profits for different values of w when $\overline{s} = 0.01$, $w^* = 0.6245$.

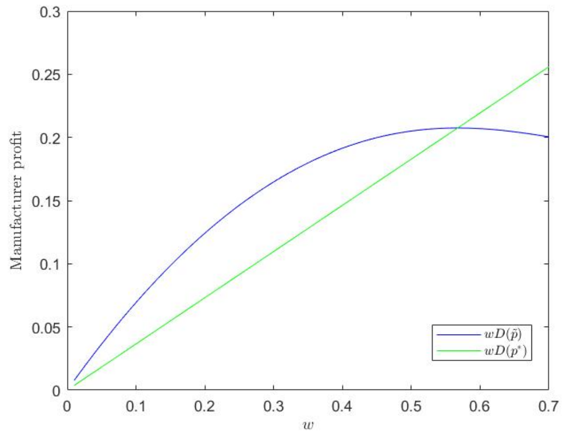

Figure 8.2. Manufacturer's profits for different values of w when $\overline{s} = 0.03$, $w^* = 0.5673$.

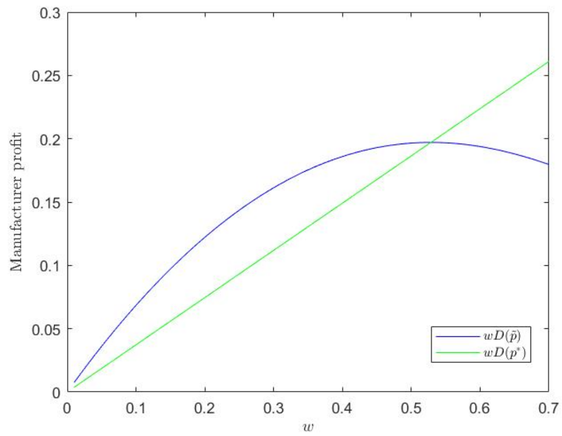

Figure 8.3. Manufacturer's profits for different values of w when $\overline{s} = 0.05$, $w^* = 0.5284$.

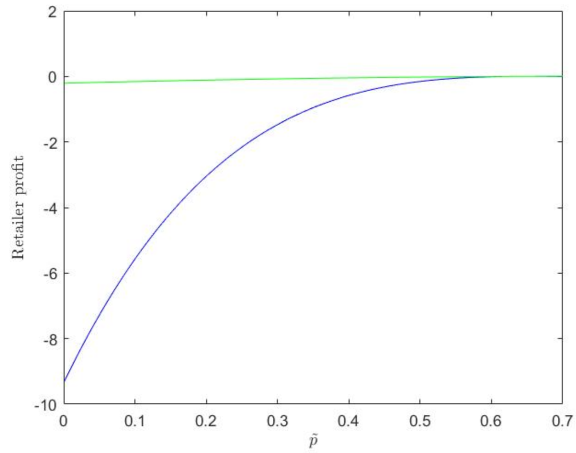

Figure 8.4. Retailer's profits for different values of $\tilde{p}$ when $\overline{s} = 0.01$, $p^* = 0.6510$.

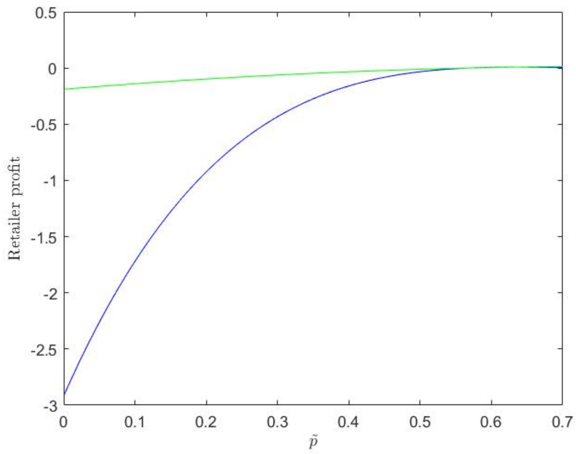

Figure 8.5. Retailer's profits for different values of $\tilde{p}$ when $\overline{s} = 0.03$, $p^* = 0.6343$.

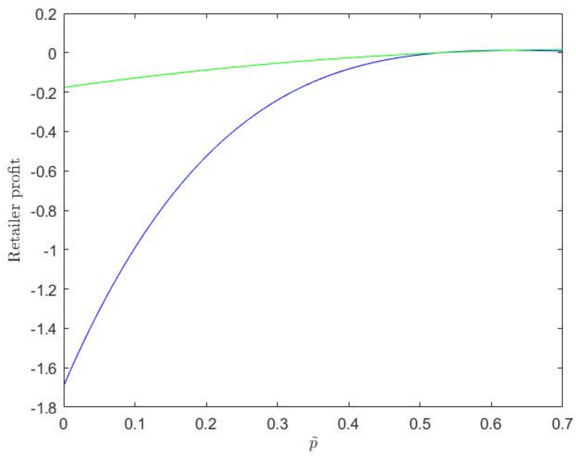

Figure 8.6. Retailer's profits for different values of $\tilde{p}$ when $\overline{s} = 0.05$, $p^* = 0.6270$.

9.3.2 Price discrimination, no commitment, linear demand, uniform distribution

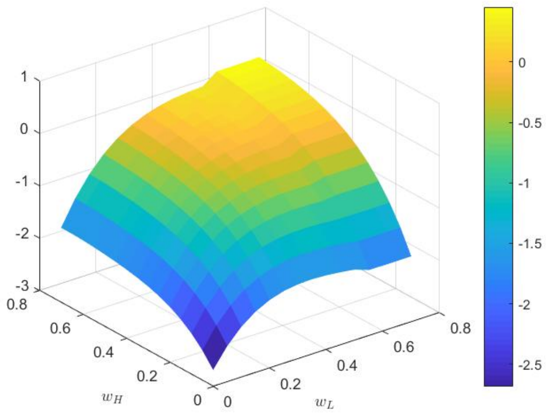

Figure 8.7. Manufacturer's profits for different values of w when $\overline{s} = 0.01$.

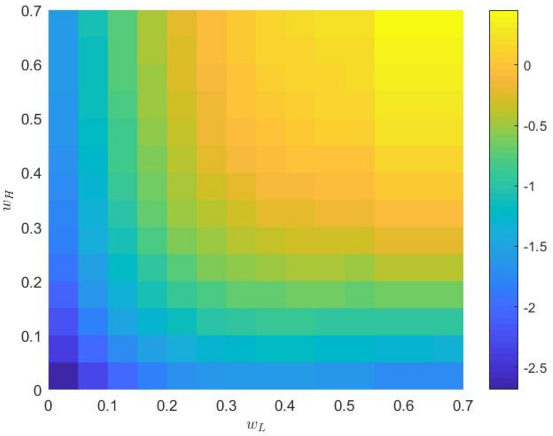

Figure 8.8. Manufacturer's profits for different values of w when $\overline{s} = 0.01$.

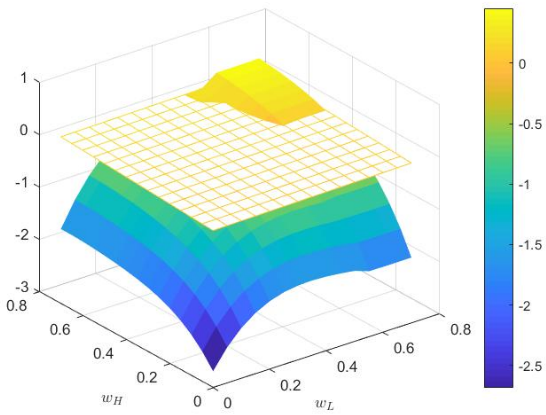

Figure 8.9. Manufacturer's profits for different values of w when $\overline{s} = 0.01$.

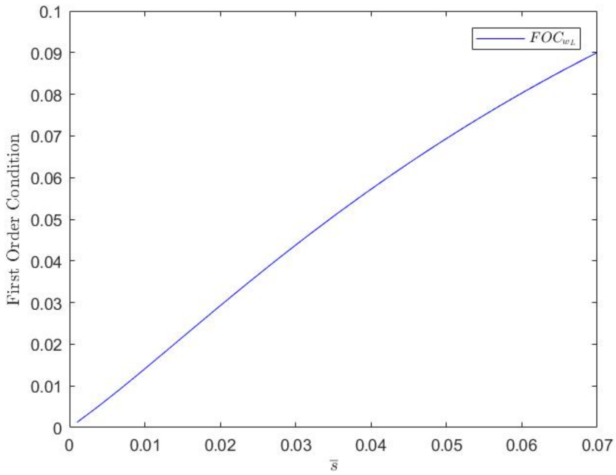

Fig 4.3 Demand given by $D(p) = (1-p)^{\beta}$, where $\beta = 0.5$.

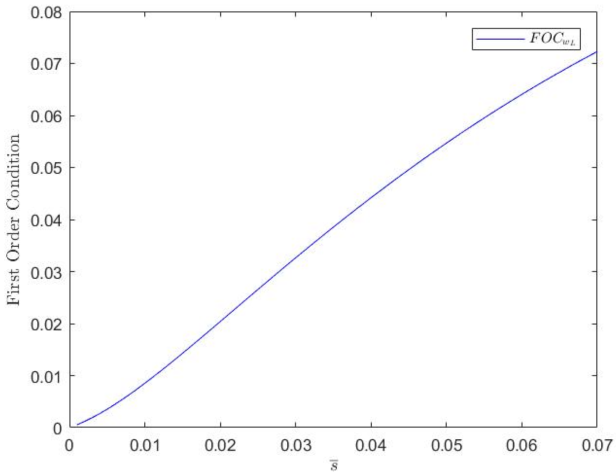

Fig 4.4 Demand given by $D(p) = (1-p)^{\beta}$, where $\beta = 1$.

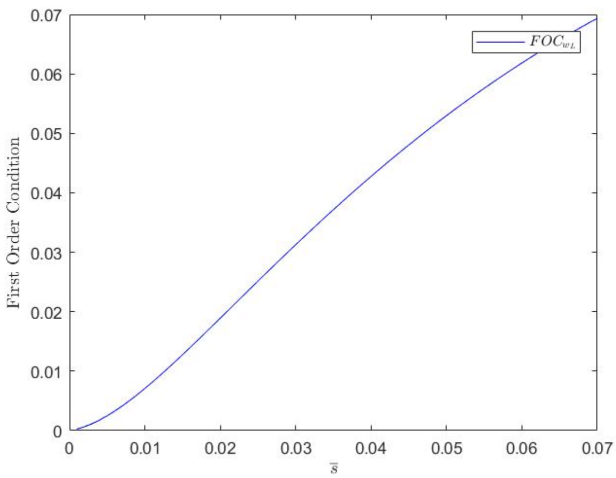

Fig 4.5 Demand given by $D(p) = (1-p)^{\beta}$, where $\beta = 1.5$.

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