Designing Innovation Contests for Diversity

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December 11, 2015

Abstract

This paper analyzes the design of innovation contests when the quality of an innovation depends on the research approach of the supplier, but the best approach is unknown. Diversity of approaches is desirable because it generates an option value. In our main model with two suppliers, the buyer optimally uses a bonus tournament, where suppliers can choose between a low bid and a high bid. This allows the buyer to implement any level of diversity with the lowest revenue for the supplier. We also compare other common contests, in particular, fixed-prize tournaments and auctions.

Keywords: Contests, tournaments, auctions, diversity, procurement.
JEL: L14, L22, L23.

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1 Introduction

The use of contests in innovation procurement has a long history, and it is becoming ever more popular. Recently, private buyers have awarded the Netflix Prize, the Ansari X Prize, and the InnoCentive prizes. Public agencies have organized, for instance, the DARPA Grand Challenges, the Lunar Lander Challenge and the EU Vaccine Prize.[1] The literature on contest design mainly focuses on the problem of providing incentives for costly innovation effort.[2] However, in addition to effort, successful innovation also requires that an adequate research approach is chosen. Ex ante, there are often many possible options. Developing more than one of these alternative research approaches generates an option value, as the best approach can be selected ex post. In this paper, we thus ask how contest design influences the diversity of research approaches developed by the contestants.

Many practical examples illustrate the importance of the issue. First, the often cited Longitude Prize of 1714 for a method to determine a ship's longitude at sea featured two competing approaches.[3] The lunar method was an attempt to use the position of the moon to calculate the position of the ship. The alternative, ultimately successful, approach relied on a clock which accurately kept Greenwich time at sea, thus allowing estimation of longitude by comparison with the local time (measured by the position of the sun). Second, when the Yom Kippur War in 1973 revealed the vulnerability of the US aircraft to Soviet-made radar-guided missiles, General Dynamics sought to resolve the issue through electronic countermeasures, while McDonnell Douglas, Northrop, and eventually Lockheed, attempted to build planes with small radar cross-section.[4] Third, the EU Vaccine Prize was announced in 2012 with the goal of improving the so-called cold-chain vaccine technology. Interestingly, the competition rules explicitly stated that diverse innovation approaches were conceivable: "It is important to note that approaches to be taken by the participants in the competition are not prescribed and may include alternate formulations, novel packaging and/or transportation techniques, or significant improvements over existing technologies, amongst others."[5] Finally, the announcement of the 2015 Horizon Prize for better use of antibiotics contains a similar statement.[6]

As illustrated by the examples above, in many innovation contests both the buyer (the contest designer) and the suppliers (contestants) are aware that there are multiple conceivable approaches to innovation. Furthermore, none of the participants knows the best approach beforehand. However, after the suppliers have followed a particular approach, it is often possible to assess the quality of innovations, for instance, by looking at prototypes or detailed descriptions of research projects. In the following, we will ask whether buyers can and should do more than to suggest that suppliers should pursue diverse approaches: Can they design contests in such a way that suppliers have incentives to provide diversity? And will they benefit from doing so?

Architectural contests share some important properties with innovation contests. A buyer

[1] See "Innovation: And the winner is...", *The Economist.* Aug 5, 2010.

[2] Section 6 discusses this literature.

[3] See, e.g., Che and Gale (2003) for a discussion of the Longitude Prize.

[4] See Paul Crickmore (2003), *Nighthawk F-117: Stealth Fighter.* Airlife Publishing Ltd.

[5] European Comission (2012), "Prize Competition Rules." August 28, 2012 (accessed on April 3, 2015). http://ec.europa.eu/research/health/pdf/prize-competition-rules_en.pdf

[6] European Commision (2015), "Better use of antibiotics." March 24, 2015 (accessed on April 3, 2015). http://ec.europa.eu/research/horizonprize/index.cfm?prize=better-use-antibiotics

who thinks about procuring a new building usually does not know what exactly the ideal building would look like, but once she examines the submitted plans, she can choose the one she prefers. Guidelines for architectural competitions explicitly recognize the need for diversity. For example, the Royal Institute of British Architects states: "Competitions enable a wide variety of approaches to be explored simultaneously with a number of designers."[7]

To our knowledge, we are the first to analyze the optimal design of innovation contests with multiple conceivable research approaches. In our main model, there are two homogeneous suppliers who decide whether to exert costly research efforts and which research approach to choose. The research approach is captured as a point on the unit interval. Crucially, the value of an innovation depends on the difference between the chosen research approach and an ideal, but initially unknown approach. The suppliers and the buyer agree about the distribution of this ideal approach, which has a strictly positive, symmetric and single-peaked density. If different suppliers try different approaches, this creates an option value for the buyer who can choose the most adequate result once uncertainty is resolved.

In line with the literature on innovation contests, we assume that neither research inputs (approaches) nor research outputs (qualities) are verifiable, because they are both difficult to evaluate, and the relation between them is stochastic. The lack of verifiability of research activity precludes any kind of contract that conditions on research inputs or outputs, and it motivates the focus on contests as the means to provide incentives for investment in innovation.[8] The notion of contest design that we use was suggested by Che and Gale (2003). The buyer prescribes a possible set of prices and commits herself to paying the price chosen by the supplier from which the innovation is procured. The class of such contests is very rich (see Che and Gale (2003) for a longer discussion). Examples include fixed-prize tournaments (when the price set is a singleton) as well as auctions (when the price set is the set of non-negative real numbers). We also allow the buyer to pay subsidies to the suppliers to induce them to participate in the contest and exert costly research effort. Contest design in this setting is the choice of the allowable price set and the subsidy. After the buyer has communicated the rules of the game, the suppliers choose their approaches, and qualities become common knowledge. We assume all approaches are equally costly, so as to focus on suppliers' incentives to diversify.

We show that the optimal contest in this setting is what we call a bonus tournament. In a bonus tournament, the price set consists of two elements — a low price and a high ("bonus") price. The crucial feature of these contests is the non-convexity of the price set. We show that, with a bonus tournament, the buyer can implement essentially any level of diversity. In particular, a bonus tournament with suitably chosen prices (and possibly a subsidy) implements the socially optimal amount of diversity. The amount of diversity implemented in a bonus tournament is determined by the difference between the bonus price and the low price. The suppliers diversify in the hope that their quality advantage over the competitor will be sufficiently high that they can bid the bonus price and win even so. At the same time, the bonus tournament minimizes rent extraction whenever innovations are similar. However, inducing diversity is costly for the buyer. We show that the optimal contest leads to just enough diversity that expected supplier revenues are equal to the cost needed to develop the innovation. This will imply lower diversity than socially optimal, except when research costs

[7]See Royal Institute of British Architects (2013), "Design competitions guidance for clients." (accessed on Apr 3, 2015); http://competitions.architecture.com/requestform.aspx.

[8]For an extensive discussion see Che and Gale (2003) and Taylor (1995).

are very high. Thus the buyer resolves a trade-off between efficiency and rent extraction in favor of the latter.

We then compare several familiar contests against the background of the optimal contest. Unrestricted auctions induce the social optimum, while auctions with price caps induce lower levels of diversity. The price ceiling determines the amount of diversity. While auctions can in general implement the same diversity as the optimal bonus tournaments, they always generate higher revenues for the suppliers. Thus the buyer prefers bonus tournaments to auctions. Fixed-prize tournaments do not induce any diversity and are therefore less efficient than auctions and optimal bonus tournaments. Nevertheless, for low research costs, the buyer prefers the inefficient fixed-prize tournaments to the socially efficient unrestricted auctions.

We then extend the analysis, and show that, with some caveats, the bonus tournaments perform well even in more general environments. In particular, we study contests with multiple suppliers, and contests with more general distributions and quality functions. In addition, we discuss heterogeneous suppliers, multiple prizes and multiple approaches developed by the same supplier. Under very general conditions, bonus tournaments still induce the social optimum. The buyer continues to prefer them to fixed-prize tournaments, even though the latter induce some diversity (but suboptimal amounts) when there are multiple suppliers. However, when research costs are sufficiently high, the buyer may prefer auctions to bonus tournaments. Moreover, she may benefit from inviting a large number of suppliers, which is a straightforward implication of the option value provided by additional suppliers.

Our analysis has potential applications beyond innovation contests organized by a single buyer. As we discuss briefly in the conclusion, our model also applies when suppliers choose products in the face of uncertain demand by a potentially large number of homogeneous buyers. Contest design then corresponds to the choice of alternative regulatory institutions. Our approach shows that unregulated markets provide incentives for suppliers to choose the socially optimal products, but at the cost of endowing them with ex-post market power. As a result, regulation may yield higher expected consumer surplus, even though it does not induce the optimal expected product quality.

In Section 2, we introduce the model. Section 3 deals with the design of optimal contests for the buyer. Section 4 compares several commonly used contests, such as fixed-prize tournaments and auctions with and without reserve prices. Section 5 presents extensions of the model. Section 6 discusses the relation of our paper to the literature. Section 7 concludes, pointing in particular to the above-mentioned re-interpretation of our model for a world with many buyers. Proofs are in the Appendix.

2 The Model

A risk-neutral buyer B needs an innovation that two risk-neutral suppliers ($i \in \{1,2\}$) can provide. Each supplier simultaneously chooses whether to carry out costly research and which approach $v_i \in [0,1]$ to pursue. The cost of approach v_i is $C(v_i) \equiv C \geq 0$. Thus all approaches are equally costly. The quality q_i of the resulting innovation depends on a state $\sigma \in [0,1]$, which is distributed with density $f(\sigma)$, and corresponds to an (ex-post) ideal approach. We thus assume that $q_i = \Psi - \delta(|v_i - \sigma|)$, where $\Psi > 0$ is large enough[9] and δ is an increasing

[9] Ψ needs to be large enough so that it is worthwhile for the buyer to hold a contest. A simple sufficient condition is $\Psi > \delta(1) + 2C$. This assumption is innocuous as none of our results depend on Ψ.

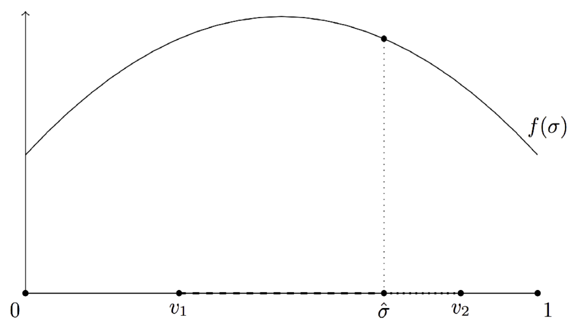

Figure 1: Illustration of an outcome given v_1 and v_2 where the ideal approach is $\hat{\sigma}$.

function.

Figure 1 illustrates one particular outcome of the model. Suppose that the uncertainty is given by the distribution $f(\sigma)$ and that the suppliers have chosen the approaches v_1 and v_2. The quality difference between the ideal approach $\hat{\sigma}$ and v_i is proportional to the distance between v_i and $\hat{\sigma}$ (the dashed line for $i = 1$, and the horizontal dotted line for supplier $i = 2$). Unless specified otherwise, we will maintain assumptions (A1) and (A2) below.

Assumption (A1) *The density function* $f(\sigma)$ *is (i) symmetric:* $f(1/2 - \varepsilon) = f(1/2 + \varepsilon)$ $\forall \varepsilon \in [0, 1/2]$*, (ii) single-peaked:* $f(\sigma) \leq f(\sigma') \ \forall \sigma < \sigma' < 1/2$*, (iii) has full support:* $f(\sigma) > 0$ $\forall \sigma \in [0, 1]$ *and (iv) satisfies* $f(x) \leq 2f(y)$ *for all* $x, y \in [0, 1]$.

Assumption (A2) $\delta(|v_i - \sigma|) = b|v_i - \sigma|$ *with* $b \in (0, \Psi]$.

A wide class of distributions satisfies (A1). For each of these distributions, there is an approach which has the highest expected quality ex ante, namely the median. Furthermore, single-peakedness makes it more difficult to induce diversity with the contest: As there is less mass on approaches that are away from the median, contestants will not choose them without additional incentives. Part (iv) excludes the possibility that some states are much less probable than others, that is, it requires that the amount of uncertainty about the ideal approach is sufficiently high. Using (A3), we denote the quality resulting from approach v_i in state σ as $q(v_i, \sigma) = \Psi - b|v_i - \sigma|$ for some $\Psi > 0$. Thus quality is bounded below by $\Psi - b$ and bounded above by Ψ.

In this setting, the buyer chooses an *innovation contest* determining the procedure for choosing and remunerating suppliers. These contests are closely related to those analyzed by Che and Gale (2003), where suppliers choose efforts rather than approaches. In line with these authors, we assume that neither v_i nor q_i is contractible.[10] The environment (b, Ψ, C) of a contest consists of the utility and cost parameters. The buyer chooses a set $\mathcal{P}$ of allowable prices (bids), where $\mathcal{P}$ is an arbitrary finite union of closed subintervals of $\mathbb{R}^+$.[11] We denote

[10] For example, Che and Gale (2003) and Taylor (1995) assume that neither inputs nor outputs of innovative activity are verifiable. As an example of the verifiability problem, Che and Gale (2003) point to the protracted battle between John Harrison, the inventor of the marine chronometer, and the Board of Longitude, over whether his invention met the requirements of the 1714 Longitude Prize. See also references in Taylor (1995).

[11] Formally, $\mathcal{P}$ is chosen from $\mathcal{I}(\mathbb{R}^+) := \{\mathcal{P} \subseteq \mathbb{R}^+ : \mathcal{P} = \cup_{k=1}^{\bar{k}}[a_k, b_k]$ or $\mathcal{P} = \cup_{k=1}^{\bar{k}}[a_k, b_k] \cup [a_{\bar{k}+1}, \infty)$ for $a_k \leq b_k \in \mathbb{R}^+, \bar{k} \in \mathbb{N}\}$.

the minimum of $\mathcal{P}$ as $\underline{P}$ and the maximum, if it exists, as $\overline{P}$. Moreover, the buyer can offer subsidies $t \geq 0$ to the suppliers. An innovation contest is thus the extensive-form game between the buyer and the suppliers given by the buyer's choice of $\{\mathcal{P}, t\}$ and the following rules:

Period 1:	Suppliers simultaneously choose whether to engage in research and they select approaches $v_i \in [0,1]$.
Period 2:	The state is realized. All players observe $v_i \in [0,1]$; thus q_1 and q_2 become common knowledge.
Period 3:	Suppliers simultaneously choose prices $p_i \in \mathcal{P}$.
Period 4:	The buyer observes prices; then she chooses a supplier $i \in \{1,2\}$. She pays $p_i + t$ to the chosen supplier and t to the other supplier.

Importantly, the suppliers receive two types of payments, namely the revenue from the contest (that is paid only to the successful supplier) and the subsidies paid to both suppliers. The assumption that all players observe v_i and σ is made purely for convenience, as it allows us to apply the subgame perfect equilibrium (SPE). The assumption is much more restrictive than necessary for the main insights of the analysis. As long as all players can observe qualities, all results still hold with the SPE replaced by a Perfect Bayesian Equilibrium with suitably specified beliefs.[12] Moreover, we provide an extensive discussion of the case when not even quality is observable in Letina and Schmutzler (2015); which we summarize briefly in Section 5.3.1.

The following are examples of innovation contests:

1. $\mathcal{P} = \mathbb{R}^+$: an *auction without a price ceiling.*
2. $\mathcal{P} = \left[0, \overline{P}\right]$: an *auction with a price ceiling* $\overline{P}$.
3. $\mathcal{P} = \{A\}$, where $A \geq 0$: a *fixed-prize tournament* (FPT).
4. $\mathcal{P} = \{A, a\}$, where $A > a \geq 0$: a *bonus tournament.*

The first three examples are well-known. The last example will turn out to be a useful alternative for the buyer.

To finish the description of the contests, we require several further conventions. First, we apply the following tie-breaking rules, which can be interpreted as second-order lexicographic preference for winning and for higher quality.

(T1) (Preference for quality) If suppliers offer the same surplus, the buyer prefers the higher quality one. If both have the same quality, the tie is randomly broken.

(T2) (Preference for winning) Given equal monetary payoffs, the suppliers prefer to participate in the contest rather than to stay out and to win the contest rather than not.

(T1) and (T2) will be shown to guarantee that the outcomes are robust to infinitesimal changes in the reward structure.

Second, we assume that, in cases where only one supplier decides to participate, the contest is called off and players obtain zero overall payoff.

Finally, we will confine our analysis to the case of pure-strategy equilibria for simplicity.

[12] This relies heavily on the fact that suppliers' bidding behavior will only depend on the observed qualities.

3 The Optimal Contest for the Buyer

In this section, we characterize the optimal contest for the buyer. We start with some auxiliary results. These results characterize the social optimum, and they deal with the pricing subgames.

3.1 Auxiliary Results

We introduce the following terminology which applies to the case that both suppliers participate.

Definition 1 *For any given* $(v_1, v_2) \in [0,1]^2$*, the (expected)* ***total surplus*** *is* $S_T(v_1, v_2) \equiv E_\sigma[\max\{q(v_1,\sigma), q(v_2,\sigma)\}] - 2C$*. The social optimum is* $(v_1^*, v_2^*) \equiv \arg\max_{(v_1,v_2)\in[0,1]^2} S_T(v_1, v_2)$*. The (expected)* ***surplus of supplier*** i *in an equilibrium* (v_1, v_2) *of a contest* $(\mathcal{P}, t)$*,* $S_i^{(\mathcal{P},t)}(v_1, v_2)$*, is the sum of the expected revenue and the subsidies, net of research costs. The (expected)* ***buyer surplus*** *in an equilibrium* (v_1, v_2) *of a contest* $(\mathcal{P}, t)$*,* $S_B^{(\mathcal{P},t)}(v_1, v_2)$*, is expected quality minus the expected revenues and subsidies of the suppliers. We usually drop the superscript* $(\mathcal{P}, t)$ *when there is no danger of confusion.*

As the costs of each approach are the same, the social optimum (v_1^*, v_2^*) maximizes the expected maximal quality $E_\sigma[\max\{q(v_1,\sigma), q(v_2,\sigma)\}]$ or, equivalently, minimizes the expected minimal distance to the ideal approach, $E_\sigma[\min\{\delta(|v_1-\sigma|), \delta(|v_2-\sigma|)\}]$. With only one potential supplier i, the optimal approach would correspond to $v_i = 1/2$, as this maximizes the expected quality. With two suppliers, the optimization needs to take into account the option value generated by having different choices once qualities have been observed. It is always socially optimal to have at least some diversification. This simple but important observation holds without the restrictions on distributions coming from (A1), as long as there is any uncertainty about the ideal approach. The intuition is simple: Starting from a situation with identical approaches, suppose one of the suppliers chooses an arbitrary alternative approach, whereas the other supplier continues to choose the same one. After this modification, the minimal distance decreases for a set of ideal states with positive measure. There can be no σ for which the expected minimal distance to the best approach increases, as the initial approach is still available. The following result provides a sharper characterization of the social optimum:

Lemma 1 *The unique social optimum with* $v_1^* \le v_2^*$ *satisfies* $F(v_1^*) = 1/4$ *and* $F(v_2^*) = 3/4$ *and thus* $v_2^* = 1 - v_1^*$*.*

Hence v_1^* and v_2^* are symmetric around $1/2$. The result relies on (A1(iv)), which states that the ideal state distribution is sufficiently dispersed.[13] Moreover, the comparative statics of

[13] The condition guarantees that the expected quality is a strictly concave function of the approaches. It is thus more restrictive than necessary. A simple *necessary* condition for the optimum to satisfy $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ is $f(1/2) < 2f(v_1^*)$; otherwise the objective function is not even locally concave. Moreover, this condition turns out to be necessary for the existence of a social optimum with $v_2^* = 1 - v_1^*$. It is simple to provide examples where $f(1/2) < 2f(v_1^*)$ is violated. For instance, consider the kinked distribution defined by the density

$$f(\sigma) = \begin{cases} 0.6 & \text{if} \quad \sigma \in [0, 0.45) \cup (0.5, 1] \\ 4.6 & \text{if} \quad \sigma \in [0.45, 0.55] \end{cases} .$$

optimal diversity are straightforward: The more the state distribution is concentrated around the median, the smaller optimal diversity $(v_2^* - v_1^*)$. Research costs, however, have no influence on the optimal diversity.

We now characterize the equilibria of the pricing subgames, using the following notation:

Notation 1 $\overline{p}(\sigma) \equiv \max\{p \in \mathcal{P} | \, p \leq |q(v_1, \sigma) - q(v_2, \sigma)| + \underline{P}\}$.

The following result is closely related to the familiar "asymmetric Bertrand" logic that inefficient firms choose minimal prices, whereas efficient firms translate their efficiency advantage into a price differential.[14]

Lemma 2 *The subgame of an innovation contest corresponding to* (q_i, q_j) *has an equilibrium such that* $p_i(q_i, q_j) = \overline{p}(\sigma)$ *if* $q_i \geq q_j$ *and* $p_i(q_i, q_j) = \underline{P}$ *if* $q_i < q_j$. *In any SPE of any contest,* $p_i(q_i, q_j) = \overline{p}(\sigma)$ *if* $q_i \geq q_j$.

Lemma 2 sharpens the Bertrand logic: The price differential will only fully reflect the quality differential when such pricing is feasible for the high-quality supplier. In many cases, the equilibrium described in Lemma 2 is unique.[15] We need further notation:

Notation 2 $\Delta q(v_i, v_j) \equiv |q(v_i, v_i) - q(v_j, v_i)|$ *is the maximum quality difference given* (v_i, v_j).

By Lemma 2, in any subgame the successful supplier chooses the highest available price below the sum of the quality differential and the minimum bid in any subgame. We now show that, for an equilibrium (v_1, v_2), the bid corresponds *exactly* to the sum of the maximum quality differential and the minimum bid. This is also the price paid in all other states resulting in the maximum quality difference.

Lemma 3 *Let* $v_1 \leq v_2$. *(i) If a contest implements* (v_1, v_2), *then* $\Delta q(v_1, v_2) + \underline{P} \in \mathcal{P}$. *(ii) If* $\sigma \in [0, v_1] \cup [v_2, 1]$, *the successful supplier bids* $p_i(q_i, q_j) = \Delta q(v_i, v_j) + \underline{P}$.

Lemma 3 is a key result. It shows that the amount of diversity that any contest can implement is limited by the highest price that the contest allows. Intuitively, (i) if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$, suppliers could increase their chances of winning by small moves towards the approach of the other party, without reducing the price in those cases where they win. (ii) shows that in all states outside the interval (v_1, v_2) the buyer pays a constant price, reflecting the (maximal) quality difference between the two suppliers. Therefore, to implement any (v_1, v_2), a buyer has to pay at least $\Delta q(v_1, v_2)(F(v_1) + 1 - F(v_2))$ in expectation to the suppliers.

[14] The adequacy of pure-strategy equilibria in asymmetric Betrand games has received some attention, in particular, but not only, because they tend to involve weakly dominated strategies (see Blume, 2003 and Kartik, 2011). In our setting, these issues are resolved by the appeal to the "preference for quality" (T1) and "preference for winning" (T2). In some of our contests (in particular, in auctions with and without price ceilings), constructions as in Blume (2003) and Kartik (2011) where the low-quality firm mixes over a small interval of prices exist.

[15] If P is convex and $\sup P > \bar{p}(\sigma)$ for all σ, then $p_i(q_i, q_j) = \underline{P}$ for $q_i < q_j$ in every equilibrium. To see this, note that, according to the lemma, $p_i = \overline{p}(\sigma) = \underline{P} + q(v_i, \sigma) - q(v_j, \sigma)$ in any equilibrium. If $p_j > \underline{P}$, then player i can choose a slightly higher prize, and he still wins. Hence, this is a profitable deviation.

3.2 Characterizing the Optimum

We now turn to our main results. Before identifying the optimal contest for the buyer, we first show that bonus tournaments can implement a wide range of allocations.

Proposition 1 *Suppose* $A = \Delta q(v_1, v_2)$ *for some* (v_1, v_2) *such that* $0 < v_1 \leq 1/2 \leq v_2 < 1$. *In the bonus tournament with* $\mathcal{P} = \{A, 0\}$ *and sufficiently high subsidies that suppliers break even, the strategy profiles* $(v_1, v_2, p_1(), p_2())$ *such that* $p_i(q_i, q_j) = A$ *if* $q_i - q_j \geq A$ *and* 0 *otherwise, form an equilibrium. In particular, the social optimum can be implemented as an equilibrium of a bonus tournament.*

Thus, the buyer can implement any desired diversity in a bonus tournament with A as the corresponding maximal quality difference. For instance, to induce the social optimum, the buyer has to set $A = \Delta q(v_i^*, v_j^*)$. The bonus tournament is thus a flexible instrument with which the buyer can fine-tune diversity with low supplier revenues. This suggests that the optimal contest is in this class. However, this intuition is incomplete, as it does not account for subsidies. We now show that it is nevertheless always optimal for the buyer to use bonus tournaments. However, she will not always implement the social optimum.

Theorem 1 *(i) The buyer optimum can be implemented with a suitable bonus tournament* $(\{A, 0\}, t)$ *where the suppliers breaks even on expectation.*
(ii) If $C \geq F(v_1^*)\, \Delta q(v_i^*, v_j^*)$, *the optimal contest for the buyer is a bonus tournament that implements* (v_1^*, v_2^*) *with* $t = C - F(v_1^*)\, \Delta q(v_i^*, v_j^*) \geq 0$ *and* $A = \Delta q(v_i^*, v_j^*)$.
(iii) If $C < F(v_1^*)\, \Delta q(v_i^*, v_j^*)$, *the optimal contest for the buyer is a bonus tournament that implements* $(\widetilde{v}_i, 1 - \widetilde{v}_i)$ *with* $t = 0$ *and* $A = \Delta q(\widetilde{v}_i, 1 - \widetilde{v}_i)$, *where* $\widetilde{v}_i = \max_{[0,1/2]} v_i$ *s.t.* $C = F(v_i)\, \Delta q(v_i, 1 - v_i)$.

Whereas (i) only states the optimality of bonus tournaments, (ii) and (iii) specify the details for the two different parameter regions. When research costs are high enough and quality differences in the social optimum are low, the buyer implements the social optimum. Otherwise, the buyer induces just enough diversity that the suppliers break even. In any event, the suppliers earn zero surplus. Contrary to the social optimum, the amount of diversity in the buyer optimum thus depends on the research cost parameter as long as $C < F(v_1^*)\, \Delta q(v_i^*, v_j^*)$: As C increases, diversity increases from 0 to the social optimum. In addition, the more concentrated the state distribution is, the smaller the difference between v_2^* and v_1^* is and thus the smaller the difference between the socially optimal diversity and the optimal diversity for the buyer.

The desirable incentive properties of bonus tournaments stem from the non-convexity of the price set. From Lemma 3 we know that in any contest implementing (v_1, v_2), the price $\Delta q(v_1, v_2)$ has to be in the price set. This fixes the price that the buyer has to pay in any state of the world when the quality difference is maximal. What contest design can achieve, then, is to reduce prices paid in those states of the world when $\sigma \in (v_1, v_2)$, so that the quality difference is not maximal. With a bonus tournament (A, a), the buyer commits herself not to pay prices between a and A in these states: Even though the quality difference is greater than a, she only pays a. Setting $a = 0$ clearly minimizes the revenues of the sellers. The only remaining question is how much diversity the buyer optimally induces. Through the option value it generates, diversity can increase efficiency. However, it is costly for the buyer to induce. The theorem shows that whenever there is a tradeoff between efficiency and rent extraction, the buyer sacrifices efficiency.

Even when subsidies are not feasible, the buyer can still implement the same outcome as with subsidies unless research costs are too high. For $C \leq F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, this is evident, as the bonus tournament in Theorem 1(iii) does not require subsidies. For slightly higher research costs the buyer can increase the low price a in order to make sure that the participation constraints of the buyers hold.

Proposition 2 *Suppose that (A1) and (A2) hold and that the buyer cannot use subsidies. If* $F(v_1^*)\,\Delta q(v_i^*, v_j^*) < C \leq F(v_2^*)\,\Delta q(v_i^*, v_j^*)$*, the buyer surplus is maximized by the bonus tournament implementing* (v_1^*, v_2^*) *with* $\mathcal{P} = \{A, a\}$*, where* $A = 2C + \Delta q(v_i^*, v_j^*)/2$ *and* $a = 2C - \Delta q(v_i^*, v_j^*)/2$*.*

The low positive price is a substitute for subsidies. However, for very large cost C, the difference between prices A and a is small relative to the absolute size of the price a. As a becomes very large in absolute terms, sellers are willing to sacrifice winning the bonus price in order to increase their probability of winning the low price. This can be overcome by using a generalized bonus tournaments with multiple intermediate prizes $\{a_0, a_1, \ldots, a_k, A\}$, such that $A - a_0 = \Delta q(v_i^*, v_j^*)$, and the intermediate prizes are chosen so that the sellers do not want to deviate from the social optimum and break even in expectation.

The buyer can increase her surplus if she is allowed to charge entry fees $e > 0$. She would charge such fees only if $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, in which case the optimal fee e^* satisfies $C + e^* = F(v_1^*)\,\Delta q(v_i^*, v_j^*)$.[16] With or without entry fees, the buyer thus designs the contest so that the suppliers exactly break even on expectation.

4 Auctions and Fixed Prize Tournaments

In Section 3.2, we characterized the optimal contest. We now study two other types of contests that are discussed in the literature, namely auctions and fixed prize tournaments. Auctions generally have good incentive properties (for example auctions are the optimal contest in the setting of Che and Gale (2003)). On the other hand, fixed prize tournaments are commonly used in innovation contests. Next, we examine how these contests perform in our setting, where the choice of research approaches is important.

Proposition 3 *(i) For any t such that the suppliers' participation constraints are met, the auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implements the social optimum. (ii) For any $A \geq 2C$, the unique equilibrium of an FPT ($\mathcal{P} = \{A\}$) is $(v_1, v_2) = (1/2, 1/2)$. (iii) Whenever $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, the buyer prefers the inefficient FPT to the efficient auction.*

Proposition 3(i) states that the auction induces the efficient amount of diversity. It is intuitively clear that an auction implements some diversity: With identical approaches, no supplier will earn a positive revenue. Any move away from the other supplier will lead to quality advantages in a measurable set of states and thereby to positive expected revenues. Auctions implement the socially efficient outcome because they align the externalities of the choice of an approach v_i with the private benefits of the choice of such an approach. For

[16] If the buyer is limited to setting fees below e^*, she will charge the maximum allowable fee.

example, fix some v_2 and consider a marginal change of v_1. Such a change generates externalities only in the states of the world for which the quality of supplier 1 is greater than the quality of supplier 2. Furthermore, the size of the externality is exactly the change in the quality difference. Since supplier 1 wins the auction only when his quality is higher and he bids exactly the quality difference, the private incentives and the externalities are aligned. While we prove Proposition 3(i) directly in Appendix 8.4, an analogous result also applies for more general state distributions and quality functions and for arbitrary numbers of suppliers. It also holds when suppliers are heterogeneous. Appendix 8.7 provides a statement and proof of such a general implementation result for the social optimum. This result extends beyond auctions to any type of institution that gives the chosen supplier a positive share of the quality difference to the next-best alternative.

Proposition 3(ii) states that an FPT induces no diversity at all. We will provide a proof of the result under more general conditions (Proposition 5(ii) below). The intuition for the absence of diversity is straightforward. As the size of the prize is independent of quality differences in an FPT, the suppliers care only about maximizing the expected winning probability. By (A1), this requires moving to the center. In particular, there is no diversity.

As to (iii), even though an action implements the social optimum, it leaves rents to the successful supplier whenever research costs are low enough. Because it avoids such rents, the buyer may prefer to use a suitable FPT. This trade-off between efficiency and rent extraction also shows up when analyzing the price ceiling in auctions.

Corollary 1 *In an auction with a price ceiling $\bar{P}$, the diversity in any equilibrium (v_i, v_j) is bounded by the price ceiling: $\bar{P} \geq \Delta q(v_i, v_j)$.*

If the maximal quality difference between the two suppliers were above the maximum feasible bid, the supplier could not charge the buyer for this quality difference. He could thus choose an approach slightly closer to the competitor to increase his chances of winning without reducing the price.

Corollary 1 embeds the auction without price ceiling and the FPT as polar cases. In an auction without price ceiling, suppliers are free to choose the bid and thus capture the benefits of diversification. This results in optimal diversity. By Corollary 1, reserve prices limit this possibility: They determine an upper bound on equilibrium diversity. As the price ceiling declines, so does the equilibrium diversity. Thus, the choice of the price ceiling involves a trade-off between efficiency-increasing diversity and market power for the suppliers. Consistent with the logic of Theorem 1 (iii) and Proposition 3(ii), the following result shows that the buyer never resolves the trade-off in favor of efficiency when costs are low.

Proposition 4 *Let $C = 0$. Among all contests where $\mathcal{P}$ is convex, the buyer's surplus is maximal in an FPT with $A = 0$.*

The proof of Proposition 4 relies heavily on the fact that higher quality suppliers bid the sum of the quality differential and the minimum $\underline{P}$ whenever available (Lemma 2). Thus the buyer surplus is the difference between the expectation of the minimum quality and the minimum bid. Her best choice is an FPT with $A = 0$, because this maximizes the minimum quality and minimizes the minimum bid.

5 Extensions

In this section, we extend the model in several directions and study the robustness of our main results. We show that, with more general distributions and quality functions and with multiple suppliers, bonus tournaments still have desirable properties. Very generally, bonus tournaments are still preferable to FPTs, and they still implement the social optimum with the lowest revenues. However, they may require higher subsidies than alternative contests. For instance, auctions may implement the social optimum with lower supplier surplus than bonus tournaments when research costs are high and when the number of suppliers is large. We also briefly sketch several other extensions.

5.1 Generalized distributions and quality functions.

In this subsection, we still assume that there only two suppliers, but we generalize the assumptions as follows:
Assumption (A1)' *The density function $f(\sigma)$ is (i) symmetric and (ii) has full support: $f(\sigma) > 0 \; \forall \sigma \in [0, 1]$.*
Assumption (A2)' $\delta(|v_i - \sigma|)$ *is increasing and continuous.*

Thus, we relax the requirements that the distribution be single-peaked and relatively flat and that the distance function be linear.

Lemmas 2, 3 and Proposition 1 also hold under the relaxed assumptions (A1)' and (A2)'. The proofs are analogous and are therefore omitted here. As a result, the main contests that we previously dealt with have the same properties as before:

Proposition 5 *Suppose that (A1)' and (A2)' hold. Then, (i) the bonus tournament ($\mathcal{P} = \{\Delta q(v_1^*, v_2^*), 0\}$) and the auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implement the social optimum with appropriate $t \geq 0$. Moreover, (ii) in any FPT ($\mathcal{P} = \{A\}$ for $A \geq 0$) where suppliers break even, the unique equilibrium is such that $v_1 = v_2$ and $F(v_i) = 1/2$ for $i = 1, 2$.*

The rankings between the contests are similar to the benchmark model of Section 2. However, there are cases where the buyer prefers auctions to bonus tournaments. Intuitively, bonus tournaments still implement the social optimum with the lowest possible supplier revenue: The price is zero except for $\sigma \leq v_1$ and for $\sigma \geq v_2$, in which case the price just compensates the quality difference to the second-best supplier. However, for high research costs, the subsidies required for break-even can be so much lower with auctions that the buyer prefers them to bonus tournaments. This reflects the fact that, in auctions, the revenues of the different suppliers are more similar than for bonus tournaments, so that subsidies for which all suppliers break even involve less rents for the suppliers whose expected revenues are highest. The following result clarifies the circumstances under which bonus tournaments are preferable even so.

Proposition 6 *Suppose that (A1)', and (A2)' hold. Then, (i) the buyer strictly prefers a suitable bonus tournament to the FPT whenever $C > 0$. (ii) The buyer weakly prefers a suitable bonus tournament to the auction if at least one of the following conditions holds: (a) $C \leq \min\{F(v_1^*)\,\Delta q(v_1^*, v_2^*),\ (1 - F(v_2^*))\,\Delta q(v_1^*, v_2^*)\}$, (b) $v_1^* + v_2^* = 1$, or (c) $f(\sigma)$ is single-peaked. Whenever (a) holds, the preference is strict.*

According to (i), a suitable bonus tournament is still always preferable to an FPT in the more general set-up. Together, the conditions in (ii) show that a suitable bonus tournament dominates an auction under quite general conditions: Counterexamples require that research costs are high, that the social optimum is not symmetric and that $f(\sigma)$ is not single-peaked.

5.2 Number of Suppliers

In innovation contests there are usually more than two suppliers. For example, there were 49 registered competitors in the EU Vaccine Prize, 12 of which submitted final designs for evaluation.[17] We therefore now deal with the possibility that there are many suppliers. For simplicity, we assume that the distribution of ideal states is uniform.

Assumption (A1)" $f(\sigma) = 1 \; \forall \sigma \in [0, 1]$.

With this assumption, we can characterize the social optimum and the equilibria of the main contests previously discussed. Though most results also apply to the case $n = 3$, an FPT does not have a pure strategy equilibrium in this case. To allow for simple formulations, we confine ourselves to $n > 3$.

Lemma 4 *Suppose there are $n > 3$ suppliers and (A1)", and (A2) hold.*
(i) The social optimum is $(v_1^*, ..., v_n^*) = (1/2n, 3/2n, 5/2n, ..., (2n-1)/2n)$.
(ii) The social optimum can be implemented with a bonus tournament where $\mathcal{P} = \{b/n, 0\}$ and $t = C$ or with an auction with appropriate $t \geq 0$.
(iii) In any equilibrium of an FPT with n suppliers, there is duplication, and the amount of diversity is inefficiently low. As n increases, the difference between the socially optimal diversity and the minimal diversity in an FPT equilibrium converges to zero.

Figure 2 illustrates the result for $n = 6$. In line with Lemma 4(i), there is no duplication in the social optimum, and the approaches are evenly spread. The buyer can implement the social optimum effort with a bonus tournament or an auction. The two other constellations describing the equilibria of the FPT highlight implications of Lemma 4(iii). First, the two most extreme approaches are not as far apart as the most extreme approaches of the social optimum; in this sense, there is less than optimal diversity. Second, there is duplication. The remaining features of the depicted FPT hold in a class of FPT equilibria given in Lemma 8 in Appendix 8.6: The two most extreme approaches are always chosen by two suppliers. Moreover, depending on the specific equilibrium, there may be additional duplication for intermediate approaches.

Lemma **4** allows us to compare different institutions.

Proposition 7 *Suppose there are $n > 3$ suppliers and (A1)" and (A2) hold.*
(i) The buyer prefers to implement the social optimum with a bonus tournament rather than an auction if and only if $C < (n-1)\, b/2n^3$.
(ii) The buyer strictly prefers the bonus tournament $(b/n, 0)$ to any FPT for $n > 4$; she is indifferent for $n = 4$.

[17] European Commision (2014), "German company has won the EU's € 2 million vaccine prize." March 10, 2014 (accessed on April 3, 2015). http://ec.europa.eu/research/health/vaccine-prize_en.html

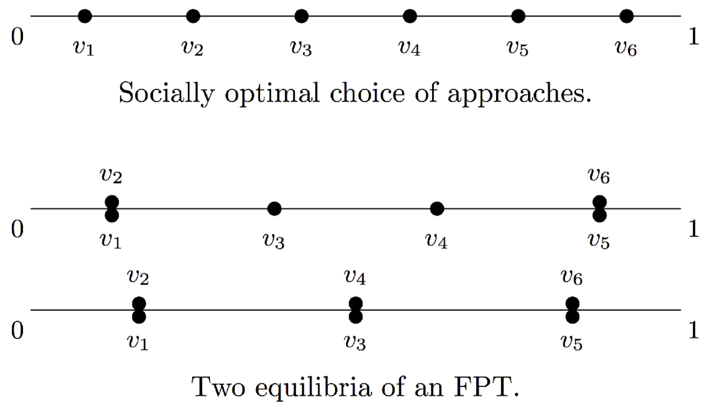

Figure 2: Equilibria when $n = 6$.

Result (i) contrasts with the case $n = 2$, for which the buyer always prefers bonus tournaments to auctions under assumptions (A1) and (A2) (which include uniform state distributions and linear quality functions). The intuition is essentially the same as for the case of two suppliers with generalized distributions: While the bonus tournament implements the social optimum with lower supplier revenues than the auction, it may require higher subsidies. Result (ii) generalizes the corresponding result for the benchmark model, with a small qualification for $n = 4$.

Lemma 4 has another simple but important implication: It may be socially optimal to invite a large number of suppliers. This differs from the case of contests that merely influence the suppliers' efforts: Several papers show that, in those settings, the optimal number of participants is typically two.

Corollary 2 *Suppose research costs are $C > 0$ and that (A1)" holds. Define $n_-(C) = \max\left\{n \in \mathbb{N} \middle| 2 \leq n \leq \sqrt{b} \middle/ 2\sqrt{C}\right\}$ and $n_+(C) = n_-(C) + 1$. Auctions or bonus tournament with $n_-(C)$ or $n_+(C)$ suppliers maximize total surplus in the set of all contests with arbitrary number of suppliers.*

The result is a straightforward implication of the previous results. Lemma 4(i) characterizes the socially optimal allocation for given n, and auctions and bonus tournaments implement this allocation. Corollary 2 describes the number of suppliers that optimally balances the gains from higher expected quality against the losses from higher research costs. While the corollary is stated for the socially optimal contest, it is simple to show that the buyer can also often benefit from inviting more than two suppliers.

5.3 Other Extensions

We now discuss to which extent several other extensions of the set-up are feasible. We deal with heterogeneous suppliers, multiple prizes and multiple research approaches of each supplier. In particular the first issue is treated in much more detail in the working paper (Letina and Schmutzler 2015).

5.3.1 Heterogeneous Suppliers

The assumption of homogeneous suppliers simplifies the analysis. In many contexts, it is nevertheless natural to allow for exogenous heterogeneity: Suppliers may differ with respect to expertise or research capabilities. Architects may have different and essentially fixed styles. In Letina and Schmutzler (2015), we extend the model to allow for such exogenous heterogeneity. To this end, we consider a two-dimensional state space to capture both exogenous and endogenous heterogeneity. We focus on uniform state distributions and the case $C = 0$. We show that the social optimum only involves diversification if exogenous heterogeneity is not too strong. As in the case of homogeneous suppliers with low research costs, however, fixed-prize tournaments do not induce any diversification, but buyers prefer them to auctions.

The framework with heterogeneous suppliers has an additional advantage: For sufficiently heterogeneous buyers, the modified framework allows us to use the alternative informational assumption that suppliers cannot observe qualities when they submit bids, which is intractable for homogeneous suppliers. We show that there is no differentiation in this equilibrium for an auction.

5.3.2 Fixed-Prize Tournaments with Multiple Prizes

In the 2005 DARPA Grand Challenge, only the winner of the contest was eligible for the prize ($2 million), while the other contestants received nothing. This corresponds to an FPT as introduced above. However, in the subsequent DARPA contest, known as the 2007 Urban Challenge, rules specified that not only would the winner receive a prize (which was again $2 million), but the next two participants would also receive prizes ($1 million and $0.5 million).[18] While a full analysis is beyond the scope of this paper, we can show that a buyer is worse off in an FPT with two prizes than with a single prize.[19] The following result shows that the buyer has nothing to gain from using multiple prizes.

Lemma 5 *Suppose that $n > 3$ and that (A1)" and (A2) hold. Further, suppose that t is sufficiently large, and that the two prizes are $A_1 > A_2 > 0$. For any equilibrium in an FPT with two prizes, there exists an equilibrium in an FPT with a single prize which makes the buyer strictly better off.*

Clearly, when there are only two suppliers, a second prize has no effect, as the suppliers would consider it as a pure subsidy, and the effective prize would be the difference between the first and the second prize. The proof of Lemma 5 shows that any equilibrium of an FPT with two prizes involves more duplication than the chosen equilibrium of an FPT with a single prize, which leads to a lower buyer surplus. This result suggests that multiple prizes do not improve diversity.[20]

[18] See Section 1.4 of the DARPA Urban Challenge Rules (2007) (accessed on June 24, 2015). http://archive.darpa.mil/grandchallenge/docs/Urban_Challenge_Rules_102707.pdf

[19] The results can be extended to more than two prizes.

[20] Of course, there may be reasons outside of the model which would make multiple prizes a desirable choice for a contest designer. For example, if suppliers are risk averse, providing multiple prizes may be a way of increasing their expected utility.

5.3.3 Multiple Designs by the Same Supplier

We have assumed so far that each supplier can only develop a single approach. However, in the 2005 DARPA Grand Challenge, vehicles designed by the Red Team from Carnegie Mellon University took the second and third place. By developing multiple designs, a supplier internalizes some of the resulting option value. It is thus natural to allow for multiple approaches of different suppliers. The modified model is analytically intractable, but a numerical analysis suggest that our main results are robust. We study the cases with $n \in \{2, ..., 5\}$ suppliers, each of which can develop $m = 2$ approaches, and the case with $n = 2$ suppliers, each of which can develop $m = 3$ approaches. We assume that (A1)" and (A2) hold and that $C = 0$. We also fix values of Ψ and b.[21]

Result 1 *If there are n suppliers and each develops m approaches, then: (i) in both a bonus tournament and an auction there exists an equilibrium which is equivalent to the socially optimal equilibrium of an auction with $n \cdot m$ suppliers, each of which develops one approach. (ii) In an FPT, there exists an equilibrium which is identical to the maximally duplicative equilibrium of an FPT with $n \cdot m$ suppliers, each of which develops one approach.*

The notion of a maximally duplicative equilibrium refers to Lemma 8 in Appendix 8.6: There, we consider a class of equilibria where maximal duplication occurs when each active research approach is chosen by two suppliers. While the analysis is clearly incomplete, Result 1 suggests that the case where n suppliers each develop m approaches can be analyzed using the framework where $n \cdot m$ suppliers each develop one approach (see Section 5.2).

6 Relation to the Literature

This paper contributes to the literature on optimal contest design, especially the design of innovation contests. The existing design literature focuses exclusively on effort incentives. In models of fixed-prize tournaments, Taylor (1995) shows that free entry is undesirable, and Fullerton and McAfee (1999) show that the optimal number of participants is two. Fullerton et al. (2002) find that buyers are better off with auctions rather than fixed-prize tournaments. In a very general framework, Che and Gale (2003) show that an auction with two suppliers is the optimal contest. Contrary to the previous literature, our paper focuses on the suppliers' choice of research approaches rather than only on effort levels. We characterize the optimal contests in such settings, highlighting in particular the useful role of bonus tournaments.

This paper extends our working paper (Letina and Schmutzler, 2015). In particular, we now characterize the optimal contest under more general conditions. Compared with the working paper, we focus more on the natural case that research costs are positive, and we identify the conditions under which suppliers exert costly research effort. We also clarify how the availability of subsidies and entry fees influences the optimal design. Finally, we extend the analysis of bonus tournaments beyond the benchmark model. Due to these extensions, we find a wide range of circumstances where auctions outperform fixed prize tournaments, and we clarify the relative merits of auctions and bonus tournaments.

[21] For details and the code used to obtain numerical results, see Supplementary Material for Section 5.3.3, available at https://sites.google.com/site/iletina.

Letina (2015) also studies the diversity of approaches to innovation, but the objects of analysis and the employed models are very different. He focuses on a market context, and he deals with comparative statics rather than optimal design. In particular, the paper finds that a merger decreases the diversity of approaches to innovation.

While we are not aware of any other paper that considers optimal contest design when diversity plays a role, some authors compare contests in related, but different settings. For instance, Schöttner (2008) considers two contestants who influence quality stochastically by exerting effort. She finds that, for large random shocks, the buyer prefers to hold a fixed-prize tournament rather than an auction to avoid the market power of a lucky seller in an auction. This resembles the trade-off underlying Proposition 3 in our setting. However, her analysis does not speak to optimal design and the role of bonus tournaments. It also does not address the setting with n suppliers.

In Ganuza and Hauk (2006), suppliers choose both an approach to innovation and a costly effort.[22] However, these authors focus exclusively on fixed-prize tournaments.[23] Erat and Krishnan (2012) study a fixed-prize tournament where suppliers can choose from a discrete set of approaches.[24] Each approach is successful with some probability that is independent of success or failure of any other approach. The qualities of successful approaches can vary. The authors find that suppliers cluster on approaches delivering the highest quality. This result is related to our result that there is duplication of approaches in the equilibria of fixed-prize tournaments. In addition to allowing for alternative contests, our model also considers correlated rather than independent qualities; it is thus meaningful to speak of similar approaches.[25]

Our paper is also related to the literature on innovation contests with exponential-bandit experimentation (see Halac, Kartik and Liu (2014) and references therein). In these models, it is uncertain whether the innovation is feasible. Suppliers participating in the contest expend costly effort to learn the state, and they also learn from the experimentation of their opponents. The goal of the contest is to induce experimentation. However, each supplier experiments in the same way. In our model, experimentation arises at the industry level for suitable contests, as the heterogeneity of approaches allows the buyer to pick the best available choice.

More broadly, our paper is related to the literature on policy experimentation. For instance, Callander and Harstad (2015) show that decentralized policy experimentation yields too much diversity. In their model, the success probabilities of different experiments are independent, no matter how similar the policies are. This assumption removes the option value of having different experiments, which is central to our model. If there existed an ideal policy (in terms of quality) as in our model, then the option value would have to be traded off against the

[22] In Ganuza and Pechlivanos (2000), Ganuza (2007) and Kaplan (2012), the buyer has to choose the design or alternatively can reveal information about the preferred design.

[23] More broadly related is Bajari and Tadelis (2001) who do not deal with innovations, but with construction projects. The issue of the right approach to the problem arises in such settings as well. The supplier obtains new information during the period when the contract is being executed, which allows him to adapt the original approach at some cost. Since the relationship is between a buyer and only one supplier, the question of diversity of approaches does not arise. This is also true for the related work by Arve and Martimort (2015) who study risk-sharing considerations in the design of contracts with ex-post adaptation.

[24] See also Terwiesch and Xu (2008) for the effect of number of suppliers when exogeneous random shocks are large. For empirical evidence see Boudreau, Lacetera and Lakhani (2011).

[25] See also Konrad (2014) for a variant of Erat and Krishnan's model where first best is restored if the tie-breaking is decided via costly competition (for example lobbying) as opposed to randomly.

benefits of convergence emphasized by Callander and Harstad (2015). It would be interesting to see whether and how centralization would help to resolve this trade off. In a related setting, Bonatti and Rantakari (2015) consider a setting where two agents choose which project to develop. To successfully develop a project, an agent exerts effort until a success occurs. For a successful project to be adopted (and yield a positive payoff) both agents have to consent to the adoption. By assumption, the agents have opposite preferences over the set of projects. The agents have an incentive to pursue extreme projects (which they like the most) but the veto power of the other agent forces them to compromise. As in Callander and Harstad (2015) the success of one approach is unrelated to the success of any other approach. This removes the option value of diversity that we identify in our paper.

7 Conclusions and Discussion

The ideal approach to solving an innovation problem is usually unknown to suppliers and buyers. Our paper investigates the implications of this uncertainty for contest design. Under very general conditions, it is socially optimal for suppliers to take diverse research approaches, and the social optimum can be obtained with both bonus tournaments and auction mechanisms. Inducing diversity of approaches to innovation is costly. To reduce supplier rents, the buyer may want to induce suboptimal diversification. Bonus tournaments are in the set of optimal contests under quite general conditions. The difference between the bonus and the low price provides incentives for sellers to diversify, which allows the buyer to fine-tune the amount of diversity induced. At the same time, bonus tournaments minimize the power of suppliers to exploit their quality advantage. The non-convexity of the price set is decisive for this feature.

Beyond innovation contests, our model can be used to analyze how institutions affect the incentives for experimentation when the optimal approach to solving a given problem is not known. A particularly promising application would be to think of our model as capturing product choice in markets with a unit mass of homogeneous buyers, each of which has unit demand. We can then interpret the uncertainty about the ideal state in two ways. First, it may reflect uncertainty about the buyers' taste. Second, it may capture an "engineering uncertainty" where the suppliers know what the buyers would like, but are uncertain about how to achieve this. Either way, the rules of the contest translate directly into a description of the regulatory constraints in a market environment. For instance, the auction corresponds to an unregulated market environment where suppliers choose products under uncertainty about the preferred product and charge prices once qualities are realized. The fixed prize tournament can be interpreted as a regulated market where prices are fixed ex ante: The prize is then the profit that the firm earns in the market from selling at the regulated price to a unit mass of consumers. Similarly, auctions with price ceilings have a natural interpretation as regulated markets with price caps.

Our results suggest that an unregulated market maximizes expected total surplus, whereas a regulated market maximizes the expected consumer surplus. The unregulated market gives incentives for firms to diversify, but leaves them with market power. The trade-off resembles the one between ex-ante incentives and ex-post monopoly power in the innovation literature. In our case, however, the higher expected quality from the unregulated market does not result from higher innovation incentives at the individual firm level, but rather from the higher diversification incentives at the market level. Price caps strike a balance between the goals of maximizing consumer surplus and total surplus. Bonus tournaments would correspond to a regulated environment where firms can select between offering two different prices depending on the realized quality levels. Our analysis suggests that, in some markets, such bonus tournaments may even be better for consumers than full price regulation. These simple considerations clearly have limitations resulting from the rather special market environment. However, the arguments suggest that the contest approach may potentially be valuable to analyze product innovations (or product selection) in market environments. A full analysis of this topic is left for future research.

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8 Appendix

8.1 Basics

In the following, we introduce some notation that we use throughout the Appendix. We also formulate the restrictions implied by subgame perfection.

8.1.1 Notation

We consistently use subscripts B for buyers, $i = 1, 2$ for suppliers and T for "total" (buyers plus suppliers). Superscripts such as fpt for fixed-price tournament, bt for bonus tournament or a for auction refer to the contest $\mathcal{P}$ under consideration. We will drop these superscripts whenever there is no danger of confusion.

1. $p_i\left(q_i, q_j\right) \in \mathcal{P}^{[\Psi-b,\Psi]^2}$ is a *price strategy function.*[26]

2. $\pi_i\left(p_i, p_j \middle| q_i, q_j\right)$ is the realized revenue that supplier i earns with prices p_1 and p_2, conditional on qualities q_1 and q_2, assuming that the buyer chooses the i sequentially rationally, i.e., the i that maximizes $q_i - p_i$ in contest $\mathcal{P}$.[27]

3. $\widehat{\Pi}_i\left(v_i, v_j, p_i, p_j\right)$ is the expectation over $\pi_i\left(p_i, p_j \middle| q_i, q_j\right)$ when suppliers choose v_1, v_2, $p_1()$ and $p_2()$, where the expectation is taken over all pairs of quality realizations for given (v_1, v_2).

4. $\Pi_i^{\mathcal{P}}\left(v_i, v_j\right) = \widehat{\Pi}_i\left(v_i, v_j, p_i, p_j\right)$, where $p_i()$ and $p_j()$ are the subgame equilibria for the contest $\mathcal{P}$ as in Lemma 2, is the *(expected) revenue* of supplier i.

5. $S_i^{\mathcal{P}}\left(v_i, v_j\right) = \Pi_i^{\mathcal{P}}\left(v_i, v_j\right) + t - C$ is the *(expected) surplus* of supplier i.

6. $S_B^{\mathcal{P}}\left(v_i, v_j\right) = E_\sigma\left[\max\left\{q\left(v_1, \sigma\right), q\left(v_2, \sigma\right)\right\}\right] - \Pi_1^{\mathcal{P}}\left(v_i, v_j\right) - \Pi_2^{\mathcal{P}}\left(v_i, v_j\right) - 2t$ is the *(expected) surplus* of the buyer.

8.1.2 Subgame-Perfect Equilibrium

A subgame-perfect equilibrium of the innovation contest given by $\mathcal{P}$ consists of supplier strategies $s_i = (v_i, p_i) \in [0,1] \times \mathcal{P}^{[\Psi-b,\Psi]^2}$ and buyer strategies $\nu \in \{v_1, v_2\}^{(\mathcal{P}\times[\Psi-b,\Psi])^2}$ such that:

(DC1) ν_1 and v_2 are sequentially rational.

(DC2) $\pi_i\left(p_i\left(q_i, q_j\right), p_j\left(q_j, q_i\right) \middle| q_i, q_j\right) \geq \pi_i\left(p_i', p_j\left(q_j, q_i\right) | q_i, q_j\right)$ for all $p_i' \in \mathcal{P}$,$(q_i, q_j) \in [\Psi - b, \Psi]^2$ (sequential rationality of supplier i)

(DC3) $\widehat{\Pi}_i\left(v_i, v_j, p_i\left(q_i, q_j\right), p_j\left(q_j, q_i\right)\right) \geq \widehat{\Pi}_i\left(v_i', v_j, \widetilde{p}_i\left(q_i, q_j\right), p_j\left(q_j, q_i\right)\right)$ for all $v_i' \in [0,1]$ and all $\widetilde{p}_i\left(q_i, q_j\right) \in \mathcal{P}^{[\Psi-b,\Psi]\times[\Psi-b,\Psi]}$ (best-response condition for supplier i).

[26] For sets X and Y, Y^X is the set of all mappings from X to Y.

[27] When $q_1 - p_1 = q_2 - p_2$, we appeal to tie-breaking rule (T1) below.

8.2 Proofs of Auxiliary Results

8.2.1 Proof of Lemma 1

Suppose, without loss of generality, that $v_1 \leq v_2$. The total surplus is

$$\begin{aligned} S_T(v_1, v_2) - 2C &= \int_0^1 \max\{q(v_1,\sigma), q(v_2,\sigma)\} dF(\sigma) - 2C = \\ & \Psi - b\left(\begin{array}{c} \int\limits_0^{v_1} (v_1 - \sigma)\, dF(\sigma) + \int\limits_{v_1}^{(v_1+v_2)/2} (\sigma - v_1)\, dF(\sigma) + \\ \int\limits_{(v_1+v_2)/2}^{v_2} (v_2 - \sigma)\, dF(\sigma) + \int\limits_{v_2}^{1} (\sigma - v_2)\, dF(\sigma) \end{array} \right) - 2C. \end{aligned}$$

This is a continuous function with a compact domain, hence it attains the maximum. Note that

$$\frac{\partial S_T(v_1, v_2)}{\partial v_1} = b\left(-2F(v_1) + F((v_1+v_2)/2)\right) \quad (1)$$

$$\frac{\partial S_T(v_1, v_2)}{\partial v_2} = b\left(1 - 2F(v_2) + F((v_1+v_2)/2)\right). \quad (2)$$

(1) and (2) imply that there are no boundary optima. To see this, first note that $\frac{\partial S_T(0,v_2)}{\partial v_1} > 0 \forall v_2 > 0$ and $\frac{\partial S_T(v_1,1)}{\partial v_2} < 0 \forall v_1 < 1$. Moreover $(v_1, v_2) = (0,0)$ and $(1,1)$ are both dominated by $(1/2, 1/2)$. Thus, the optimum must satisfy

$$-2F(v_1) + F((v_1+v_2)/2) = 0 \quad (3)$$

$$1 - 2F(v_2) + F((v_1+v_2)/2) = 0. \quad (4)$$

Together these conditions imply $F(v_2^*) = 1/2 + F(v_1^*)$.

For $v_1 \in [0, 1/2]$, let $g(v_1) = F^{-1}\left(F(v_1) + \frac{1}{2}\right)$. F^{-1} is well-defined because of (A1)(iii). Inserting $v_2 = g(v_1)$ in (3) and (4), the first-order conditions hold for $(v_1, v_2) = (v_1, g(v_1))$ if

$$v_1 = F^{-1}\left(\frac{F((v_1 + g(v_1))/2)}{2}\right). \quad (5)$$

(5) has at least one solution $v_1^* \in (0, 1/2)$. This holds because both sides of (5) are strictly increasing, and the r.h.s. is positive for $v_1 = 0$ and strictly less than $1/2$ for $v_1 = 1/2$. Now consider $(v_1^*, v_2^*) = (v_1^*, g(v_1^*))$ such that $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Thus $F(v_2^*) = F(v_1^*) + 1/2$. Moreover, symmetry implies $v_1^* + v_2^* = 1$ and thus the r.h.s. of (5) is $F^{-1}\left(\frac{1}{4}\right)$, so that the first-order condition holds for (v_1^*, v_2^*).

Finally, consider the Hessian matrix

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial^2 S_T}{\partial v_1^2} & \frac{\partial^2 S_T}{\partial v_1 \partial v_2} \\ \frac{\partial^2 S_T}{\partial v_1 \partial v_2} & \frac{\partial^2 S_T}{\partial v_2^2} \end{bmatrix} \\ &= \begin{bmatrix} -2f(v_1) + \frac{1}{2}f((v_1+v_2)/2) & \frac{1}{2}f((v_1+v_2)/2) \\ \frac{1}{2}f((v_1+v_2)/2) & -2f(v_2) + \frac{1}{2}f((v_1+v_2)/2) \end{bmatrix}. \end{aligned}$$

First, H is negative definite at (v_1^*, v_2^*) if and only if $f(1/2) < 2f(v_1^*)$. To see this, note that $f(v_1^*) = f(v_2^*)$ and $f((v_1^* + v_2^*)/2) = f(1/2)$. Hence,

$$-2f(v_1^*) + \frac{1}{2} f((v_1^* + v_2^*)/2) = -2f(v_1^*) + \frac{1}{2} f(1/2) < 0 \Leftrightarrow f(1/2) < 4f(v_1^*).$$

In addition,

$$|H| = 4f(v_1^*) f(v_2^*) - (f(v_1^*) + f(v_2^*)) f((v_1^* + v_2^*)/2) = 4f(v_1^*)^2 - 2f(v_1^*) f(1/2).$$

This condition holds if and only if $f(1/2) < 2f(v_1^*)$, which holds by (A1)(iv).

Second, H is negative definite $\forall (v_1, v_2)$ if $f(1/2) < 2f(0)$. To see this, note that $f(v)$ is minimized at $v = 0$ and maximized at $v = 1/2$. Hence, $f(1/2) < 2f(0) < 4f(0)$ implies

$$-2f(v_i) + \frac{1}{2} f\left(\frac{v_1 + v_2}{2}\right) \leq -2f(0) + \frac{1}{2} f\left(\frac{1}{2}\right) < 0 \; \forall i \in \{1, 2\}.$$

and

$$|H| = f(v_1)\left(2f(v_2) - f\left(\frac{v_1 + v_2}{2}\right)\right) + f(v_2)\left(2f(v_1) - f\left(\frac{v_1 + v_2}{2}\right)\right) > 0.$$

Therefore, $f(1/2) < 2f(0)$, which holds by (A1)(iv), is a sufficient condition for (v_1^*, v_2^*) to be the unique global optimum.

8.2.2 Proof of Lemma 2

Consider the equilibrium for the subgame defined by (v_1, v_2, σ) and the resulting quality vector (q_1, q_2). If $q_1 = q_2$, the standard Bertrand logic implies that $(\overline{p}(\sigma), \overline{p}(\sigma)) = (\underline{P}, \underline{P})$ is the unique equilibrium. Now suppose $q_i > q_j$. Clearly, the suggested strategy profile is a subgame equilibrium. To see that i must bid $\overline{p}(\sigma)$ in equilibrium, first suppose $p_i > \overline{p}(\sigma)$. If $p_i > p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier j wins. By setting $p_i = \overline{p}(\sigma) \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier i can ensure that he wins, which is a profitable deviation by (T2). If $p_i > \overline{p}(\sigma)$ and $p_i \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier i wins. By setting $p_j = \underline{P}$, supplier j can profitably deviate. If $p_i < \overline{p}(\sigma)$, supplier i can deviate upwards to $\overline{p}(\sigma)$. He then still wins by (T1), and revenues are higher.

8.2.3 Proof of Lemma 3

(i) The result is trivial for $v_1 = v_2$. For $v_1 < v_2$, we show that supplier 1 can profitably deviate to some $v_1' > v_1$ if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$. Before the deviation, by Lemma 2, if $\sigma \in [0, v_1]$, supplier 1 wins and $\overline{p}(\sigma) < \Delta q(v_1, v_2) + \underline{P}$. By continuity, $\exists \; v_1' \in (v_1, v_2]$ such that $\overline{p}(\sigma) < \Delta q(v_1', v_2) + \underline{P} < \Delta q(v_1, v_2) + \underline{P}$. By deviating to v_1', supplier 1 wins whenever $\sigma < (v_1' + v_2)/2$ rather than when $\sigma < (v_1 + v_2)/2$. The set of states in which supplier 1 wins after the deviation thus is a strict superset of the set of states in which the supplier wins before the deviation. For $\sigma \in [0, v_1]$, the price is unaffected. For $\sigma \in (v_1, (v_1' + v_2)/2]$, the price is at least as high as before the deviation. Thus, v_1' is a profitable deviation by (T2). (ii) follows directly from Lemmas 2 and 3 (i).

8.3 Proofs of Main Results

8.3.1 Proof of Proposition 1

Sequential rationality of $p_i()$ follows from Lemma 2. We now show that $(v_1, p_1())$ is a best response of supplier 1 to $(v_2, p_2())$; the argument for supplier 2 is analogous. For $A = 0$, only $(v_1, v_2) = (1/2, 1/2)$ satisfies the above conditions. Thus, the statement for $A = 0$ will follow from Proposition 3(ii). If $v_1 < v_2$, $\Delta q(v_1, v_2) > 0$, and the probability that supplier 1 wins with a positive prize is $F(v_1)$. Deviating to $v_1' < v_1$ is not profitable, because the winning probability falls to $F(\widehat{v}_1)$, with $\widehat{v}_1 < v_1$ implicitly defined by $q(v_1', \widehat{v}_1) - q(v_2, \widehat{v}_1) = \Delta q(v_1, v_2)$, and the prize does not rise. It is not profitable to deviate to $v_1'' \in (v_1, \widetilde{v})$, where $\widetilde{v} = \min(2v_2 - v_1, 1) \geq 1/2$: For such deviations, $\Delta q(v_1'', v_2) < \Delta q(\widetilde{v}, v_2) \leq \Delta q(v_1, v_2)\, \forall \sigma$, so that the probability of winning a positive prize is 0. Finally, if $\widetilde{v} < 1$, deviating to $v_1''' \in [\widetilde{v}, 1]$ is not profitable, because $\widetilde{v} \geq 1/2 + v_2 - v_1$ implies $1 - \widetilde{v} \leq 1/2 - (v_2 - v_1) \leq v_2 - (v_2 - v_1) = v_1$ and therefore, by symmetry of the state distribution, $1 - F(v_1''') \leq 1 - F(\widetilde{v}) \leq F(v_1)$. By analogous arguments, there are no profitable deviations for supplier 2.

By Lemma 1, the social optimal satisfies $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Clearly, it must be that $0 < v_1^* \leq 1/2 \leq v_2^* < 1$, and the social optimum can be implemented.

8.3.2 Proof of Theorem 1

The buyer optimally chooses $(v_1, v_2, p_1, p_2, \mathcal{P}, t) \in [0,1]^2 \times \mathcal{P}^{[\Psi - b, \Psi]^2} \times \mathcal{I}(\mathbb{R}^+) \times [0, +\infty)$ so as to maximize

$$S_T(v_1, v_2) - \widehat{\Pi}_1(v_1, v_2, p_1, p_2) - \widehat{\Pi}_2(v_1, v_2, p_1, p_2) - 2t$$

such that, for all $i \in \{1,2\}$ and $j \neq i$, (DC1)-(DC3) hold and

$$\widehat{\Pi}_i(v_i, v_j, p_i, p_j) + t - C \geq 0 \text{ for all } i, j \in \{1,2\} \text{ and } i \neq j. \tag{6}$$

(i) The statement follows from two lemmas. Lemma 6 shows that allocations maximizing buyer surplus satisfy the conditions of Proposition 1 and can thus be implemented by a bonus tournament. Lemma 7 shows that implementation requires lower expected transfer than any alternative; hence buyer surplus is maximal.

Lemma 6 *If (v_1^B, v_2^B, p_1, p_2) is an equilibrium of a contest that maximizes buyer surplus, then $0 < v_1^B \leq \frac{1}{2} \leq v_2^B < 1$.*

We prove this lemma in two steps.

Step 1: *If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is an equilibrium where w.l.o.g. $v_1^B \leq v_2^B$, then $v_1^B \leq 1/2 \leq v_2^B$.*

Proof: We will show that $v_1 \leq 1/2 \leq v_2$ must hold in any contest equilibrium. Suppose, to the contrary, that $v_1 \leq v_2 < 1/2$. The case that $1/2 < v_1 \leq v_2$ follows analogously. Let p_1, p_2 be the associated pricing strategies. Then, the expected revenue of supplier 1 is $\Pi_1(v_1, v_2) = \int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$. Consider the deviation $v_1' = 2v_2 - v_1 < 1$ with the same pricing function. Supplier 1 now wins whenever $\sigma > (v_2 + v_1')/2$. We can write the expected revenue as $\Pi_1(v_1', v_2) = \int_{\frac{v_1'+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma) + \int_{2v_2}^{1} p_1(q_1(\sigma), q_2(\sigma))\, d\sigma$. Clearly, $(v_1 + v_2)/2 = 2v_2 - \frac{v_1'+v_2}{2}$. Moreover, there exists a bijective mapping $[0, (v_1 + v_2)/2] \to [(v_1' + v_2)/2, 2v_2]$; $\sigma' \mapsto \sigma''$ such that $q(v_1, \sigma') - q(v_2, \sigma') = q(v_1', \sigma'') - q(v_2, \sigma'')$ and $f(\sigma') \leq$

$f(\sigma'')$. Thus $\int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma))\,dF(\sigma) \le \int_{\frac{v_1'+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma))\,dF(\sigma)$. As a result, $\Pi_1(v_1, v_2) \le \Pi_1(v_1', v_2)$ and v_1' leads to strictly higher probability of winning, hence v_1' is a profitable deviation.[28] Thus, $v_1 \le 1/2 \le v_2$ must hold in any equilibrium; in particular, therefore $v_1^B \le 1/2 \le v_2^B$.

Step 2: *If* $\left(v_1^B, v_2^B, p_1^B, p_2^B\right)$ *is an equilibrium maximizing buyer surplus, then* $0 < v_i^B < 1$ *for* $i \in \{1,2\}$.

Proof: By Step 1, we know that $v_1 \le 1/2 \le v_2$. Suppose $v_1^B = 0$ and $v_2^B = 1$. We will distinguish two cases, $C = 0$ and $C > 0$. First suppose $C = 0$. By single-peakedness (A1), $v_1 = v_2 = 1/2$ results in weakly higher total surplus than $\left(v_1^B, v_2^B\right)$. As the allocation $(v_1, v_2) = (1/2, 1/2)$ can be implemented with an FPT and $A = 2C$ by Proposition 3(ii), the buyer would be strictly better off than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$ where the suppliers earn positive surplus. Finally, observe that $v_1^B = 0$ and $v_2^B = 1$ cannot be implemented so that the suppliers earn zero surplus, as the suppliers could increase their probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. Next suppose $C > 0$. There exists some small ε such that $S_T\left(v_1^B = 0, v_2^B = 1\right) < S_T(\varepsilon, 1-\varepsilon)$ and $F(\varepsilon)\Delta q(\varepsilon, 1-\varepsilon) < C$. But then a bonus tournament with subsidy $t' = C - F(\varepsilon)\Delta q(\varepsilon, 1-\varepsilon)$, and $\mathcal{P} = \{\Delta q(\varepsilon, 1-\varepsilon), 0\}$ implements $(\varepsilon, 1-\varepsilon)$, achieves higher total surplus, and the supplier surplus not higher than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$. Hence, the buyer surplus is higher, which is a contradiction.

Next suppose $v_1 = 0$ and $v_2 < 1$ (the case that $v_1 > 0$ and $v_2 = 1$ follows analogously). By Lemma 2, the revenue is $\Pi_1(0, v_2) = \int_0^{\frac{v_2}{2}} \bar{p}(q_1(\sigma), q_2(\sigma))\,dF(\sigma)$ for supplier 1 and $\Pi_2(v_2, 0) = \int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(\sigma), q_1(\sigma))\,dF(\sigma) + \int_{v_2}^{1} \bar{p}(q_2(\sigma), q_1(\sigma))\,dF(\sigma)$ for supplier 2. Moreover, $\Pi_1(0, v_2) > 0$, because otherwise supplier 1 could increase his probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. By single-peakedness (A1) it holds $\int_0^{\frac{v_2}{2}} \bar{p}(q_1(\sigma), q_2(\sigma))\,dF(\sigma) \le \int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(\sigma), q_1(\sigma))\,dF(\sigma)$. Suppose that this equilibrium is implemented with transfers t such that $t + \Pi_1(0, v_2) \ge C$. This implies $t + \Pi_2(v_2, 0) > C$. Further, using (1), $dS_T\left(v_1^B, v_2^B\right)/dv_1^B\big|_{v_1^B=0} = bF(v_2/2) > 0$, so that there exists some $\bar{\varepsilon} > 0$ such that $S_T\left(\varepsilon, v_2^B\right) > S_T\left(0, v_2^B\right)$ for every $\varepsilon \in (0, \bar{\varepsilon})$. Fix ε such that $F(\varepsilon)\Delta q(\varepsilon, v_2) \le \Pi_1(0, v_2)$ and $F(\varepsilon) < 1 - F(v_2)$. Let $t' = t + \Pi_1(0, v_2) - F(\varepsilon)\Delta q(\varepsilon, v_2)$. Now consider a bonus tournament with subsidy t' and $\mathcal{P} = \{\Delta q(\varepsilon, v_2), 0\}$. By Proposition 1, this bonus tournament will implement (ε, v_2) if the participation constraint is met. This condition holds for both suppliers, because $t' + (1 - F(v_2))\Delta q(\varepsilon, v_2) > t' + F(\varepsilon)\Delta q(\varepsilon, v_2) \ge C$. Compared to the original situation with $v_1 = 0$ and $v_2 < 1$, the rent of supplier 1 is unchanged, but the rent of supplier 2 decreases since $\int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(\sigma), q_1(\sigma))\,dF(\sigma) + t > t'$ and $\int_{v_2}^{1} \bar{p}(q_2(\sigma), q_1(\sigma))\,dF(\sigma) > (1 - F(v_2))\Delta q(\varepsilon, v_2)$. Since the total surplus increases and the sellers' surplus decreases, the buyer's surplus must increase. Therefore, the bonus tournament that implements (ε, v_2) increases the buyer surplus, which is a contradiction.

Lemma 7 *If* $\left(v_1^B, v_2^B, p_1^B, p_2^B\right)$ *is an equilibrium of a contest maximizing buyer surplus, then it can be implemented by a contest with* $\mathcal{P} = \{A, 0\}$.

Proof: From Proposition 1 and Lemma 6, we know that the bonus tournament with

[28] Given the tie-breaking rule T2, this is even true for $p = 0$.

$A=\Delta q\left(v_1^B,v_2^B\right)$ implements $\left(v_1^B,v_2^B\right)$. It remains to be shown that the buyer cannot implement $\left(v_1^B,v_2^B\right)$ with lower expected total transfers with any other contest. First, suppose that $v_1^B+v_2^B=1$. By Lemmas 2 and 3, in any contest that implements $\left(v_1^B,v_2^B\right)$ the price paid by the buyer is exactly $\Delta q(v_1^B,v_2^B)+\underline{P}$ if $\sigma\in[0,v_1^B]\cup[v_2^B,1]$ and it is at least 0 if $\sigma\in\left(v_1^B,v_2^B\right)$. Thus, if $\Delta q(v_1^B,v_2^B)F(v_1^B)>C$, a bonus tournament implements $\left(v_1^B,v_2^B\right)$ with the lowest possible expected total transfers. If $\Delta q(v_1^B,v_2^B)F(v_1^B)\le C$, a bonus tournament with an appropriate t implements $\left(v_1^B,v_2^B\right)$ with zero expected supplier surplus. Next, consider an arbitrary contest implementing $\left(v_1^B,v_2^B\right)$ with $v_1^B+v_2^B<1$ with subsidy t (the case $v_1^B+v_2^B>1$ is analogous). The surplus of supplier 1 is then $S_1=\Delta q(v_1^B,v_2^B)F(v_1^B)+\int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}}\bar{p}\left(q_1\left(\sigma\right),q_2\left(\sigma\right)\right)dF\left(\sigma\right)+t-C$, and for supplier 2 it is $S_2=\Delta q(v_1^B,v_2^B)(1-F(v_2^B))+\int_{\frac{v_1^B+v_2^B}{2}}^{v_2}\bar{p}\left(q_1\left(\sigma\right),q_2\left(\sigma\right)\right)dF\left(\sigma\right)+t-C$. By similar arguments as in Lemma 6, $\Delta q(v_1^B,v_2^B)F(v_1^B)<\Delta q(v_1^B,v_2^B)(1-F(v_2^B))$ and $\int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}}\bar{p}\left(q_1\left(\sigma\right),q_2\left(\sigma\right)\right)dF\left(\sigma\right)\le\int_{\frac{v_1^B+v_2^B}{2}}^{v_2}\bar{p}\left(q_1\left(\sigma\right),q_2\left(\sigma\right)\right)dF\left(\sigma\right)$. Now consider a bonus tournament with $\mathcal{P}=\{\Delta q\left(v_1^B,v_2^B\right),0\}$ and $t'=\int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}}\bar{p}\left(q_1\left(\sigma\right),q_2\left(\sigma\right)\right)dF\left(\sigma\right)+t$. The surplus of supplier 1 now becomes $S_1'=S_1$ by construction. On the other hand, $S_2'\le S_2$, but $S_2'>S_1'$. Thus, the proposed bonus tournament implements $\left(v_1^B,v_2^B\right)$ with lowest possible net supplier surplus, which implies that the buyer surplus is maximized.

(ii) Suppose $C\ge F\left(v_1^*\right)\Delta q\left(v_1^*,v_2^*\right)$. From Proposition 1 we know that for the proposed $\mathcal{P}=\{A,0\}$, (v_1^*,v_2^*) emerges in equilibrium; and the result also gives the pricing strategies p_1 and p_2. For $t=C-F\left(v_1^*\right)\Delta q\left(v_1^*,v_2^*\right)$, the buyer surplus in the proposed equilibrium is

$$\begin{aligned}&S_T\left(v_1^*,v_2^*\right)-\Pi_1\left(v_1^*,v_2^*\right)-\Pi_2\left(v_1^*,v_2^*\right)+2t\\=\;&S_T\left(v_1^*,v_2^*\right)-2F\left(v_1^*\right)\Delta q\left(v_1^*,v_2^*\right)+2\left(F\left(v_1^*\right)\Delta q\left(v_1^*,v_2^*\right)-C\right)=S_T\left(v_1^*,v_2^*\right)-2C\end{aligned}\tag{7}$$

This is the highest surplus that the buyer can achieve without violating the suppliers' participation constraints.

(iii) Suppose $C<F\left(v_1^*\right)\Delta q\left(v_1^*,v_2^*\right)$. The proof for this case relies on the fact that implementation with minimal revenues uses bonus tournaments. It shows that $\left(v_1^B,v_2^B\right)$ must satisfy $v_1^B+v_2^B=1$. Among all the bonus tournaments implementing (v_1,v_2) with $v_1\le v_2$ and $v_1+v_2=1$, the buyer has highest surplus (ignoring participation constraint) at $(1/2,1/2)$. Using these facts, the proof shows that the buyer always chooses the minimal value of the subsidy t, and she just implements enough diversity so that the suppliers (who benefit from some diversity) break even on expectation.

Step 1: *The outcome of an optimal contest can be implemented by* $\mathcal{P}=\{A,0\}$ *for some* $A\ge 0$.

This follows from Part (i).

Step 2: *In an optimal contest* $v_1^B+v_2^B=1$**.**

Consider any (v_1,v_2) such that $v_1+v_2<1$. We show that $(v_1,v_2)\neq\left(v_1^B,v_2^B\right)$; the case $v_1+v_2>1$ follows analogously. By Step 1, the optimal outcome can be implemented by some $\mathcal{P}=\{A,0\}$ and $t\ge 0$. The equilibrium values of p_i in this contest are zero whenever $\sigma\in(v_1,v_2)$. Hence, the participation constraint for supplier 1 implies that $F\left(v_1\right)A+t\ge C$; thus $v_1+v_2<1$ implies $\left(1-F\left(v_2\right)\right)A+t>C$. Now suppose the buyer implements $\left(v_1+\varepsilon,v_2+\varepsilon\right)$, where ε is sufficiently small. We know that $(v_1+\varepsilon, v_2+\varepsilon)$ can also be implemented with $\mathcal{P}=\{A,0\}$. Thus, we can write the buyer surplus as

$$S_B(\varepsilon) = S_T(v_1+\varepsilon, v_2+\varepsilon) - F(v_1+\varepsilon)A - (1-F(v_2+\varepsilon))A - 2t$$

for $\varepsilon \geq 0$. Thus

$$\frac{dS_B(\varepsilon)}{d\varepsilon} = dS_T(v_1+\varepsilon, v_2+\varepsilon)/d\varepsilon - Af(v_1+\varepsilon) + Af(v_2+\varepsilon).$$

Since $v_1+v_2<1$, single-peakedness and symmetry (A1) imply $f(v_1+\varepsilon)<f(v_2+\varepsilon)$. Thus $dS_B(\varepsilon)/d\varepsilon > dS_T(v_1+\varepsilon, v_2+\varepsilon)/d\varepsilon$. We will show that $dS_T(v_1+\varepsilon, v_2+\varepsilon)/d\varepsilon > 0$; because $F(v_1+\varepsilon)A+t>C$ and (for sufficiently small ε) $(1-F(v_2))A+t \geq C$, the buyer will thus be better off implementing $(v_1+\varepsilon, v_2+\varepsilon)$ than (v_1,v_2). Maximizing total surplus is equivalent to minimizing the expected distance

$$\begin{aligned} D(v_1+\varepsilon, v_2+\varepsilon) = &\int_0^{v_1+\varepsilon}(v_1+\varepsilon-\sigma)f(\sigma)d\sigma + \int_{v_1+\varepsilon}^{\frac{v_1+v_2}{2}+\varepsilon}(\sigma-v_1-\varepsilon)f(\sigma)d\sigma \\ &+\int_{\frac{v_1+v_2}{2}+\varepsilon}^{v_2+\varepsilon}(v_2+\varepsilon-\sigma)f(\sigma)d\sigma + \int_{v_2+\varepsilon}^{1}(\sigma-v_2-\varepsilon)f(\sigma)d\sigma. \end{aligned}$$

From this we obtain

$$\begin{aligned} \frac{dD(v_1+\varepsilon, v_2+\varepsilon)}{d\varepsilon} &= \int_0^{v_1+\varepsilon} f(\sigma)d\sigma - \int_{v_1+\varepsilon}^{\frac{v_1+v_2}{2}+\varepsilon} f(\sigma)d\sigma + \int_{\frac{v_1+v_2}{2}+\varepsilon}^{v_2+\varepsilon} f(\sigma)d\sigma - \int_{v_2+\varepsilon}^{1} f(\sigma)d\sigma \\ &= 2F(v_1+\varepsilon) + 2(F(v_2+\varepsilon)) - 2F\left(\frac{v_1+v_2}{2}+\varepsilon\right) - 1. \end{aligned}$$

We will show that this expression is negative for $v_1+v_2<1$ and sufficiently small ε. To see this, fix any v_2 such that $1/2 \leq v_2 < 1$. Note that $h(v_1,v_2) := \left.\frac{dD(v_1+\varepsilon,v_2+\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = 0$ for $v_1 = 1-v_2$. Furthermore

$$\frac{\partial h}{\partial v_1} = 2f(v_1) - f\left(\frac{v_1+v_2}{2}\right) > 0,$$

were the last inequality follows by (A1)(iv). Thus, $v_1+v_2<1$ implies $2F(v_1)+2(F(v_2)) - 2F((v_1+v_2)/2)-1<0$ and thus $dD(v_1+\varepsilon,v_2+\varepsilon)/d\varepsilon<0$ for small enough ε. This in turn implies that $S_T(v_1+\varepsilon,v_2+\varepsilon)$ increases in ε so that buyer surplus also increases in ε.

Step 3: *For $v_1 \in [0,1/2]$ and for fixed t, buyer surplus $S_B^{bt}(v_1, 1-v_1)$ increases in v_1.*

For any $\sigma \in [0,v_1]$, buyer surplus equals $q_2(1-v_1,\sigma)$, which increases in v_1. For any $\sigma \in (v_1, 1/2]$, buyer surplus equals $q_1(v_1,\sigma)$; for a small marginal change, the expected payoff from states on $(v_1,1/2]$ thus also increases in v_1. By similar arguments, buyer surplus increases for any $\sigma \in (1/2, 1-v_1]$ and any $\sigma \in (1-v_1, 1]$.

Step 4: *Suppose $C < F(v_1^*)\Delta q(v_1^*, v_2^*)$. There exists $v_1 \in (v_1^*, 1/2]$ such that $C = F(v_1)\Delta q(v_1, 1-v_1)$.*

The result follows from $F(1/2)\Delta q(1/2,1/2) = 0$ because $F(v_1)\Delta q(v_1, 1-v_1)$ is a continuous function.

Step 5: Fix $\widetilde{v}_1 = \max_{[0,1/2]} v_1$ s.t. $C = F(v_1)\Delta q(v_1, 1-v_1)$. Then $v_1^B \geq \widetilde{v}_1$.

Suppose not. According to Step 1, $\left(v_1^B, v_2^B\right)$ can be implemented in a bonus tournament with a subsidy $t \geq 0$. By Step 4, $\widetilde{v}_1$ exists. Suppose $v_1^B < \widetilde{v}_1$. By Step 3, moving from $\left(v_1^B, v_2^B\right)$ to $(\widetilde{v}_1, 1 - \widetilde{v}_1)$ the buyer could increase her payoff while the participation constraint would remain satisfied.

Step 6: *For $v_2 = 1 - v_1$ and $v_1 > v_1^*$, $S_T\left(v_1, 1 - v_1\right)$ decreases in v_1.*

Using the same argument as in Step 2, the derivative of the expected distance is

$$\frac{dD\left(v_1, 1 - v_1\right)}{dv_1} = \int_0^{v_1} f\left(\sigma\right) d\sigma - \int_{v_1}^{\frac{1}{2}} f\left(\sigma\right) d\sigma - \int_{\frac{1}{2}}^{1-v_1} f\left(\sigma\right) d\sigma + \int_{1-v_1}^{1} f\left(\sigma\right) d\sigma = 4F(v_1) - 1$$

This function is monotonic and positive for all $v_1 > v_1^*$. Hence the total expected distance increases in v_1, and the total expected surplus decreases.

Step 7: Fix $\widetilde{v}_1$ as in Step 5. Then $v_1^B \leq \widetilde{v}_1$.

Suppose not. By Step 1, $\left(v_1^B, v_2^B\right)$ can be implemented in a bonus tournament with a subsidy $t \geq 0$. Suppose $v_1^B > \widetilde{v}_1$. By Step 6, moving from $\left(v_1^B, v_2^B\right)$ to $(\widetilde{v}_1, 1 - \widetilde{v}_1)$ increases total surplus. Since $(\widetilde{v}_1, 1 - \widetilde{v}_1)$ can be implemented with $t' = 0$ because $C = F\left(\widetilde{v}_1\right) \Delta q\left(\widetilde{v}_1, 1 - \widetilde{v}_1\right)$, buyer surplus increases.

Step 8: *The optimal $\left(v_1^B, v_2^B\right)$ for the buyer (i) satisfies $v_1^B = \widetilde{v}_1$ and $v_2^B = 1 - v_1^B$. It can implemented with (ii) a bonus tournament such that $t = 0$.*

The first part of the statement follows by combining Steps 5 and 7. The second part follows from (i) and the fact that the suggested contest implements $(\widetilde{v}_1, 1 - \widetilde{v}_1)$ with minimal subsidies required to satisfy the participation constraint.

8.3.3 Proof of Proposition 2

Proof. As $F\left(v_1^*\right) \Delta q(v_i^*, v_j^*) < C$ by assumption, both prizes are positive. For firm 1, the expected profit of following the candidate equilibrium is $\Pi_1\left(v_1^*, v_2^*\right) = F\left(v_1^*\right) A + (1/2 - F\left(v_1^*\right)) a - C$. Inserting the values of A and a and $F\left(v_1^*\right) = 1/4$, $\Pi_1\left(v_1^*, v_2^*\right) = 0$. By symmetry, both suppliers break even on expectation. Thus, the suggested allocation maximizes total surplus, with full rent appropriation by the buyer. It thus suffices to show that $\{A, a\}$ implements (v_1^*, v_2^*). Consider supplier 1. First, any deviation $v_1 = v_2^* + \varepsilon$ is dominated by $v_1' = v_2^* - \varepsilon$. Next, a deviation to $v_1' < v_1^*$ cannot increase expected supplier profit, as the probability of winning decreases and the price charged in any state of the world does not increase. Thus, the only remaining case is a deviation to $v_1' \in (v_1^*, v_2^*]$. The expected gross profit can be written as $\Pi_1\left(v_1', v_2^*\right) = aF\left(\left(v_1' + v_2^*\right)/2\right)$. This is clearly increasing in v_1' and the profit of supplier 1 is at most $\Pi_1\left(v_1', v_2^*\right) = aF\left(v_2^*\right)$. The expected profit of following the candidate equilibrium is $\Pi_1\left(v_1^*, v_2^*\right) = F\left(v_1^*\right) A + \left(1/2 - F\left(v_1^*\right)\right) a$. Thus there is no profitable deviation to values just below v_2^* if and only if $F\left(v_1^*\right) A + \left(1/2 - F\left(v_1^*\right)\right) a \geq aF\left(v_2^*\right)$. Inserting the values of A and a and $F\left(v_1^*\right) = 1/4$ and $F\left(v_2^*\right) = 3/4$ shows that (v_1^*, v_2^*) is an equilibrium. ■

8.4 Proofs on Auctions and Tournaments

8.4.1 Proof of Proposition 3

(i) By Lemma 2, the unique equilibrium of the pricing subgame induced by q_1 and q_2 is $p_i = \max\left\{q_i - q_j, 0\right\}$ for $i, j \in \{1, 2\}$; $j \neq i$. Suppose that an auction does not implement the social optimum (v_1^*, v_2^*). Then, for some i, there exists $\bar{v}_i \neq v_i^*$ such that $\Pi_i(\bar{v}_i, v_j^*) > \Pi_i(v_i^*, v_j^*)$.

Let $\Theta_i(v_i, v_j) = \{\sigma \in [0,1] | \, q(v_i, \sigma) \geq q(v_j, \sigma)\}$ and $\Theta_{-i}(v_i, v_j) = [0,1] \setminus \Theta_i(v_i, v_j)$. Thus $\Pi_i(\bar{v}_i, v_j^*) > \Pi_i(v_i^*, v_j^*)$ if and only if

$$\int_{\Theta_i\left(\bar{v}_i, v_j^*\right)} \left(q(\bar{v}_i, \sigma) - q\left(v_j^*, \sigma\right)\right) dF(\sigma) > \int_{\Theta_i\left(v_i^*, v_j^*\right)} \left(q(v_i^*, \sigma) - q\left(v_j^*, \sigma\right)\right) dF(\sigma),$$

or equivalently

$$\begin{aligned} &\int_{\Theta_i\left(\bar{v}_i, v_j^*\right)} \left(q(\bar{v}_i, \sigma) - q\left(v_j^*, \sigma\right)\right) dF(\sigma) + \int_0^1 q\left(v_j^*, \sigma\right) dF(\sigma) > \\ &\int_{\Theta_i\left(v_i^*, v_j^*\right)} \left(q(v_i^*, \sigma) - q\left(v_j^*, \sigma\right)\right) dF(\sigma) + \int_0^1 q\left(v_j^*, \sigma\right) dF(\sigma) \end{aligned}$$

Splitting $[0,1]$ into $\Theta_i\left(\bar{v}_i, v_j^*\right)$ and $\Theta_{-i}\left(\bar{v}_i, v_j^*\right)$ in the first line and into $\Theta_i\left(v_i^*, v_j^*\right)$ and $\Theta_{-i}\left(v_i^*, v_j^*\right)$ in the second line and simplifying, this is equivalent with

$$\begin{aligned} &\int_{\Theta_i\left(\bar{v}_i, v_j^*\right)} q(\bar{v}_i, \sigma)\, dF(\sigma) + \int_{\Theta_{-i}\left(\bar{v}_i, v_j^*\right)} q\left(v_j^*, \sigma\right) dF(\sigma) > \\ &\int_{\Theta_i\left(v_i^*, v_j^*\right)} q(v_i^*, \sigma)\, dF(\sigma) + \int_{\Theta_{-i}\left(v_i^*, v_j^*\right)} q\left(v_j^*, \sigma\right) dF(\sigma). \end{aligned}$$

and thus

$$\int_0^1 \max\{q(\bar{v}_i, \sigma), q(v_j^*, \sigma)\} dF(\sigma) > \int_0^1 \max\{q(v_i^*, \sigma), q(v_j^*, \sigma)\} dF(\sigma),$$

contradicting optimality of (v_1^*, v_2^*).

(ii) This follows from the more general statement in Propostion 5(ii).

(iii) Using Proposition 3(ii), any FPT such that the supplier breaks even has a unique equilibrium with $(v_1, v_2) = (1/2, 1/2)$. For $A = 2C$ and $t = 0$, the participation constraint of the suppliers binds. Hence, buyer surplus is maximized in the class of FPTs. It is

$$\begin{aligned} S_B^{fpt} &= \int_0^{1/2} \left(\Psi - b\left(\frac{1}{2} - \sigma\right)\right) f(\sigma)\, d\sigma + \int_{1/2}^1 \left(\Psi - b\left(\sigma - \frac{1}{2}\right)\right) f(\sigma)\, d\sigma - 2C \\ &= \Psi + \int_0^{1/2} b\sigma f(\sigma)\, d\sigma - \int_{1/2}^1 b\sigma f(\sigma)\, d\sigma - 2C \end{aligned}$$

The surplus of supplier 1 (supplier 2 follows by symmetry) is

$$\begin{aligned} S_1^a &= F(v_1^*)\, \Delta q(v_1^*, v_2^*) + \int_{v_1^*}^{1/2} (q(v_1^*, \sigma) - q(v_2^*, \sigma))\, f(\sigma)\, d\sigma \\ &= \frac{b(v_2^* - v_1^*)}{4} + \int_{v_1^*}^{1/2} (q(v_1^*, \sigma) - q(v_2^*, \sigma))\, f(\sigma)\, d\sigma. \end{aligned}$$

Thus whenever $C < b(v_2^* - v_1^*)/4$, the participation constraint of the suppliers is satisfied even with $t = 0$. By Lemma 2, in an auction the winning supplier bids exactly the quality difference. This implies that the value the buyer receives, in any state of the world, is equal to the quality of the losing supplier. Then, the buyer surplus in an auction with $t = 0$ is

$$\begin{aligned} S_B^a &= \int_0^{1/2} \left(\Psi - b\left(v_2^* - \sigma\right)\right) f\left(\sigma\right) d\sigma + \int_{1/2}^1 \left(\Psi - b\left(\sigma - v_1^*\right)\right) f\left(\sigma\right) d\sigma \\ &= \Psi + \int_0^{1/2} b\sigma f\left(\sigma\right) d\sigma - \int_{1/2}^1 b\sigma f\left(\sigma\right) d\sigma - \frac{bv_2^*}{2} + \frac{bv_1^*}{2} \end{aligned}$$

The buyer prefers FPT to the auction if $S_B^{fpt} - S_B^a > 0$, which holds whenever $\frac{bv_2^*}{2} - \frac{bv_1^*}{2} - 2C > 0$ or equivalently $\frac{b\left(v_2^* - v_1^*\right)}{4} > C$.

When $\frac{b\left(v_2^* - v_1^*\right)}{4} < C$, the participation constraints require positive subsidies. In this case, the buyer implements the social optimum by using an auction with $t = C - \Pi_1^a$ with zero supplier surplus. Obviously this outperforms the inefficient FPT.

8.4.2 Proof of Proposition 4

Denote the minimum allowable price with $\underline{P}$. If $v_1 \neq v_2$ in equilibrium, by Proposition 3(ii), the contest is not an FPT. Suppose that $v_1 < v_2$. By Lemmas 2 and 3, the buyer pays $q_i - q_j + \underline{P}$ to the supplier with $q_i \geq q_j$ in equilibrium. Thus, for any σ, the buyer surplus is $\min\{q_1, q_2\} - \underline{P}$. Hence, the surplus of a buyer who induces $v_1 < v_2$ with $\underline{P}$ is

$$\begin{aligned} S_B\left(v_1, v_2; \underline{P}\right) &= \int_0^1 \min\{q_i\left(v_i, \sigma\right), q_j\left(v_j, \sigma\right)\} dF(\sigma) - \underline{P} \\ &= \int_0^{\frac{v_1+v_2}{2}} q_2\left(v_2, \sigma\right) dF(\sigma) + \int_{\frac{v_1+v_2}{2}}^1 q_1\left(v_1, \sigma\right) dF(\sigma) - \underline{P} \end{aligned}$$

Thus

$$\frac{dS_B}{dv_1} = \int_{\frac{v_1+v_2}{2}}^1 \frac{\partial q_1}{\partial v_1} dF(\sigma) > 0; \; \frac{dS_B}{dv_2} = \int_0^{\frac{v_1+v_2}{2}} \frac{\partial q_2}{\partial v_2} dF(\sigma) < 0.$$

Thus, the buyer surplus is maximal for $v_1 = v_2$ and $\underline{P} = 0$. Given $v_1 = v_2$, the buyer surplus is maximal for $v_1 = v_2 = 1/2$, the unique equilibrium of an FPT with A arbitrarily close to 0. Given (T2), it is an equilibrium for $A = 0$.

8.5 Extensions: Generalizations for n=2

8.5.1 Proof of Proposition 5

Proof. (i) The result for the auction mechanism follows from Lemma 10. By the generalized Proposition 1, the social optimum can be implemented with a bonus tournament if $0 < v_1^* \leq 1/2 \leq v_2^* < 1$. Thus, we only need to show that the social optimum always satisfies these conditions. Therefore, first consider any $v_1 = 0$ ($v_2 = 1$ is analogous). Clearly, $\left.\frac{\partial S_T(v_1,v_2)}{\partial v_1}\right|_{v_1=0} > 0$. Hence, in the social optimum $v_1^* > 0$. Next, consider (v_1, v_2) such that $v_1 \leq v_2 < 1/2$ (the case $1/2 < v_1 \leq v_2$ is analogous). Supplier 2 offers higher quality than supplier 1 in the interval $\left[\frac{v_1+v_2}{2},1\right]$. We can write the total surplus from this interval as

$$S_T(v_1,v_2)|_{\sigma\geq\frac{v_1+v_2}{2}} =$$
$$\Psi\left(1-F\left(\frac{v_1+v_2}{2}\right)\right)-\int_{\frac{v_1+v_2}{2}}^{v_2}\delta(|v_2-\sigma|)f(\sigma)\,d\sigma-\int_{v_2}^{1/2}\delta(|v_2-\sigma|)f(\sigma)\,d\sigma$$
$$-\int_{1/2}^{1-v_2}\delta(|v_2-\sigma|)f(\sigma)\,d\sigma-\int_{1-v_2}^{1-\frac{v_1+v_2}{2}}\delta(|v_2-\sigma|)f(\sigma)\,d\sigma-\int_{1-\frac{v_1+v_2}{2}}^{1}\delta(|v_2-\sigma|)f(\sigma)\,d\sigma$$

Consider a deviation to $v_2'=1-v_2$. Symmetry of $f(\sigma)$ implies that $\int_{\frac{v_1+v_2}{2}}^{1-\frac{v_1+v_2}{2}}\delta(|v_2-\sigma|)f(\sigma)\,d\sigma=\int_{\frac{v_1+v_2}{2}}^{1-\frac{v_1+v_2}{2}}\delta(|v_2'-\sigma|)f(\sigma)\,d\sigma$. As the highest available quality determines the total surplus, it follows $S_T(v_1,v_2)|_{\frac{v_1+v_2}{2}\leq\sigma\leq1-\frac{v_1+v_2}{2}}\leq S_T(v_1,v_2')|_{\frac{v_1+v_2}{2}\leq\sigma\leq1-\frac{v_1+v_2}{2}}$. Observe that $\int_{1-\frac{v_1+v_2}{2}}^{1}\delta(|v_2-\sigma|)f(\sigma)\,d\sigma<\int_{1-\frac{v_1+v_2}{2}}^{1}\delta(|v_2'-\sigma|)f(\sigma)\,d\sigma$, because δ is increasing. Thus $S_T(v_1,v_2)|_{\sigma\geq\frac{v_1+v_2}{2}}<S_T(v_1,v_2')|_{\sigma\geq\frac{v_1+v_2}{2}}$. For $\sigma<\frac{v_1+v_2}{2}$, the highest quality always comes from v_1. Thus $S_T(v_1,v_2)|_{\sigma<\frac{v_1+v_2}{2}}=S_T(v_1,v_2')|_{\sigma<\frac{v_1+v_2}{2}}$. Thus, we obtain $S_T(v_1,v_2)<S_T(v_1,v_2')$. Thus, there can be no social optimum with $v_1^*\leq v_2^*<1/2$.
(ii) *The unique equilibrium in an FPT is such that* $v_1=v_2$ *and* $F(v_i)=1/2$ *for* $i=1,2$.
First, we show that the suggested (v_1,v_2) emerges as an equilibrium. Denote the prize with A. Let v_j be such that $F(v_j)=1/2$. Since f is everywhere positive, such a v_j is unique. Now if supplier $i\in\{1,2\}$ plays $v_i=v_j$, his revenue is $\Pi_i(v_i,v_j)=A/2$. For any $v_i<v_j$ the revenue is $\Pi_i(v_i,v_j)=AF((v_i+v_j)/2)<A/2$. Similarly, for any $v_i>v_j$ the revenue is $\Pi_i(v_i,v_j)=A(1-F((v_i+v_j)/2))<A/2$. Thus, $v_i=v_j$ is an equilibrium. Second, $v_i'=v_j'$ is an equilibrium only if $F(v_j')=1/2$. Suppose not. Then, a supplier i can profitably deviate to v_i such that $F(v_i)=1/2$, since his revenue will be $\Pi_i(v_i,v_j)>A/2$. Third, $v_i\neq v_j$ is never an equilibrium. Suppose it was. Let $v_1<v_2$. Then, the revenue of supplier 1 is $\Pi_1(v_1|v_2)=AF((v_1+v_2)/2)$, while deviating to $(v_1+v_2)/2$ leads to a revenue of $AF((v_1+3v_2)/4)>AF((v_1+v_2)/2)$. ■

8.5.2 Proof of Proposition 6

(i) By Proposition 5(ii), the FPT uniquely implements $v_1=v_2=1/2$ and $F(v_i)=1/2$ for $i=1,2$. Then there exists $\varepsilon>0$, such that $F(1/2-\varepsilon)\Delta q(1/2-\varepsilon,1/2+\varepsilon)=(1-F(1/2-\varepsilon))\Delta q(1/2-\varepsilon,1/2+\varepsilon)<C$. Then, by the generalized version of Proposition 1, a bonus tournament with $\mathcal{P}=\{\Delta q(1/2-\varepsilon,1/2+\varepsilon),0\}$ and $t=C-F(1/2-\varepsilon)\Delta q(1/2-\varepsilon,1/2+\varepsilon)$ implements $(v_1,v_2)=(1/2-\varepsilon,1/2+\varepsilon)$. This yields strictly greater total surplus, with weakly lower supplier surplus than any FPT. Hence, buyer surplus is strictly greater in such a bonus tournament than in any FPT.

(iia) By Proposition 5(i), both the auction and the bonus tournament implement the social optimum with $t=0$. When $\sigma\in[0,v_1^*]\cup\{(v_1^*+v_2^*)/2\}\cup[v_2^*,1]$, the price paid is equal in both the auction and the bonus tournament. Everywhere else the price paid is in the bonus tournament is zero, while in the auction it is strictly positive. Hence, the buyer is strictly better off in the bonus tournament than in the auction.

(iib) If $v_1^*+v_2^*=1$, then $F(v_1^*)\Delta q(v_1^*,v_2^*)=(1-F(v_2^*))\Delta q(v_1^*,v_2^*)$. Moreover, $\int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}}(q(v_1^*,\sigma)-q(v_2^*,\sigma))\,f$

$\int_{\frac{v_1^*+v_2^*}{2}}^{v_2^*} \left(q\left(v_2^*,\sigma\right)-q\left(v_1^*,\sigma\right)\right) f\left(\sigma\right) d\sigma$. By (iia), we can focus on the case that $F\left(v_1^*\right) \Delta q\left(v_1^*,v_2^*\right) < C$. If, in addition, $C < F\left(v_1^*\right) \Delta q\left(v_1^*,v_2^*\right) + \int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}} \left(q\left(v_1^*,\sigma\right)-q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma$, the auction implements the social optimum with positive supplier surplus. The bonus tournament implements the social optimum (with appropriate choice of t) in such a way that the suppliers make zero surplus. Hence, the buyer is strictly better off in a bonus tournament. If $C \geq F\left(v_1^*\right) \Delta q\left(v_1^*,v_2^*\right) + \int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}} \left(q\left(v_1^*,\sigma\right)-q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma$, both contests implement the social optimum with zero supplier surplus and the buyer is indifferent.

(iic) Suppose that neither (a) nor (b) hold and suppose (w.l.o.g.) that $v_1^*+v_2^* < 1$. We can write the revenues for each buyer as

$$\begin{aligned}
\Pi_1^a &= F\left(v_1^*\right) \Delta q\left(v_1^*,v_2^*\right) + \int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}} \left(q\left(v_1^*,\sigma\right)-q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma \\
\Pi_2^a &= \left(1-F\left(v_2^*\right)\right) \Delta q\left(v_1^*,v_2^*\right) + \int_{\frac{v_1^*+v_2^*}{2}}^{v_2^*} \left(q\left(v_2^*,\sigma\right)-q\left(v_1^*,\sigma\right)\right) f\left(\sigma\right) d\sigma
\end{aligned}$$

From $v_1^*+v_2^* < 1$, it follows that $F\left(v_1^*\right) \Delta q\left(v_1^*,v_2^*\right) < \left(1-F\left(v_2^*\right)\right) \Delta q\left(v_1^*,v_2^*\right)$. Furthermore, symmetry and single-peakedness of $f\left(\sigma\right)$ implies that $\int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}} \left(q\left(v_1^*,\sigma\right)-q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma < \int_{\frac{v_1^*+v_2^*}{2}}^{v_2^*} \left(q\left(v_2^*,\sigma\right)-q\left(v_1^*,\sigma\right)\right) f\left(\sigma\right) d\sigma$. Let $t' \geq 0$ be the lowest subsidy needed to satisfy participation constraints in an auction; t' guarantees that supplier 1 breaks even. Then, the bonus tournament with $\mathcal{P} = \{\Delta q(v_1^*,v_2^*), 0\}$ and $t = t' + \int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}} \left(q\left(v_1^*,\sigma\right)-q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma$ implements the social optimum. Again, the participation constraint of supplier 1 binds, while supplier 2 obtains positive surplus. However, the surplus of supplier 2 is lower in the bonus tournament than in the auction since $\int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}} \left(q\left(v_1^*,\sigma\right)-q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma < \int_{\frac{v_1^*+v_2^*}{2}}^{v_2^*} \left(q\left(v_2^*,\sigma\right)-q\left(v_1^*,\sigma\right)\right) f\left(\sigma\right) d\sigma$ implies that $t+\left(1-F\left(v_2^*\right)\right) \Delta q\left(v_1^*,v_2^*\right) < t'+\int_{\frac{v_1^*+v_2^*}{2}}^{v_2^*} \left(q\left(v_2^*,\sigma\right)-q\left(v_1^*,\sigma\right)\right) f\left(\sigma\right) d\sigma+\left(1-F\left(v_2^*\right)\right) \Delta q\left(v_1^*,v_2^*\right)$. Hence, the buyer is better off in the bonus tournament than in the auction.

8.6 Extensions: n>3

8.6.1 Proof of Lemma 4

(i) Arguing as for two suppliers, $v_i^* \neq v_j^*$ for all $i \neq j \in \{1,...,n\}$. Thus

$$S_T\left(\mathbf{v}\right) = \int_0^{\frac{v_1+v_2}{2}} q_1\left(v_1,\sigma\right) d\sigma + \sum_{k=2}^{n-1} \int_{\frac{v_{k-1}+v_k}{2}}^{\frac{v_k+v_{k+1}}{2}} q_k\left(v_k,\sigma\right) d\sigma + \int_{\frac{v_{n-1}+v_n}{2}}^{1} q_n\left(v_n,\sigma\right) d\sigma$$

The maximum of this function exists and it obviously does not involve corner solutions. Hence, it is given by the first order conditions

$$\begin{aligned}\frac{\partial S_T(\mathbf{v})}{\partial v_1} &= -bv_1 + b\frac{v_2-v_1}{2} = 0 && (8)\\ \frac{\partial S_T(\mathbf{v})}{\partial v_k} &= -b\frac{v_k-v_{k-1}}{2} + b\frac{v_{k+1}-v_k}{2} = 0 && (9)\\ & \text{for } k\in\{2,...,n-1\} \\ \frac{\partial S_T(\mathbf{v})}{\partial v_n} &= -b\frac{v_n-v_{n-1}}{2} + b(1-v_n) = 0 && (10)\end{aligned}$$

(9) can be rearranged to give $v_k - v_{k-1} = v_{k+1} - v_k \equiv \Delta^v$ for $k = 2, ..., n-1$. (8) and (10) give $v_1 = 1 - v_n = \Delta^v/2$. Inserting these equations into $v_1 + (v_2 - v_1) + ... + (v_n - v_{n-1}) + (1 - v_n) = 1$ gives $\Delta^v = \frac{1}{n}$. Thus, $v_1 = \frac{1}{2n}$ and $v_k = \frac{1}{2n} + \frac{k-1}{n} = \frac{2k-1}{2n}$ for $k \in \{2, ..., n\}$..

(ii) The result on auctions follows from Lemma 10 below. Consider the bonus tournament. If suppliers $1, ..., n$ choose $v_1^*, v_2^*, ..., v_n^*$, then suppliers $2, .., n-1$ receive no revenues, but they break even because of the subsidy. There are no feasible deviations for which they can earn a positive price. Consider supplier 1 (supplier n is analogous): His surplus is $\frac{1}{2n}\left(\frac{b}{n}\right) + C - C = \frac{b}{2n^2}$. Deviating to $v_1 < v_1^*$ would reduce the probability of winning the prize, with no compensating benefits. Deviating to $v_n > v_n^*$ would mean that supplier 1 would only win the low prize 0. This is clearly not profitable.

(iii) Let $\mathbf{v} = [v_1, ..., v_n]$ be the vector of approaches, ordered so that $v_1 \le ... \le v_n$. In Step 1-5, we show that diversity is less than socially optimal in the FPT. In Step 6, we consider the effect of increasing n.

Step 1: *In any equilibrium of the FPT, $v_1 = v_2$ and $v_{n-1} = v_n$. This implies that there are at most $n-2$ active approaches.*

Suppose $v_1 < v_2$. Then the revenue of supplier 1 is $A\frac{v_1+v_2}{2}$. For $v_1' = v_1 + \varepsilon$, $\varepsilon > 0$, such that $v_1' < v_2$, the revenue is $A\frac{v_1'+v_2}{2} > A\frac{v_1+v_2}{2}$. A similar argument holds for $v_{n-1} < v_n$.

We prove the second claim (that there is an inefficiently low amount of diversity) in several steps. For any supplier i, let $P^i_{\sigma<v_i}$ ($P^i_{\sigma>v_i}$) be the probability that supplier i wins and, in addition, $\sigma < v_i$ ($\sigma > v_i$). Let $P^i = P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$ be the total probability that supplier i wins.

Step 2: *If for suppliers i and j there exist $k \neq i$* and *$l \neq j$* such that *$v_i = v_k$* and *$v_j = v_l$*, then *$P^i_{\sigma<v_i} = P^i_{\sigma>v_i} = P^j_{\sigma<v_j} = P^j_{\sigma>v_j}$ in any equilibrium.*

Suppose first that $P^i_{\sigma<v_i} \neq P^i_{\sigma>v_i}$ for some supplier i using the same approach as another one. Suppose that $P^i_{\sigma<v_i} > P^i_{\sigma>v_i}$ (the opposite case is analogous). Then, a deviation to $v_i - \varepsilon$ for some sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P^i_{\sigma<v_i} > P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$,[29] which is a profitable deviation. Next, suppose that $P^i_{\sigma>v_i} < P^j_{\sigma<v_j}$(the opposite case is analogous). Then, a deviation of supplier i to $v_j - \varepsilon$ for sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P^j_{\sigma<v_j} > P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$,[30] which is a profitable deviation.

Step 3: *In any equilibrium of an FPT with n suppliers, $\overline{P} := P^1 = P^2 = P^{n-1} = P^n \geq \frac{1}{2(n-2)}$.*

By Step 2, all extreme approaches are duplicate. The three equalities thus follow from Step 1. Suppose that the inequality does not hold. Then $P^1 + P^2 + P^{n-1} + P^n < \frac{2}{n-2}$ which

[29] The winning probability is approximately $2P^i_{\sigma<v_i}$ if $v_i = \min\{v_1, ..., v_n\}$.

[30] The winning probability is approximately $2P^i_{\sigma<v_j}$ if $v_j = \min\{v_1, ..., v_n\}$.

in turn implies that $\sum_{j=3}^{n-2} P^j \geq \frac{n-4}{n-2}$. But then there exist at least one $k \in \{3, ..., n-2\}$ such that $P^k \geq \frac{1}{n-2}$. By deviating to v_k, each supplier $1, 2, n-1$ or n would win with a probability of at least $\frac{1}{2(n-2)}$, which would be a profitable deviation.

Step 4: *Any equilibrium of an FPT with n suppliers satisfies* $\max_i v_i^T - \min_i v_i^T \leq \frac{n-3}{n-2}$.

Suppose not. As $\frac{2(n-2)-1}{2(n-2)} - \frac{1}{2(n-2)} = \frac{n-3}{n-2}$, there exists an equilibrium of an FPT such that either $\max_i v_i^T > \frac{2(n-2)-1}{2(n-2)}$ or $\min_i v_i^T < \frac{1}{2(n-2)}$ or both. If $\max_i v_i^T > \frac{2(n-2)-1}{2(n-2)}$, then Steps 1 and 2 imply $P^n < \frac{1}{2(n-2)}$, which is impossible by Step 3. If $\min_i v_i^T < \frac{2-1}{2(n-2)}$, then $P^1 < \frac{1}{2(n-2)}$ by Steps 1 and 2, which is again impossible by Step 3.

Step 5: *The diversity in an FPT is lower than socially optimal.*

By (i), the socially optimal diversity is $\frac{n-1}{n}$. By Step 4, the diversity in an FPT is at most $\frac{n-3}{n-2} < \frac{n-1}{n}$.

Step 6: *The difference between the FPT and the social optimum converges to zero as the number of suppliers increases.*

By Step 3, we know that each supplier $1, 2, n-1, n$ wins with probability $\overline{P}$. Then in any equilibrium of an FPT, there exists a supplier j such that $P^j \leq \frac{1-4\overline{P}}{n-4}$. A deviation to $v_1 - \varepsilon$ would result in a probability of winning approximately $\overline{P}$. Then, a necessary condition for an equilibrium is that $\overline{P} \leq \frac{1-4\overline{P}}{n-4}$, which implies that $\overline{P} \leq 1/n$ and consequently $v_1 \leq 1/n$ and $v_n \geq (n-1)/n$. Then, $\max_i v_i^T - \min_i v_i^T \geq \frac{n-2}{n}$ in any equilibrium of an FPT. By (i), the socially optimal diversity is $(n-1)/n$, so the difference between the socially optimal diversity and diversity in any equilibrium of an FPT is at most $\frac{n-1}{n} - \frac{n-2}{n} = 1/n$. Thus, the difference converges to zero as n increases.

8.6.2 Sufficient Conditions for FPT equilibria

We now provide sufficient conditions for equilibria in the FPT. These conditions hold in the equilibria described in Figure 2.

Lemma 8 *An outcome with k active approaches* $(r_1, ..., r_k)$ *can be supported in an equilibrium if the following conditions both hold:*
(a) $k \in \{\underline{k}, ..., \bar{k}\}$, *where* $\bar{k} = n-2$ *and* $\underline{k} = n/2$ *if n is even and* $\underline{k} = (n+1)/2$ *if n is odd;*
(b) $(r_1, ..., r_k) = (1/2k, 3/2k, 5/2k, ..., (2k-1)/2k)$.
Two suppliers choose the extreme approaches r_1 *and* r_k*; each of the intermediate approaches* $r_2, ..., r_{k-1}$ *is chosen by one or two suppliers.*

Proof. Step 1: *Suppose n is even and* $k = n/2$. *Then any choice of* $r_1, ..., r_k$ *as stated in part (b) of the lemma can be supported as an equilibrium.*
In the suggested equilibria, the active approaches are equidistant. Also, $r_1 = 1/n$ and $r_{n/2} = 1 - 1/n$. For any $1 < m < n/2$, $r_m - r_{m-1} = 2/n$, any of the active approaches offers the highest quality with probability $1/k = 2/n$. Now suppose each approach $r_1, ..., r_k$ is chosen by exactly two suppliers. Then each supplier has a revenue of $\Pi_i = A/n$. Deviating to any other active approach leads to payoff of $2A/3n$; hence it is not profitable. A deviation to $[0, r_1)$ or $(r_{n/2}, 1]$ results in a winning probability strictly lower than $1/n$, so this is not a profitable deviation either. Finally, consider a deviation to $v \in (r_{m-1}, r_m)$, $m \in \{2, ..., n/2\}$. The deviating supplier wins if and only if σ is in the set $[\frac{v+r_{m-1}}{2}, \frac{v+r_m}{2}]$, so that the winning probability is $1/n$ and this is also not a profitable deviation.

Step 2: *Now suppose n is even or odd and $k > n/2$. Then any choice of $r_1, ..., r_k$ as stated in part (b) of the lemma is an equilibrium.*
Arguing as in Step 1, any of the active approaches offers the highest quality with probability $1/k$. Suppose two suppliers choose r_1 and r_k, respectively. Moreover, suppose that each of the approaches $r_2, ..., r_{k-1}$ is chosen by one or two suppliers. Thus, if there are two suppliers using an approach, each of them wins with probability $1/2k$, and if there is only one supplier using this approach, he wins with probability $1/k$. Consider a supplier who wins with probability $1/2k$. By the same argument as in Step 1, if he deviates to $[0, r_1)$ or $(r_k, 1]$, he wins with probability strictly lower than $1/2k$. Deviating to any approach in some interval (r_l, r_{l+1}); $l \in \{1, ..., k-1\}$, he wins with probability of at most $1/2k$; hence such a deviation is not profitable either. If he deviates to any active approach, he wins with a probability of at most $1/2k$. Thus, such suppliers do not have profitable deviations. Finally consider a deviation by a supplier who is the only one to choose some r_m, where $1 < m < k$. Any deviation to $[0, r_{m-1}]$ or $[r_{m+1}, 1]$ leads to strictly lower revenues, by the same argument as above. For any approach $v \in (r_{m-1}, r_{m+1})$, he wins whenever $\sigma \in [\frac{v+r_{m-1}}{2}, \frac{v+r_{m+1}}{2}]$, so that the winning probability is $\frac{v+r_{m+1}}{2} - \frac{v+r_{m-1}}{2} = \frac{r_{m+1}-r_{m-1}}{2} = 1/k$. Hence, this is not a profitable deviation either. ■

8.6.3 Proof of Proposition 7

(i) Arguing as in Lemma 3, the bonus prize of b/n is necessary for implementation of the social optimum with a bonus tournament. Thus, the total expected transfer from the buyer to the suppliers is $b/n^2 + nC$. In an auction, the conditional transfers to suppliers 1 and n differ from those for the remaining suppliers. The revenue of supplier 1 is

$$\Pi_1 = \frac{b}{2n^2} + \int_{1/2n}^{2/2n} \left(\Psi - b\left(\sigma - \frac{1}{2n}\right) - \left(\Psi - b\left(\frac{3}{2n} - \sigma\right)\right)\right) d\sigma = \frac{3b}{4n^2}$$

For supplier 2 it is

$$\Pi_2 = 2\int_{2/2n}^{3/2n} \left(\Psi - b\left(\frac{3}{2n} - \sigma\right) - \left(\Psi - b\left(\sigma - \frac{1}{2n}\right)\right)\right) d\sigma = \frac{2b}{4n^2}$$

By symmetry, $\Pi_1 = \Pi_n$ and $\Pi_2 = \Pi_j$ for all $j \in \{2, ...n-1\}$. As $\Pi_1 > \Pi_2$, the participation constraint of suppliers $j \in \{2, ...n-1\}$ will be binding. Suppose first $C > \Pi_2$. Then, the buyer optimally sets $t = C - \Pi_2$ in the auction. The total transfers of the buyer to the suppliers are thus $\Sigma\Pi_i + nt = 2(\Pi_1 - \Pi_2) + nC = \frac{b}{2n^2} + nC$. In this case, the total transfers of the buyer are strictly greater in the bonus tournament than in the auction. Since both contests implement the social optimum, the buyer is better off in an auction.
Next, suppose $C \leq \Pi_2$. Then, the buyer optimally sets $t = 0$. The total transfers of the buyer to the suppliers are therefore $\Sigma\Pi_i = 2\frac{3b}{4n^2} + (n-2)\frac{2b}{4n^2} = \frac{b}{2n^2}(1+n)$. The buyer prefers the bonus tournament iff $\frac{b}{n^2} + nC < \frac{b}{2n^2}(1+n)$ or, equivalently, $C < \frac{b}{2}\left(\frac{n-1}{n^3}\right)$.
(ii) According to Lemma 4(iii), an FPT can implement at most $n-2$ different approaches. By Lemma 8, an FPT implementing $n-2$ approaches exists. The FPT implementing maximum diversity (hence maximizing total surplus) thus implements $k = n-2$ with $A = 0$ and $t = C$. The participation constraint of all suppliers binds, so this is the best outcome for the buyer. In the FPT, the buyer has expected costs from suboptimal quality of $\frac{b}{4k}$. Moreover, she pays

subsidies nC. In the bonus tournament, the buyer has expected costs from suboptimal quality of $\frac{b}{4n}$, pays revenues $\frac{b}{n^2}$ and subsidies nC; together $\frac{b}{n^2} + \frac{b}{4n} + nC = \frac{b}{4}\frac{n+4}{n^2} + nC$. Thus, the buyer is better of in the bonus tournament if $\frac{b}{4}\frac{n+4}{n^2} \le \frac{b}{4k}$ or $k \le \frac{n^2}{n+4}$. The maximum value of k in any tournament equilibrium is $n-2$. For $k = n-2$ and $n > 4$, this condition holds strictly; for $k = n-2$ and $n = 4$ it holds with equality.

8.6.4 Proof of Corollary 2

According to Lemma 4(i), the social optimum is given by the choices $v_k^* = (2k-1)/2n$ ($k \in \{1, ..., n\}$). The average quality in the social optimum is thus $\Psi - b/4n$. Therefore the total surplus is $\Psi - b/4n - nC$. The maximum of this expression in $\mathbb{R}^+$ is $n = \sqrt{b}/2\sqrt{C}$. By concavity of the objective function, the optimal choice of $n \in \mathbb{N}$ is thus given by $n_-(C)$ or $n_+(C)$. According to Lemma 4(ii), the social optimum for any given number of suppliers can be implemented with an auction.

8.6.5 Proof of Lemma 5

This section provides the proof of Lemma 5 from Section 5.3.2. Suppose that there are n suppliers and that assumption (A1)" holds. Consider an FPT with two prizes $A_1 > A_2 > 0$, where the supplier with the highest quality receives A_1 and the supplier with the second-highest quality receives A_2.[31] For notational convenience, suppose that $v_1 \le v_2 \le \cdots \le v_n$. We first provide an intermediate result.

Lemma 9 *If $v_1, v_2, \ldots, v_n$ is an equilibrium of an FPT with two prizes, then $v_1 = v_2 = v_3$ and $v_{n-2} = v_{n-1} = v_n$.*

Proof. We will prove that $v_1 = v_2 = v_3$. The other claim follows by an analogous argument.
Step 1: $v_1 = v_2$. Suppose not. Then $v_1 < v_2$. Thus, the revenue of supplier 1 is

$$\Pi_1(v_1, v_{-1}) = \frac{v_1 + v_2}{2} A_1 + \frac{v_3 - v_2}{2} A_2.$$

Therefore, a deviation to any $v_1' \in (v_1, v_2)$ increases the probability of winning the first prize, while not affecting the probability of winning the second prize. Hence, it is profitable.
Step 2: $v_1 = v_2 < v_3 = v_4$ cannot be an equilibrium. Denote with $P^{i,1}_{\sigma<v_i}$ the probability that supplier i wins the first prize when $\sigma < v_i$. Analogously define the probabilities of winning when the state is greater than the chosen approach and the probabilities of winning the second prize. By random tie breaking we have $P^{1,1}_{\sigma<v_1} = P^{2,1}_{\sigma<v_2} = P^{1,2}_{\sigma<v_1} = P^{2,2}_{\sigma<v_2}$ and $P^{1,1}_{\sigma>v_1} = P^{2,1}_{\sigma>v_2} = P^{1,2}_{\sigma>v_1} = P^{2,2}_{\sigma>v_2}$. We will show that $P^{1,1}_{\sigma<v_1} = P^{1,1}_{\sigma>v_1}$. Suppose that this was not true. First, suppose $P^{1,1}_{\sigma<v_1} > P^{1,1}_{\sigma>v_1}$. Then, there exist $\varepsilon, \varepsilon', \varepsilon'' > 0$ arbitrarily small such that a deviation $v_1' = v_1 - \varepsilon$ leads to revenues

$$\Pi_1(v_1', v_{-1}) = 2\left(P^{1,1}_{\sigma<v_1} - \varepsilon'\right) A_1 + 2\left(P^{1,1}_{\sigma>v_1} - \varepsilon''\right) A_2.$$

For sufficiently small ε this constitutes a profitable deviation. The case $P^{1,1}_{\sigma<v_1} < P^{1,1}_{\sigma>v_1}$ follows by an analogous argument, but the incentives to deviate are even stronger.

[31] Ties are broken randomly, with equal chance of winning for each firm with the respective quality.

Now suppose that $P^{1,1}_{\sigma<v_1} = P^{1,1}_{\sigma>v_1}$ and $v_1 = v_2 < v_3 = v_4$. We will show that this cannot be an equilibrium. In the proposed equilibrium $P^{1,1}_{\sigma<v_1} = v_1/2$ and $P^{1,1}_{\sigma<v_1} + P^{1,1}_{\sigma>v_1} = P^{1,2}_{\sigma<v_1} + P^{1,2}_{\sigma>v_1} = v_1$. Hence, the expected revenue is

$$\Pi_1(v_1, v_{-1}) = v_1 A_1 + v_1 A_2.$$

For any deviation $v_1' \in (v_2, v_3)$ the probability of winning the first prize is

$$\frac{v_1' + v_3}{2} - \frac{v_1' + v_2}{2} = \frac{v_3 - v_2}{2} = v_1$$

where the last equality follows from $P^{1,1}_{\sigma<v_1} = P^{1,1}_{\sigma>v_1}$. Using $v_3 = v_4$, the probability of winning the second prize is

$$\frac{v_2 + v_1'}{2} > v_1$$

thus it follows that $\Pi_1(v_1', v_{-1}) > \Pi_1(v_1, v_{-1})$.

Step 3: $v_1 = v_2 < v_3 < v_4$ cannot be an equilibrium. The revenue of supplier 1 is

$$\Pi_1(v_1, v_{-1}) = \frac{v_1}{2}A_1 + \frac{v_3 - v_1}{4}A_1 + \frac{v_3 + v_1}{4}A_2 + \frac{v_4 - v_3}{4}A_2. \tag{11}$$

Consider a deviation to $v_1' \in (v_1, v_3)$. The revenue is

$$\Pi_1(v_1', v_{-1}) = \frac{v_3 - v_1}{2}A_1 + \frac{v_1' + v_1}{2}A_2 + \frac{v_4 - v_3}{2}A_2.$$

If $\Pi_1(v_1', v_{-1}) > \Pi_1(v_1, v_{-1})$, then this is a profitable deviation. If $\Pi_1(v_1', v_{-1}) \le \Pi_1(v_1 | v_{-1})$ is equivalent with

$$\frac{v_1}{2}A_1 - \frac{v_3 - v_1}{4}A_1 + \frac{v_3 - v_1}{4}A_2 - \frac{v_1'}{2}A_2 - \frac{v_4 - v_3}{4}A_2 \ge 0 \tag{12}$$

But consider in that case a deviation to $v_1'' = v_1 - \varepsilon$ for small positive ε. The expected revenue is

$$\Pi_1(v_1'', v_{-1}) = \frac{v_1'' + v_1}{2}A_1 + \frac{v_3 - v_1}{2}A_2$$

and $\lim_{\varepsilon\to 0}\Pi_1(v_1'', v_{-1}) = v_1 A_1 + \frac{v_3 - v_1}{2}A_2$. Together with (11), this implies

$$\lim_{\varepsilon\to 0}\Pi_1(v_1'', v_{-1}) - \Pi_1(v_1, v_{-1}) = \frac{v_1}{2}A_1 + \frac{v_3 - v_1}{4}A_2 - \frac{v_3 - v_1}{4}A_1 - \frac{v_1}{2}A_2 - \frac{v_4 - v_3}{4}A_2.$$

Since $v_1' > v_1$, (12) implies $\lim_{\varepsilon\to 0}\Pi_1(v_1'', v_{-1}) - \Pi_1(v_1, v_{-1}) > 0$. Hence, there always exists $\varepsilon > 0$ small enough such that $\Pi_1(v_1'', v_{-1}) - \Pi_1(v_1, v_{-1}) > 0$. ■

The lemma implies that the maximal number of active approaches in an FPT with two prizes is $n-4$. By Lemma 8 an FPT with a single prize implements an equilibrium with $n-2$ active approaches. By Lemma 4(ii), it is possible to implement the socially optimal allocation with $n-2$ approaches in an FPT with a single prize. Implementing this equilibrium in a single-prize FPT, where the prize size is the sum of the two prizes in an FPT with two prizes, strictly increases the total payoff. On the other hand, the payoff of the sellers remains the same, as the total size of the fixed prize remains the same. Hence, the expected buyer payoff strictly increases.

8.7 General Implementation of the Social Optimum

We prove, in a general setting, that a mechanism which i) selects the highest quality ex-post and ii) transfers a positive fraction of the difference between best and second-best quality to the winner always implements the socially optimal choice of approaches.

Let $N = \{1, ..., n\}$ be the set of suppliers. Let $V \subseteq \mathbb{R}$ be a compact and convex set of possible approaches. Let $\Theta \subseteq \mathbb{R}$ be a compact and convex set of possible supplier characteristics. The actual supplier characteristics $\theta_1, ..., \theta_n \in \Theta$ are commonly known. Let the ideal approach $\sigma \in V$ and the ideal supplier characteristic $\theta \in \Theta$ be distributed according to continuous probability density functions $f_\sigma(\sigma)$ and $f_\theta(\theta)$, respectively, with corresponding cumulative distribution functions F_σ and F_θ. For any combination of v_i and θ_i, and any (θ, σ), the quality of supplier i is given by the function $q(v_i, (\theta, \sigma)) := \widetilde{q}(\delta_\theta(|\theta_i - \theta|), \delta_\sigma(|v_i - \sigma|))$ where δ_θ and δ_σ are differentiable functions satisfying $\delta_\theta' > 0$, $\delta_\sigma' > 0$, and q is a differentiable function satisfying $\partial q / \partial \delta_\theta < 0$ and $\partial q / \partial \delta_\sigma < 0$. Here, $\delta_\theta(\cdot)$ is the reduction in quality due to suboptimal supplier characteristic and and $\delta_\sigma(\cdot)$ is the reduction in quality due to a suboptimal approach. Let q, δ_θ and δ_σ be continuous functions.

Let the vector of approaches be $\mathbf{v} = [v_1, ..., v_n]$. Then the total surplus is given by $S_T(\mathbf{v}) = \int_V \int_\Theta \max_{i \in N} \{q(v_i, (\theta, \sigma))\} dF_\theta dF_\sigma$. Denote with $\tilde{\Theta}_i(\mathbf{v}, \sigma) \subseteq \Theta$ the set of states θ for which supplier i offers the highest quality. Ties are broken so that the demand is assigned to the supplier with a lower index.[32] Thus, $\tilde{\Theta}_i(\mathbf{v}, \sigma) \cap \tilde{\Theta}_j(\mathbf{v}, \sigma) = \emptyset$ for any $i \neq j$ and $\cup_N \tilde{\Theta}_i(\mathbf{v}, \sigma) = \Theta$.

Definition 2 *In the context just described, a generalized innovation contest is a game as follows: The suppliers simultaneously select approaches $v_i \in [0, 1]$. Then the buyer selects a supplier $i \in N$. When choosing his approach, the selected supplier correctly anticipates a transfer payment $p_i(q_i, \mathbf{q}_{-i})$ conditional on being selected; the transfer is zero otherwise.*

A generalized innovation contest is thus sufficiently general to encompass innovation contests and negotiations as discussed in this paper.

Lemma 10 is a statement on the SPE of a generalized innovation contest as defined above.

Lemma 10 *Any generalized innovation contest that always selects the ex-post optimal quality and uses the payment rule*

$$p_i(q_i, \mathbf{q}_{-i}) = c\left(q_i - \max_{j \neq i}\{q_j\}\right) \tag{13}$$

if $q_i > \max_{j \neq i}\{q_j\}$ for all $j \in N$,and $t_i(q_i, \mathbf{q}_{-i}) = 0$ otherwise, (weakly) implements the social optimum for any $c > 0$.

Proof. Step 1: *S_T has a maximum $v^* \in V^n$.*
V^n is compact and S_T is continuous. Thus, a maximum exists.

[32] This tie breaking assumption is different from the one made in baseline model, where given equal qualities and prices, the ties are broken randomly. This is done in order to simplify the definition of sets $\tilde{\Theta}$. The tie breaking is immaterial in this case since, in the institution we will consider, all suppliers make positive profits in expectation, while the transfer when qualities are equal are zero. Hence, incentives do not change regardless of the tie breaking rule.

Step 2: $\mathbf{v}^*$ *is an equilibrium of the game defined by the proposed procurement institution.* Suppose not. Then, there exists $v_i \in V$ such that

$$\Pi_i(v_i, \mathbf{v}^*_{-i}) = \int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}^*_{-i}, \sigma)} c\left(q_i(v_i|\theta,\sigma) - \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\}\right) dF_\theta dF_\sigma > \tag{14}$$
$$\int_V \int_{\tilde{\Theta}_i(v^*_i, \mathbf{v}^*_{-i}, \sigma)} c\left(q_i(v^*_i|\theta,\sigma) - \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\}\right) dF_\theta dF_\sigma = \Pi_i(v^*_i|\mathbf{v}^*_{-i}).$$

Multiply by $1/c$ and add the maximum expected quality of suppliers other than i to both sides:

$$\begin{aligned}
\int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}^*_{-i}, \sigma)} q_i(v_i|\theta,\sigma) \quad &- \quad \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \\
+ \int_V \int_\Theta \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \quad &> \quad \int_V \int_{\tilde{\Theta}_i(v^*_i, \mathbf{v}^*_{-i}, \sigma)} q_i(v^*_i|\theta,\sigma) \\
- \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \quad &+ \quad \int_V \int_\Theta \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma.
\end{aligned}$$

We can choose to split the set Θ in a convenient way:

$$\begin{aligned}
\int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}^*_{-i}, \sigma)} q_i(v_i|\theta,\sigma) - \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \quad &+ \\
\int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}^*_{-i}, \sigma)} \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \quad &+ \\
\int_V \int_{\tilde{\Theta}_{-i}(v_i, \mathbf{v}^*_{-i}, \sigma)} \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \quad &> \\
\int_V \int_{\tilde{\Theta}_i(v^*_i, \mathbf{v}^*_{-i}, \sigma)} q_i(v^*_i|\theta,\sigma) - \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \quad &+ \\
\int_V \int_{\tilde{\Theta}_i(v^*_i, \mathbf{v}^*_{-i}, \sigma)} \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \quad &+ \\
\int_V \int_{\tilde{\Theta}_{-i}(v^*_i, \mathbf{v}^*_{-i}, \sigma)} \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma. &
\end{aligned}$$

Simplifying, we obtain:

$$\begin{aligned}
\int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}^*_{-i}, \sigma)} q_i(v_i|\theta,\sigma) dF_\theta dF_\sigma \quad &+ \\
\int_V \int_{\tilde{\Theta}_{-i}(v_i, \mathbf{v}^*_{-i}, \sigma)} \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma \quad &> \\
\int_V \int_{\tilde{\Theta}_i(v^*_i, \mathbf{v}^*_{-i}, \sigma)} q_i(v^*_i|\theta,\sigma) dF_\theta dF_\sigma \quad &+ \\
\int_V \int_{\tilde{\Theta}_{-i}(v^*_i, \mathbf{v}^*_{-i}, \sigma)} \max_{j\neq i}\left\{q_j(v^*_j|\theta,\sigma)\right\} dF_\theta dF_\sigma. &
\end{aligned}$$

By definition of $\tilde{\Theta}$ we have:

$$\int_V \int_\Theta \max \left\{ q_i(v_i|\theta, \sigma), \max_{j \neq i} \left\{ q_j(v_j^*|\theta, \sigma) \right\} \right\} dF_\theta dF_\sigma > \tag{15}$$
$$\int_V \int_\Theta \max \left\{ q_i(v_i^*|\theta, \sigma), \max_{j \neq i} \left\{ q_j(v_j^*|\theta, \sigma) \right\} \right\} dF_\theta dF_\sigma$$

Thus $S_T \left(v_i, \mathbf{v}_{-i}^*\right) > S_T \left(v_i^*, \mathbf{v}_{-i}^*\right)$, contradicting the optimality of $\mathbf{v}^*$. ■