Segmentation versus Agglomeration: Competition between Platforms with Competitive Sellers*

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Abstract

Two competing non-differentiated platforms bring together sellers and buyers who face the discrete choice problem which platform to visit. Platforms charge listing fees to sellers for their service. If competition between sellers is soft, only agglomeration equilibria exist, i.e. all sellers and buyers locate on one platform. By contrast, if competition between sellers is moderate or fierce, in the unique equilibrium, buyers and sellers segment, and sellers enjoy a monopoly position vis-a-vis buyers. This allows platforms to obtain strictly positive profits in equilibrium. The segmentation equilibrium often features dispersion of listing fees. We characterize the equilibrium and discuss implications for price structure and market structure.

keywords: intermediation, two-sided markets, price competition, imperfect and perfect competition

1 Introduction

In many industries, platforms play the essential role to enable transactions between buyers and sellers. As trade migrates from physical venues to the Internet, platforms increasingly serve as intermediaries for purchase decisions. For example, in the rental

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market, the main bulk of matching of landlords and tenants is done via Internet platforms such as Craigslist in the US, Rightmove and Zoopla in the UK, or Immobilienscout24 and Immowelt in German speaking countries. Another example is the used car market, where a large fraction of transactions is initiated via portals.

However, the market structure differs considerably across industries and space. For example, in the US, Craigslist dominates the market in several cities, foremost in the bay area. Buyers and sellers almost exclusively choose this portal, leaving other platforms only specialized market niches. By contrast, in the UK or Germany, the market is more segmented and two (or more) platforms have non-negligible market shares.

Due to positive cross-group external effects between buyers and sellers, it has been recognized that platforms have the tendency to tip (as shown by Caillaud and Jullien, 2001, 2003). This explains the phenomenon of market agglomeration, where all buyers and sellers choose one platform over the other. However, as the examples above indicate, in several industries more than one platform has positive market shares. A possible explanation is that platforms offer differentiated matching services and, therefore, are active in the market (e.g., Rochet and Tirole, 2003, and Armstrong, 2006). However, in the above examples (and more broadly for many Internet platforms) there appears to be little room for differentiation, that is, platforms offer homogeneous services to their customers. In this case, it is unclear how multiple platforms can survive and it is puzzling that they compete with each other for several years without tipping taking place. In this paper, we resolve the puzzle how multiple homogeneous platforms can survive in an industry that exhibits strong positive network externalities.

Our explanation is based on endogenous differentiation of competitive sellers via their platform choice. We present a theoretical model in which sellers and buyers decide on which platform to be active.[1] If sellers locate on the same platform, it is optimal for buyers to do the same. An agglomeration equilibrium arises. Buyers are then informed about all offers, implying that sellers are in competition with each other. However, if competition between sellers in the same industry is sufficiently intense, they prefer to be active on different platforms. Then, consumers also prefer to be active on different platforms, which implies that single-homing consumers do not become informed about all offers on the market. This relaxes seller competition on each platform. Hence, platforms allow for segmentation of the product market and obtain positive profit for playing this role.

We show that using the natural selection criterion that sellers choose their profit-dominant equilibrium, a unique equilibrium exists. This allows us to make predictions under which conditions an agglomeration and under which conditions a separation equi-

[1] In the basic model, agents on both side single-home. We explain below that our results carry over to the case of multi-homing buyers and sellers.

librium exists. First, we obtain that if the degree of competition between sellers is low, segmentation cannot occur and tipping prevails. Sellers obtain a higher demand from consumers in the agglomeration equilibrium because all consumers are exposed to all offers by sellers. This increased-demand effect dominates the increased-competition effect. Platforms compete fiercely to win the market. This leads to a Bertrand-style competition between platforms, and their listing fees are driven down to zero.

By contrast, if the degree of competition between sellers is high, segmentation occurs. Sellers use the platforms to avoid competition with their rivals in the product market. Platforms serve the role of segmenting the market and receive positive margins for providing this service. Thus, depending on the degree of competition between sellers, very different market structures can emerge and our paper provides clear predictions how the competitive environment between sellers drives the market structure.

If the degree of competition between sellers is moderate, we show that a mixed-strategy equilibrium in listing fees occur. This equilibrium might consist of two disjoint segments of fees. The upper segment of fees are charged if a platform intends to segment the market. By contrast, a listing fee in the lower segment is charged if the platform intends to agglomerate agents on its platform. In this mixed-strategy equilibrium, platforms segment the market with positive probability. We demonstrate that the probability for segmentation taking place increases if the degree of competition between sellers gets larger. Our result therefore contributes to the explanation of why different market structures emerge in industries with similar competitive conditions.

Interestingly, the mixed-strategy equilibrium contains one or two mass points. The logic behind the best-response dynamic in our model is similar to that of Bertrand-Edgeworth cycles. In our case, if a platform sets a high listing fee, the rival's optimal response is to set a fee which is lower by a discrete amount to induce agglomeration. The best response of the platform is then to lower its fee slightly to induce segmentation. The rival's optimal response to lower its fee slightly to induce agglomeration again, and so on. This tendency goes on until the fee of the platform with the lower fee is so low that it prefers to set a discretely higher fee than the other platform in order to induce segmentation instead of lowering its fee further. In contrast to Bertrand-Edgeworth cycles, there is no marginal undercutting of the rival's fee but a discrete one. Because there is a continuum of fees between the best responses, mass points occur.

To illustrate segmentation in the real world, consider a search on the platforms Immobilienscout24 and Immowelt. We searched for apartments to rent in the city of Frankfurt am Main, Germany. Our search criteria were "at least 3 rooms", "at least 100 m^2", and "distance no less than 1 kilometer to the centre". A search on November 23, 2015 gave 12 matches on each portal. We report the matches in Table 1 in ascending order of the rental price by stating the square meters of the apartment and the rental price in Euros.

Out of these 12 offers, only 2 could be found on both platforms.[2]

	Immobilienscout24	Immowelt
1.	m^2:103.78; Rent:1.350	m^2:111.00; Rent:1.285
2.	m^2:110.00; Rent:1.450	m^2:104.00; Rent:1.290
3.	m^2:100.00; Rent:1.450	m^2:117.00; Rent:1.350
4.	m^2:105.90; Rent:1.450	m^2:103.56; Rent:1.490
5.	m^2:129.02; Rent:1.548	m^2:114.00; Rent:1.550
6.	m^2:124.74; Rent:1.597	m^2:145.00; Rent:1.650
7.	m^2:142.00; Rent:1.700	m^2:100.00; Rent:1.800
8.	m^2:136.00; Rent:1.890	m^2:140.00; Rent:1.970
9.	m^2:137.48; Rent:2.007	m^2:140.00; Rent:2.450
10.	m^2:173.00; Rent:2.290	m^2:160.00; Rent:2.800
11.	m^2:140.00; Rent:2.450	m^2:152.00; Rent:2.830
12.	m^2:152.00; Rent:2.830	m^2:200.00; Rent:3.200

Table 1: Apartment offers in ascending order of the rental price

Although some consumers may multi-home on both platforms, we expect that many of them single-home, as it is time-consuming to conduct searches on various platforms. As a consequence, the listing behavior of sellers gives rise to a segmentation of the market, which dampens competition.[3]

While our base model features single-homing on both sides of the market, we allow for multi-homing sellers and buyers in our extensions and show that our solution to the puzzle that multiple platforms are active (and profitable) carries over to those settings. First, we consider the case in which some (but not all) buyers multi-home. The intuition for the existence of the segmentation equilibrium remains: the remaining single-homing consumers do not observe all offers, which dampens competition on the product market. Hence, if the degree of competition is fierce, firms prefer segmentation over agglomeration and platforms can demand positive listing fees in equilibrium. The sellers' and platform's profit from multi-homing consumers depends on the trade-off between increased competition brought about by multi-homing and a higher demand because more consumers are informed. We show profits may change non-monotonically in the mass of multi-homing consumers. A few multi-homers lead to increased profit whereas profits fall if the number of multi-homers is above a certain threshold.

Finally, we analyze multi-homing of sellers. We find that platforms can be hurt by the possibility that sellers multi-home. This result is in contrast to the existing literature which shows that multi-homing benefits platforms because platforms do no

[2]Offer 11 on Immobilienscout24 is the same apartment as offer 9 on Immowelt, and offer 12 on Immobilienscout24 is the same apartment as offer 11 on Immowelt.

[3]We note that the segmentation equilibrium is inefficient because it leads to higher product market prices and implies that consumers do not observe all offers.

longer compete for multi-homing agents. Our result shows, however, that this is not true if agents (here sellers) compete against each other. The intuition is as follows: consider listing fees that lead to a segmentation equilibrium with single-homing sellers. If sellers can multi-home, a profitable deviation from the segmentation equilibrium may exits for each seller. This deviation is to multi-home and to serve buyers on both platforms. This possibly breaks the segmentation equilibrium, in which platforms can earn positive profits. Then, only an agglomeration equilibrium exists, in which buyers observe all offers, and platforms end up competing à la Bertrand and receive zero profits.

Our paper contributes to the literature on competition in two-sided markets, pioneered by Caillaud and Jullien (2001), Rochet and Tirole (2003, 2006), and Armstrong (2006). These papers focus on the cross-group externalities between agents on both sides and did not consider competition between agents on the same side (as the sellers do in our model).

Several papers in the two-sided markets literature analyze competition between sellers. Nocke, Peitz and Stahl (2007) and Galeotti and Moraga-González (2009) consider the effect of platform ownership on prices and of search for sellers' products, respectively, but focus on a monopoly platform. Hagiu (2006) is primarily interested in price commitment by platforms when agents of the two sides make their decision in a sequential order. However, he shows that if commitment is not possible and agents single-home, an agglomeration equilibrium with zero profits emerges. Belleflamme and Toulemonde (2009) show how a fee-setting platform can gain market shares from a platform with zero fees by applying a divide-conquer-strategy (i.e., negative prices on one side and positive prices on the other). Hagiu (2009) considers the effects of product variety on platform prices in a model with differentiated platforms, implying that both platforms are active in equilibrium.[4] None of these papers has considered the effect identified in our paper and analyzed how the market structure depends on seller competition.

In an early paper, Gehrig (1998), considering Hotelling competition between platforms and competition on the circle (Salop, 1979) between sellers, analyzes the effect of transportation costs on entry and location of platforms. In contrast to our analysis, he is mainly interested in agglomeration equilibria.[5] Armstrong and Wright (2007) endogenize the decision of agents to single-home or to multi-homes (thereby endogenizing the market structure in a different way than we do) and determine how differentiation between platform affects this choice.

Ellison, Fudenberg, and Möbius (2004) consider competition between two auction

[4]Belleflamme and Peitz (2010) consider congestion externalities between sellers, which lead to similar effects as competition. They focus on how investment incentives are influenced by the platform prices.

[5]See also Gal-Or and Dukes (2003) for an analysis how competition between sellers affects platforms' location decision.

sides and analyze market thickness of the platforms. They show that concentration tends to be optimal but under some conditions sellers may prefer different platforms because this lowers the seller-buyer ratio on each platform and leads to (higher) expected prices. Hence, platforms can co-exist in equilibrium. In contrast to our paper, they do not consider how platforms can influence the market structure via their fees, and determine how the homing decision of agents affects the equilibrium. Lee (2014) considers a model with non-atomistic sellers and determines how bilateral contracting between platforms and sellers affects the market structure. He shows that even without competition between sellers, platforms may achieve segmentation, which leads to co-existence of platforms.

Our paper also contributes to the literature on competition in the Internet, in particular, on price comparison websites. For example, Baye an Morgan (2001) show how sellers can obtain positive profits, even if a website informs buyers about their prices. The intuition is that sellers still sell on their local market where buyers are not informed about all prices. This leads to price dispersion in equilibrium. Ronayne (2015) uses this framework and demonstrates that due to the website's fee, all prices increase in expectation, leading to lower surplus for buyers. Instead, our paper analyzes competition between websites and we obtain that price dispersion can occur not for sellers' prices but for the platforms.

Finally, our paper is connected to the literature obtaining mixed-strategy equilibria in price competition, as is often the case in the seach literature (Varian, 1980; Janssen and Moraga-Gozález, 2004). If firms are symmetric, the prices in the mixing domain are usually atomless as the best response is to slightly undercut the rival's price. If firms are asymmetric, this is no longer true. The distribution of the firm with lower quality or higher cost often entails a mass point at the price where its profit equals zero (see e.g., Narasimhan, 1988, or De Corniére and Taylor, 2014). In our equilibrium, mass points also exist with symmetric firms because the best responses involve a discrete increase or decrease in the price relative to the one of the rival.

The rest of the paper is organized as follows: The next Section sets out the model. Section 3 determines the equilibrium. Section provides an extension to multi-homing firms and Section 5 analyzes the effects of multi-homing sellers. Section 6 concludes. All proofs are in the Appendix.

2 The Model

There are three types of agents in our model: platforms, firms, and consumers. We describe the agents in turn.

Platforms

Two homogeneous platforms A and B offer listing services. The platforms bring together sellers and buyers of products. To be active on platform i, a firm has to pay a listing fee f_i, $i \in \{A, B\}$.[6] In the base model, consumers can access platforms for free.

Firms

Firms (or sellers) have to decide which, if any, platform to join. In the base mode, they cannot be active on both platforms (i.e., firms single-home).[7] The product of each firm belongs to a product category. There is a mass 1 of such categories, indexed by $k \in [0, 1]$. There are two sellers per product category and a platform can accommodate up to two sellers per product category.

For simplicity, we assume that the two sellers in each category are symmetric. Firms set uniform prices to consumers. We denote the symmetric equilibrium duopoly price by p^d and the monopoly price by p^m. Equilibrium profits per consumer in duopoly are denoted by π^d. If consumers can buy from only one of the firms because only one firm is listed on the platform that consumers are visiting, this firm makes monopoly profits π^m per consumer. Our formulation implies that per-consumer profit in duopoly and monopoly are independent of the number of consumers. At the end of this section, we provide two illustrations that generate π^d and π^m from two widely-used demand functions. However, as will become clear later, our qualitative results do not depend on π^d and π^m being independent of the number of consumers, but hold more generally.

Consumers

Each consumer (or buyer) single-homes, that is, she decides to be active on (up to) one platform.[8] On the chosen platform, each consumer makes a choice among the products encountered on the platform; this includes the option not to buy. She is interested in a single product category and derives a positive gross utility from products in this category; products in all other segments give zero utility.[9] There is mass 1 of consumers per product category. When visiting a platform, a consumer becomes informed about the product category and the price of all products listed on the platform. If a platform lists all products from a fraction $\alpha \in [0, 1]$ of all product categories, then consumers expect to find a product from the liked category with probability α.

A consumer obtains a different (indirect) utility if one or two sellers are listed in her preferred category. Consider a consumer who has found at least one listed seller in

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[6] Such listing or posting fees are prevalent in markets in which the platform cannot or does not monitor the sale of a product. For example, in the housing or the rental market, platforms posting ads usually charge listing fees. Also, the portal craigslist.org charges listing fees for posting ads for cars/trucks or therapeutic services.

[7] In Section 5, we sow hat our main results carry over to the case with multi-homing firms.

[8] In Section 4, we provide the analysis with multi-homing consumers and demonstrate that our main insights remain valid.

[9] See, e.g., Burguet, Caminal, and Ellman (2014) for a similar structure.

the category she is interested in. Prior to observing the idiosyncratic taste realization within this category, this consumer obtains expected utility $V(p^d)$ because sellers charge duopoly prices if two products are listed in the category. By contrast, if only one product is listed so that the seller charges the monopoly price the consumer obtains an expected utility of $V(p^m) < V(p^d)$.

Timing

The timing is as follows:

1. Platforms A and B set listing fees f_A and f_B, respectively.

2. Firms and consumers make a discrete choice between platform A and B, and the outside option (normalized to zero).

3. Firms in each category set product prices p.[10]

4. Consumers observe all offers on the platform they are visiting and make their buying decisions.

We make two observations regarding our setup: First, according to our timing firms decide where to list before setting their prices on the product market. This is the relevant timing in most applications because the decision on which platform to list is typically more long term than the pricing decision. In our setting, firms set prices after learning about the number of competitors in the product market.[11] Second, listing fees do not enter the pricing decisions of the firms in the third stage because they are "fixed" costs for firms (which are, in addition, sunk when firms set prices). Hence, the market for listing services is in fact two-sided.[12]

Payoffs

For simplicity, we assume that all platform costs are zero. The profit of platform i is then the number of sellers active on platform i multiplied by the listing fee f_i. The profit of a firm which is listed on platform i is $x_i\pi - f_i$, where x_i is the fraction of consumers in the firm's category, who are active on platform i, and π is either π^m if the rival seller is not listed on platform i or π^d if the rival also lists on platform i. As mentioned above, the utility of a consumer is $V(p^d)$ if both sellers of the preferred product category are listed on the platform she has joined, $V(p^m)$ if only one of those two sellers is listed on the platform, and 0 if none of those sellers is listed on the platform.

[10] Firms set their prices independently at stage 3; for ease of exposition, we dropped the index of the firm.

[11] In addition, listing fees are often paid on a subscription basis. This makes them lumpy. By contrast, prices charged by the firms are flexible.

[12] Allowing for fees on both sides of the market and holding retail prices fixed, lowering the price on one side and reducing it on the other side affects the rents on both sides of the market. However, sellers will indeed not change retail prices in response and, therefore, they will not neutralize the change in fees. Hence, the market is two-sided in the terminology of Rochet and Tirole (2006).

Solution Concept

Our solution concept is subgame perfect equilibrium. We impose a selection rule to choose among multiple equilibria at stage 2. Specifically, in the second stage, firms and consumers coordinate on the equilibrium that is preferred by firms at this stage. A justification of this refinement is that the outcome is equivalent to the outcome of a sequential game in which sellers decide which platform to join before consumers do, as considered in the models by Hagiu (2006) and Lee (2014), and sellers select the coalition-proof equilibrium.[13]

Because of symmetry, if consumers expects one firm in each category to list on platform A and the other firm to list on platform B, consumers are indifferent between the two platform. We assume that in this case half of the consumers in each category join platform A and the other half platform B. A natural interpretation is that each consumer mixes with equal probability to be active either on platform A or B. Since there is a continuum of consumers, both platforms will in fact be patronized by 1/2 of the consumers. Another interpretation is that platforms are differentiated by different platform designs (with half of the consumers in each category preferring platform A and the other half platform B) but that this differentiation is negligibly small.[14] This means that consumers ex ante have lexicographic preferences in the sense that they prefer the platform that has a higher probability to list a product in the consumer's preferred category. Only if consumers expect these probabilities to be the same across platforms, they decide according to their preference for different platform designs.

Examples on buyer-seller interaction

We provide two examples of widely-used demand functions (i.e., Hotelling and linear demand as in Singh and Vives, 1984), to provide explicit expression for π^d and π^m.

Example 1: Hotelling.

Consider Hotelling competition in each product category. Each firm is located at one of the extreme points of the unit interval in a particular category; i.e., a firm is characterized by its category and its location on the unit interval, $(i, l_i) \in [0, 1] \times \{0, 1\}$. The consumers' valuation of an product at the ideal locaton in the preferred category equals v. The utility of a consumer who likes category k and is located at x_k, $(k, x_k) \in [0, 1] \times [0, 1]$, for a product which belongs to category i and is located at l_i, $(i, l_i) \in [0, 1] \times \{0, 1\}$, is $= v - t|x_k - l_i| - p_{i,l_i}$ for $i = k$ and 0 otherwise, where the parameter t captures the degree of product differentiation. The higher is t the more products are differentiated. Price competition among Hotelling duopolists leads to equilibrium prices $c + t$ and equilibrium profits $\pi^d = t/2$ per unit mass of consumers. The monopoly seller

[13] We provide the details in the appendix.

[14] For example, platforms are differentiated along a Hotelling line and the transport cost parameter t goes to zero.

sets price $(v+c)/2$ and its profit is $\pi^m = (v-c)^2/(4t)$ per unit mass of consumers, if the market is not fully covered. This is the case if $2t \geq v-c$. If the reverse inequality $2t < v-c$ holds, there is full coverage under monopoly and the monopolist sets $\pi^m = v-t$. Its profit is $\pi^m = v-t-c$. Hence, for $2t < v-c$, the monopoly profit decreases linearly in the degree of product differentiation.

Example 2: Linear demand for differentiated products by representative consumer.

Consider that consumers with the same preferred category have utility function $v = q_1+q_2-1/2\beta(q_1^2+q_2^2)-\gamma q_1 q_2 - p_1 q_1 - p_2 q_2$. This is representative consumer setting, where each consumer obtains utility from positive quantities of each product in her preferred category. Maximizing this utility function with respect to q_1 and q_2, we obtain the indirect demand functions $p_i = 1-\beta q_i - \gamma q_{-i}$, $i = 1,2$. Inverting this demand system yields the direct demand functions

$$q_i = \frac{\beta-\gamma-\beta p_i + \gamma p_{-i}}{\beta^2-\gamma^2}, \quad i = 1,2.$$

Duopoly equilibrium profits are

$$\pi^d = \frac{\beta(\beta-\gamma)(1-c)^2}{(\beta+\gamma)(2\beta-\gamma)^2}$$

per consumer.

For a monopolist, the direct demand is

$$q_i = \frac{1-p_i}{\beta}.$$

and monopoly profits are

$$\pi^m = \frac{(1-c)^2}{4\beta}.$$

Joint profits under duopoly are larger than monopoly profits if and only if $4\beta^3 + 3\beta\gamma^2 > 8\beta^2 + \gamma^3$, which holds is γ is sufficiently small.

3 Equilibrium Analysis

In our setting, two types of equilibria can arise at stage 2. First, firms and consumers may all choose the same platform giving rise to an agglomeration equilibrium. Second, firms in each product category may list with different platforms, inducing consumers to split between the two platforms. We call such a situation a segmentation equilibrium. From the firms' perspective, this equilibrium has the advantage that each firm is in a monopoly position vis-a-vis consumers. It has the disadvantage that each firm can only reach half of the consumers. Therefore, when co-locating with their competitors, firms sacrifice some profits per consumer but reach more consumers. If the first effect dominates, market tipping may not occur in equilibrium. This effect may break the Bertrand logic that platforms undercut each other until price equals marginal costs, and gives rise to the possibility of strictly positive equilibrium profits in a segmentation equilibrium.

We solve the model by backward induction. Consumers' choices and firms' pricing decisions in any subgame reached at stage 3 are straightforward: in the fourth stage, consumers buy a product in their preferred product category given that there is one according to their demand function. In the third stage, firms listed on a platform know whether they face a competitor in their product category or not. They therefore set price p^d in case of competition and p^m in case of monopoly, giving rise to π^d and π^m, respectively.

In stage 2, the two equilibrium configurations of agglomeration and segmentation are possible. In an agglomeration equilibrium, all consumers are informed about the offers of both firms. Agglomeration on platform i then gives rise to a firm profit of $\pi^d - f_i$ and a consumer utility of $v(p^d)$. Instead, in a segmentation equilibrium half of the consumers are active on platform i and the other half on platform $-i$ because firms in each category also locate on different platforms. Therefore, a firm's profit when being active on platform i is $\pi^m/2 - f_i$, whereas the utility of a consumer is $v(p^m)$. Because of our selection criterion, for any combination of fess (f_A, f_B), the equilibrium that is preferred from the firm's perspective will be played.

As an example, suppose that platforms charge listing fees of $f_A = 0$ and $f_B = \pi^d$. Then, agglomeration on platform B can never occur in equilibrium since agglomeration on A gives higher profits to firms. With agglomeration on platform A, the profit of each firm is π^d. Instead, in an equilibrium in which firms segment, the profit of a firm locating on platform B is $\pi^m/2 - f_B = \pi^m/2 - \pi^d$.[15] Therefore, firms will segment if $\pi^m/2 - \pi^d > \pi^d$ or $\pi^m > 4\pi^d$.

We now turn to the first stage. Proposition 1 describes the equilibrium listing fees and the associated (expected) profits for all parameter ranges:

Proposition 1

(i) *Agglomeration.* If $\pi^d \geq 1/2\pi^m$, in the unique equilibrium, the equilibrium listing fees are $f_A^\star = f_B^\star = 0$, platforms' profits are $\Pi_A^\star = \Pi_B^\star = 0$.

(ii) *Segmentation or agglomeration with listing fees chosen from a a convex set.* If $3/8\pi^m \leq \pi^d < 1/2\pi^m$, under the refinement, there is a unique symmetric mixed-strategy

[15]Note that consumers correctly anticipate that in equilibrium firms segment, which implies that half of consumers are active on platform B.

equilibrium, in which platforms set fees in the domain $f_i \in [\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$. The expected profit is $\Pi_A^\star = \Pi_B^\star = 3\pi^m/2 - 3\pi^d$.
(iii) *Segmentation or agglomeration with listing fees chosen from a non-convex set.* If $1/4\pi^m \leq \pi^d < 3/8\pi^m$, under the refinement, there is a unique symmetric mixed-strategy equilibrium, in which platforms set fees in the domain $f_i \in [\pi^m/4, \pi^d) \cup [3\pi^m/4 - \pi^d, \pi^m/2]$. The expected profit is $\Pi_A^\star = \Pi_B^\star = 3\pi^m/4 - \pi^d$.
(iv) *Segmentation with deterministic listing fees.* If $\pi^d < 1/4\pi^m$, under the refinement, the equilibrium listing fess are $f_A^\star = f_B^\star = \pi^m/2$, platforms profits' are $\Pi_A^\star = \Pi_B^\star = \pi^m/2$.

The proposition shows that there are four different regimes. We start by considering the extreme regimes (i) and (iv), in which platforms choose pure strategies in equilibrium at stage 1. If duopoly profits are relatively large such that industry profits under duopoly exceed monopoly profits (i.e, $\pi^d \geq 1/2\pi^m$), regime (i) applies. From a firm's point of view, the effect that agglomeration reduces profits due to firm competition is dominated by the demand expansion effect that all consumers instead of half of them consider the firm's offer. Recall that firms coordinate on the profit-dominant equilibrium at stage 2. Since each platform can obtain all demand by setting a lower fee than its rival, platforms engage in Bertrand competition and equilibrium fees of zero. Thus, in region (i) the classic Bertrand argument applies and competing homogeneous platforms obtain zero profits in equilibrium.

If instead duopoly profits are particularly low (i.e., $\pi^d < 1/4\pi^m$), regime (iv) applies. Firms avoid competition by choosing segmentation. This can be exploited by platforms. To see this, suppose that both platforms charge a fee of zero. If π^d is lower than $\pi^m/2$, firms choose to segment. But then a platform can raise its fee slightly without loosing any firms. Thus, the platform with the higher fee remains active and raises strictly positive profits. In regime (iv), platforms can extract the full surplus from firms. The argument is as follows. When a platform deviates from the equilibrium listing fee $f_i^\star = \pi^m/2$ to a listing fee slightly below π^d, this induces firms and buyers to agglomerate on this deviating platform. The deviant platform then makes profit $2\pi^d$. The profit in equilibrium is instead equal to $\pi^m/2$ which is greater than $2\pi^d$. Hence, in regime (iv) no platform has an incentive to deviate from the subscription fee $\pi^m/2$. To sum up, if competition between firms is sufficiently intense, platforms obtain positive profits by inducing firms to segment the market. This result obtains despite the fact that platforms offer the same matching service. Interestingly, fierce competition among firms enables platforms to sustain high profits in equilibrium.

We now turn to regimes (ii) and (iii), in which platforms randomize over subscription fees. The intuition for the non-existence of a pure-strategy equilibrium in the associated range of π^m and π^d is as follows: For any fee set by platform i, the competing platform $-i$'s best response is to either set a lower fee to induce agglomeration on its platform or set a higher fee, which leads to firm segmentation, where platform $-i$ receives higher profits than platform i. Suppose that platform i sets a relatively high fee. The competing platform $-i$ then optimally sets a fee that is discontinuously lower, so as to just induce agglomeration. The optimal response of platform i is to lower its fee slightly and induce agglomeration again. This sequence of best responses continues until the fee of platform i reaches a level that further adjustments by platform $-i$ to induce agglomeration is no longer its best response, but instead platform $-i$ prefers to set a fee that is discontinuously higher than the one of platform i, so as to just induce segmentation. In turn, it is then the best response of platform i to reduce its fee slightly to induce agglomeration and the sequence continues and does not converge.

From the argument above, it is evident that the range of subscription fees over which platforms mix can be divided into two intervals, a lower and an upper interval. In the lower interval, fees are set with the intention to induce agglomeration. In the upper interval, fees are set with the intention to induce segmentation. In region (ii), the upper bound of the lower interval coincides with the lower bound of the upper interval and platforms randomize over the interval $[\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$. The lower interval is $[\pi^m - 2\pi^d, 3/2\pi^m - 3\pi^d)$ and the upper interval is $[3/2\pi^m - 3\pi^d, 2\pi^m - 4\pi^d]$. Setting a fee of $3/2\pi^m - 3\pi^d$ therefore induces segmentation with probability 1. As will become clear later, there is a mass point on this fee $3/2\pi^m - 3\pi^d$. Since the event that both firms choose this fee occurs with strictly positive probability, the expected equilibrium profit in regime (ii) must equal $3/2\pi^m - 3\pi^d$.

Let us now turn to regime (iii). The maximal fee that platforms can possibly charge to obtain positive demand is $f_i = \pi^m/2$. As π^d decreases and reaches $3/8\pi^m$, the upper bound in the upper interval of regime (ii), $2\pi^m - 4\pi^d$, reaches this level. For higher π^d, the two intervals that form the support of the price distibution become separate. This implies that the support becomes non-convex and we enter regime (iii). Here, platforms set listing fees in the set $[\pi^m/4, \pi^d) \cup [3\pi^m/4 - \pi^d, \pi^m/2]$. As in region (ii), the fee $3\pi^m/4 - \pi^d$ induces segmentation with probability 1, and this fee is chosen with strictly positive probability (i.e., $3\pi^m/4 - \pi^d$ is a mass point in the distribution over subscription fees). Therefore, the expected equilibrium profit in regime (iii) must equal $3\pi^m/4 - \pi^d$.

The argument why only a mixed-strategy equilibrium exists in regions (ii) and (iii) is reminiscent of Bertrand-Edgeworth-cycles. However, different from these cycles, in our equilibrium, platforms charge different subscriptions fees because one of them intends to induce agglomeration whereas the other intends to induce segmentation. This can lead to two disjoint intervals from which subscription fees are chosen, something which cannot happen in a Bertrand-Edgeworth cycle.

In the two examples given in the previous section, the degree of competition determines π^d relative to π^m. Therefore, the boundaries of the regions can be expressed with the parameter indicating how differentiated firms are.

In the Hotelling example, this degree is given by t, that is, a higher t represents more differentiation. We obtain that region (i) occurs for $t \geq (v-c)/2$, region (ii) if $3(v-c)/7 \leq t < (v-c)/2$, region (iii) if $(v-c)/3 \leq t < 3(v-c)/7$, and region (iv) if $t < (v-c)/3$.

In example 2, the boundaries of the regions can be determined with the help of $\gamma \in [0, \beta]$. A higher γ means fiercer competition. Region (i) occurs approximately for $\gamma \leq 0.62\beta$, region (ii) if $0.62\beta < \gamma \leq 0.74\beta$, region (iii) if $0.74\beta < \gamma \leq 0.85\beta$, and region (iv) if $\gamma \geq 0.85\beta$.

We note that the (expected) equilibrium platform profit as a function of π^d is continuous and has three kinks. The profit is 0 for $\pi^d \geq 1/2\pi^m$. For $3/8\pi^m \leq \pi^d < 1/2\pi^m$, the profit is $3\pi^m/2 - 3\pi^d$. At the boundaries, this implies that the profit is 0 as π^d approaches $\pi^m/2$. At $\pi^d = 3/8\pi^m$, the profit is $3\pi^m/8$. For $1/4\pi^m < \pi^d < 3/8\pi^m$, the profit is $3\pi^m/4 - \pi^d$, which goes to $3\pi^m/8$ as π^d approaches $3\pi^m/8$, and is $\pi^m/2$ for $\pi^d \to 1\pi^m/4$. Finally, if $\pi^d \leq 1/4\pi^m$, the profit is $\pi^m/2$.

The mixed-strategy equilibrium implies that the market outcome in industries with similar conditions might be very different. Specifically, in some markets agglomeration takes place and one platform has the lion's share of demand. By contrast, as pointed out in the introduction, there are other markets which look very different. In many European countries two (or more) platforms are active in the rental market for flats and compete on almost on equal grounds. For example, in Germany Immobilienscout 24 and Immowelt are widely used by buyers and firms and often have exclusive offers (and, thus, induce segmentation).

Regarding welfare properties, we find that the market equilibrium may not be welfare maximizing in a second-best sense. Suppose that the social planner cannot control firm prices but the market structure. This brings out the trade-off between the agglomeration and the segmentation equilibrium. Welfare is the sum of platforms' and firms' profits and consumer welfare. Since platforms charge listing fees, the fees are just transfers from firms to platforms. They therefore do not enter the welfare function directly, but affect welfare as they determine whether agglomeration or segmentation prevails. Welfare in the agglomeration equilibrium is then given by $v(p^d) + 2\pi^d$ and welfare in segmentation equilibrium is $v(p^m) + \pi^m$. There are two inefficiencies in the segmentation equilibrium. First, because consumers are not informed about all prices, there is mismatch between a consumer's preference and the firm's offer. Ann agglomeration equilibrium avoids such mismatch. Second, because $p^m < p^d$, the quantity bought by consumers in an agglomeration equilibrium is (weakly) higher. Both effects imply that welfare in the agglomeration equilibrium is higher than in the segmentation equilibrium. However, platforms induce segmentation if competition between firms is fierce because their incentives are driven by p^m versus p^d. Instead, welfare considerations are driven by $v(p^m)$ versus $v(p^d)$.

So far, we focused on the listing fees charged in equilibrium but did pin down the distribution over subscription fees in the mixed-strategy equilibria of regimes (ii) and (iii). Proposition 2 complements Proposition 1 by characterizing the distribution.

Proposition 2
In region (ii), the mixing probability is characterized by a cdf of

$$G(f) = \begin{cases} \frac{f-(\pi^m-2\pi^d)}{f+1/2(\pi^m-2\pi^d)}, & \text{if } f \in [\pi^m - 2\pi^d, 3/2\pi^m - 3\pi^d); \\ \frac{2f-5/2(\pi^m-2\pi^d)}{f-1/2(\pi^m-2\pi^d)}, & \text{if } f \in [3/2\pi^m - 3\pi^d, 2\pi^m - 4\pi^d]. \end{cases}$$

There is a mass point at $f = 3/2\pi^m - 3\pi^d$ with point mass $1/4$. The corresponding generalized density is given by[16]

$$g(f) = G'(f) + \frac{1}{4}\delta^D(f - (3/2\pi^m - 3\pi^d)),$$

where $\delta^D(f - f_0)$ denotes Dirac's delta function which is 0 everywhere except for f_0 where it is ∞. Furthermore, $\int \delta^D(f - f_0)df = 1$.
In region (iii), the mixing probability is characterized by a cdf of

$$G(f) = \begin{cases} \frac{f-1/4\pi^m}{f+1/2(\pi^m-2\pi^d)}, & \text{if } f \in [\pi^m/4, \pi^d); \\ \frac{2f-1/4\pi^m-3/2(\pi^m-2\pi^d)}{f-1/2(\pi^m-2\pi^d)}, & \text{if } f \in [3\pi^m/4 - \pi^d, \pi^m/2); \\ 1, & \text{if } f = \pi^m/2. \end{cases}$$

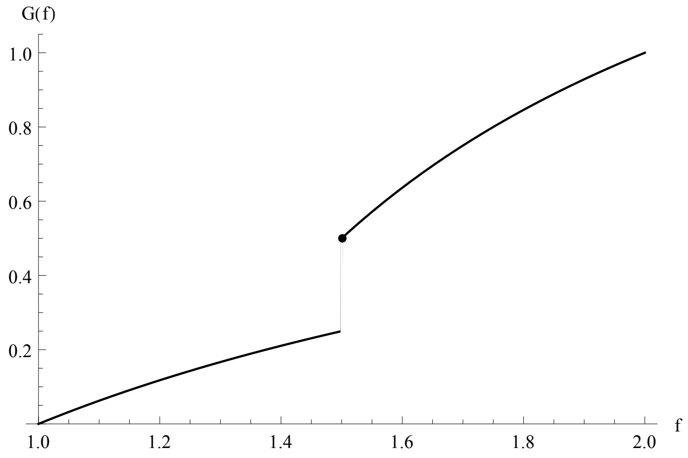
There are mass points at $f = 3\pi^m/4 - \pi^d$ and $f = \pi^m/2$. The respective point masses are $(3/4\pi^m - 2\pi^d)/\pi^d$ and $(2\pi^d - 1/2\pi^m)/\pi^m$. The corresponding generalized density is given by

$$g(f) = G'(f) + \frac{3/4\pi^m - 2\pi^d}{\pi^d}\delta^D(f - (3\pi^m/4 - \pi^d)) + \frac{2\pi^d - 1/2\pi^m}{\pi^m}\delta^D(f - \pi^m/2).$$

The cdf and the corresponding generalized density in regime (ii) are illustrated in Figures 1 and 2, respectively. There is a mass point at $3/2\pi^m - 3\pi^d$. [17] The mass point

[16]Because the distribution is not absolutely continuous with respect to the Lebesgue measure, it fails to have a density. Nevertheless, we define a generalized density, which is a generalized function (since it will be comprised of a dirac delta function) such that integration against this generalized function yields the correct desired probabilities.

[17]Mixed-strategy equilibria in symmetric oligopoly models typically do not feature mass points (e.g., Varian, 1980, or Moraga-Gonzales and Janssen, 2004). A number of contributions find mass points in asymmetric oligopoly models in which one firm is disadvantaged and therefore obtains zero profits in



Parameters are $\delta = 1/2$ which leads to $f \in [1, 2]$.

Figure 1: First Mixed-Strategy Equilibrium: Cumulative Distribution Function

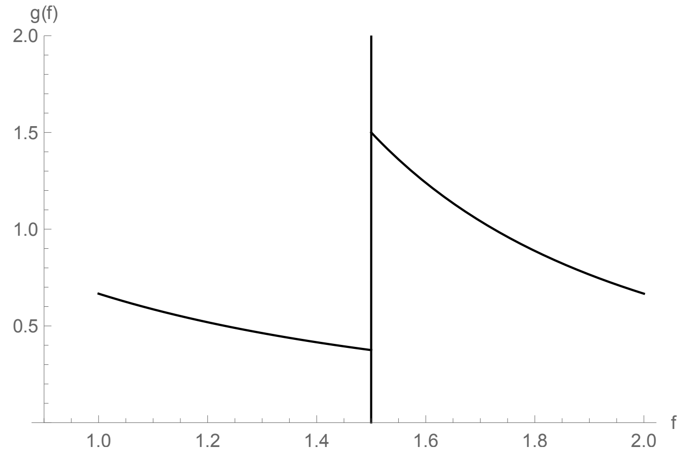
is at the fee which separates the interval of fees that are intended to induce agglomeration from those that are intended to induce segmentation. As explained above, the fee always leads to segmentation, and the platform's profit when charging this fee is certain.

The question arises why there is no mass point in the best response to $3/2\pi^m - 3\pi^d$. In Varian's (1980) seminal model of sales indeed mass points can be excluded because of such a type of best response. The answer is that the best response is a downward jump in the fee (given that firms play the agglomeration equilibrium in case they are indifferent) and not just a marginal undercutting. But this implies that there is a continuum of fees between $3/2\pi^m - 3\pi^d$ and the best response $\pi^m - 2\pi^d$, to which $\pi^m - 2\pi^d$ does not constitute a best response. Because this continuum of fees has positive probability, there is no mass point at $\pi^m - 2\pi^d$.

Figures 3 and 4 illustrate $G(f)$ and $g(f)$ of regime (iii). It is evident that in this regime the equilibrium strategy features two mass points, one at the highest fee in the support and the other at the lower bond of the upper interval. The intuition for the latter mass point (at $f = 3\pi^m/4 - \pi^d$) is the same as the one given in regime (ii). This fee induces segmentation with probability 1 and a platform sets this fee with positive probability.

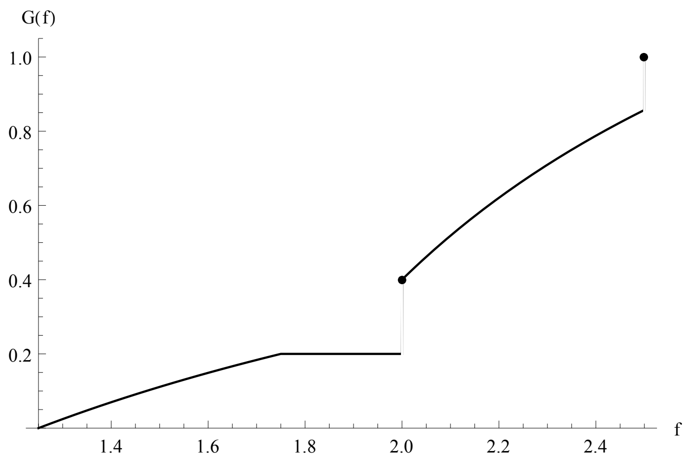
The intuition for the mass point at $f = \pi^m/2$ is different. It is rooted in the fact that charging a fee equal to upper bound of the lower interval (i.e., π^d) to induce agglomeration is only optimal if the rival platform charges $\pi^m/2$. For all other fees in the upper interval, charging a fee equal (or close to) π^d does not lead to agglomeration, implying that a lower or a higher fee does strictly better. As a consequence, to render the fee π^d optimal, the rival platform must set the highest fee with a strictly positive probability. Otherwise, a

equilibrium; see, among others, Narasimhan (1988) and de Corniére and Taylor (2015).



Parameters are $\delta = 1/2$ which leads to $f \in [1, 2]$.

Figure 2: First Mixed-Strategy Equilibrium: Generalized Density



Parameters are $\delta = 1/2$ and $\eta = 3/4$ which leads to $f \in [5/4, 7/4) \cup [2, 5/2]$.

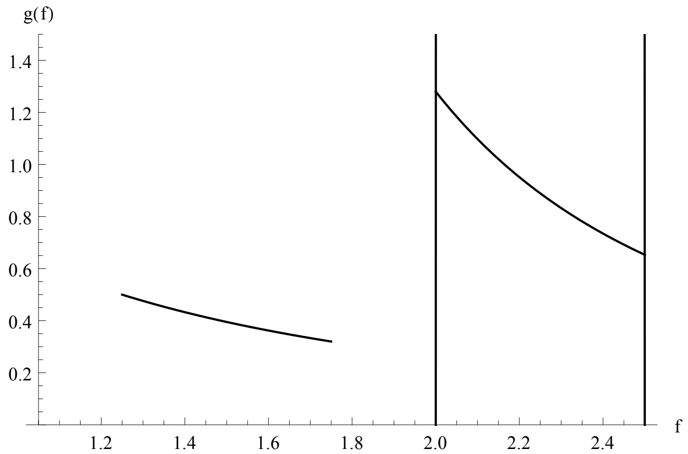
Figure 3: Second Mixed-Strategy Equilibrium: Cumulative Distribution Function

fee equal to π^d can never be part of the mixing domain, and an equilibrium would fail to exist. Again, reacting to a fee of $\pi^m/2$ by also playing the best response with strictly positive probability cannot be optimal because this best response is not the optimal reaction to all fees between $\pi^m/2$ and the best response.

4 Multi-Homing of Consumers

So far, we focused on the case in which consumers are single-homing. In this section, we show that our qualitative results extend to multi-homing consumers, as long as not all consumers multi-home.

To this end, assume that a fraction $\alpha \in [0, 1]$ of consumers join both platforms.



Parameters are $\delta = 1/2$ and $\eta = 3/4$ which leads to $f \in [5/4, 7/4) \cup [2, 5/2]$.

Figure 4: Second Mixed-Strategy Equilibrium: Generalized Density

An interpretation is that consumers incur some time cost to be active on the second platform. Consumers are heterogeneous with respect to these time costs, implying that only those consumers with low enough time costs are active on both platforms. A higher α can then be interpreted as a reduction in time costs.[18]

The profits of the firms need then to be modified. In fact, a firm will never obtain the full monopoly profit because there are always some consumers who have seen the offers of both firms in each category. In particular, in the segmentation equilibrium where one firm lists on platform A and the other on platform B, half of the single-homing consumers are active on platform A and the other half on platform B. Because there is a mass $1-\alpha$ of single-homing consumers, each firm has a mass of $(1-\alpha)/2$ of single-homing consumers and a mass α of multi-homing consumers. The total consumer mass is then $(1+\alpha)/2$.

Firms do not know which consumer is a single-homing and which one is a multi-homing one and set a single price in the product market. Since single-homers are less price-sensitive than the multi-homers, the equilibrium price with only single-homers is larger than that with only multi-homers. Hence, with a single price in the product market, the equilibrium price depends on α. We can therefore write the expected profit that a firm obtains from a consumer as $\pi(\alpha)$. In particular, $\pi(0) = \pi^m$ and $\pi(1) = \pi^d$. As α gets larger, we obtain $\pi'(\alpha) \leq 0$; hence, for all $\alpha \in [0,1]$, $\pi(\alpha) \in [\pi^d, \pi^m]$. Below, we will show how a change in α plays out in the two examples of demand functions.[19]

Deriving the equilibrium with multi-homing consumers, we the obtain the following

[18]For example, if distribution S of time costs among the consumers first-order stochastically dominates distribution S', then the latter distribution leads to a higher fraction α of multi-homing consumers.

[19]In any agglomeration equilibrium, a firm's profit is unchanged since all consumers see both offers. This leads to a profit of π^d for each firm.

proposition:

Proposition 3
All results of Propositions 1 and 2 carry over to the case of consumer multi-homing with the exception that $\pi^m/2$ needs to be replaced by

$$\pi(\alpha)\frac{1+\alpha}{2}.$$

The proposition shows that the qualitative result of Propositions 1 and 2 remains valid if consumers can multi-home. With multi-homing consumers, segmentation does not give firms monopoly power over their consumers. Segmentation nevertheless lowers the competitive pressure because some consumers are still only informed about one firm's offer, and this will be exploited by platforms.

The question arises if platforms benefit from multi-homing of consumers. If we are in the range of the agglomeration equilibrium, nothing changes compared to single-homing consumers because platforms are in Bertrand competition. However, this is not true for the regions in which the segmentation equilibrium occurs with some or full probability. There are two countervailing forces. First, platforms have more consumers, which leads to a larger demand for firms. This allows platforms to charge higher listing fees and is therefore beneficial to platforms' profits. This effect can also be seen in the formulas: Instead of serving a consumer mass of 1/2 (as with single-homing consumers), platforms now have a mass of $(1+\alpha)/2$ of consumers. However, the countervailing force is that firms make smaller profits in the product market because some consumers are now informed about both offers. As can be seen in the formulas, the profit is now $\pi(\alpha) < \pi^m$. It follows that platforms are hurt by the possibility of consumers to multi-home if the competition effect dominates the demand-enhancing effect.

We can illustrate this result with the example of the concrete demand functions. In the case of Hotelling demand, $\pi(\alpha)$ is given by

$$\frac{(2-\alpha)(\alpha t + 2(v-c)(1-\alpha))^2}{2t(4-3\alpha)^2}.$$

Comparing $\pi(\alpha)(1+\alpha)/2$ with $\pi^m/2$, we obtain a non-monotonic effect in α. Taking the derivative of $\pi(\alpha)(1+\alpha)/2 - \pi^m/2$ with respect to α at $\alpha = 0$ yields $(v-c)/8 > 0$. This implies that platforms benefit from a small fraction of multi-homing consumers. However, as α gets larger the difference between $\pi(\alpha)(1+\alpha)/2$ and $\pi^m/2$ falls in α. (In the limit, as $\alpha \to 1$, $\pi(\alpha)(1+\alpha)/2 \to \pi^d$, implying that for the whole parameter range, only the agglomeration equilibrium with zero profits for platforms exist.)

A similar picture arises with the linear demand example. Here,

$$\pi(\alpha) = \frac{(\beta-\gamma)(\beta-\gamma^2(1-\alpha)(\beta-\gamma(1-\alpha))^2(1-c)^2}{(\beta+\gamma)(2(\beta-\gamma^2)-\alpha\gamma(1-2\gamma))^2}.$$

Taking the derivative of $\pi(\alpha)(1+\alpha)/2-\pi^m/2$ with respect to α at $\alpha=0$ yields $(\alpha-c)^2/(8(\beta+\gamma)>0$. Therefore, a small fraction of multi-homing consumers increases platforms' profits in this case as well.

5 Multi-Homing of Firms

In this section we consider the effects of multi-homing of firms. In contrast to consumers, firms need to pay for being active on the platforms. Therefore, even without any costs for using a second platform, firms do not necessarily find it profitable to join both platforms.

We therefore proceed in a different way than in the last section by assuming that firms decide to single-home or to multi-home (or not to participate at all). They do not incur any intrinsic costs from doing so but need to pay the listing fees. We are particularly interested if platforms benefit from the possibility that firms can multi-home. The literature on two-sided markets predicts that platforms can exploit the multi-homing side because they do not compete for agents on this side (see e.g., Armstrong, 2006, or Hagiu, 2006). The question is if this is also true in our framework in which agents compete against each other.

With multi-homing firms, new possibilities for the distribution of firms come into play. First, both firms in a segment may multi-home. In that case, all consumers are exposed to the offer of both firms, implying that each firm receives the duopoly profit π^d per consumer. But the profit is then equivalent to that in the situation where both firms agglomerate on one platform. In the latter case, however, firms only have to pay the listing fee of one platform. Therefore, the agglomeration equilibrium configuration is weakly preferred by firms. In fact, firms are indifferent only if both listing fees are zero. We assume that firms choose the agglomeration equilibrium then. This assumption is without loss of generality because both configurations give rise to the same surplus for all agents.

Second, a configuration is possible in which one firm in a category single-homes and the other one multi-homes.[20] Competition in the product market works then differently to the situation described above. In particular, denote the mass of consumers on the

[20]This situation can never occur in equilibrium because the best response of consumers is then to join the platform on which both firms are active. Multi-homing is then not a best response because the firm receives no consumers on one of the platforms where it is active. However, the configuration can occur as a potential deviation.

platform on which both firms are active by x and the mass on the platform in which only one firm is active by $1-x$. Then, there is asymmetric competition in the product market. A mass $1-x$ of consumers can only buy from the multi-homing firm whereas the remaining mass can buy from both firms. Let us denote the profit of the multi-homing firm by $\pi^{MH}(1-x)$ and the profit of the single-homing firm by $\pi^{SH}(x)$. To save on notation we denote $\pi^{SH}(1/2)$ by π^{SH} and $\pi^{MH}(1/2)$ by π^{MH}.[21]

Finally, with multi-homing firms, the situation can occur in which there are multiple equilibria in the fee-setting game between platforms. If this occurs, we use as a selection criterion that platforms coordinate on the profit-dominant equilibrium.

We can now establish the equilibrium with multi-homing firms.

Proposition 4

- In regions (i) and (iv), the equilibrium is the same as the one characterized in Propositions 1 and 2.

- In region (ii), the equilibrium is the same as the one characterized in Propositions 1 and 2 if $\pi^{MH} < 3/2\pi^m - 2\pi^d$.
  Similarly, in region (iii), the equilibrium is the same as the one characterized in Propositions 1 and 2 if $\pi^{MH} < 3/4\pi^m$.

- In regions (ii) and (iii), for $\pi^{MH} \geq 3/2\pi^m - 2\pi^d$ and $\pi^{MH} \geq 3/4\pi^m$, respectively, in equilibrium platforms set fees of $f_A^\star = f_B^\star = 0$ and firms play an agglomeration equilibrium.

The proposition shows that for some parameter constellations, the equilibrium derived in Propositions 1 and 2 stays unchanged. Foremost, if competition between firms is relatively fierce, the segmentation equilibrium still exists. Although firms can multi-home this reduces their profits by too large an amount and so they prefer segmentation. Again platforms exploit this by charging high listing fees. Therefore, our insight that segmentation leads to high platform profits although platforms are homogeneous, is robust to multi-homing of firms.

The proposition also shows that the equilibrium with segmentation occurs for a smaller parameter range than in case of single-homing firms. In this range, platforms charge zero listing fees and obtain zero profits. We therefore obtain the result that platforms can exploit agents less if they multi-home—a result in contrast to the standard insight derived on two-sided markets.

The intuition behind the result is as follows: homogeneous platforms make positive profits because they allow firms to segment themselves and thereby reduce competition

[21]We will provide the formulas in the examples with our demand functions below.

in the product market. If firms can multi-home, segmentation may break down because firms have an incentive to deviate from the segmentation equilibrium. As can be seen in the proposition, segmentation is more likely to break down if the profit of a multi-homing firm π^{MH} becomes large. As a result, platforms can no longer charge high fees and exploit the possibility that they grant monopoly power to firms. The homogeneity of the platforms then drives fees and profits down to zero and, due to the indirect network effects, firms choose agglomeration.

We can illustrate the result with the help of the two examples of Hotelling and linear demand. Determining π^{MH} for Hotelling demand yields

$$\pi^{MH} = \frac{3(2(v-c)+t)^2}{100t}.$$

Comparing this with half of the monopoly profit $(v-c)^2/(8t)$ yields that $\pi^{MH} > \pi^m/2$ if and only if $t > (v-c)(5/\sqrt{6}-2) \approx 0.412(v-c)$. The threshold for t is then in region (iii), which is relevant for values of t between $(v-c)/3 \approx 0.333(v-c)$ and $(v-c)3/7 \approx 0.429(v-c)$. Therefore, for the whole region (ii), the possibility of firms to multi-home destroys the segmentation equilibrium.

The result is even more extreme for the linear demand example. Here,

$$\pi^{MH} = \frac{(\beta-\gamma)(2\beta-\gamma^2)(2\beta+\gamma)(1-c)^2}{2(\beta+\gamma)(4\beta-\gamma-2\gamma^2)^2}.$$

Comparing this with $\pi^m/2 = (\alpha-c)^2/(8\beta)$ yields that $\pi^{MH} > \pi^m/2$ for all $\gamma < 0.89\beta$. Since region (ii) and (iii) are relevant for γ between 0.62β and 0.85β, we obtain the the segmentation equilibrium does not exist in these regions.

6 Conclusion

We propose a model of competing platforms that bring together buyers and sellers. Platforms are homogeneous and set listing fees to sellers who compete and against each other in the product market. Generally speaking, we have analyzed how the competitive environment between agents on one side of the market affects the platform market structure. Based on the Bertrand logic adjusted to platform markets, one may expect that, due to positive network effects, only one platform will be active in the market. As argued in the introduction, this is not what we frequently observe in real life.

Can multiple intermediaries exist and make positive profits, given there is no differentiation between them? We show that the function of multiple intermediaries as a differentiation device for competitive sellers explains such an outcome. To obtain such an outcome, sellers must choose to be active on different platforms and thereby avoiding fierce competition with each other. Platforms can exploit this by demanding positive fees and thus obtain strictly positive equilibrium profits. Thus imperfect competition among sellers can explain that homogeneous platforms can survive in the market and make positive profits. Such a segmentation equilibrium exists if competition between sellers is sufficiently strong. If, by contrast, there is little competition between sellers the standard intuition is confirmed and the equilibrium features agglomeration; i.e., all buyers and sellers go to the same platform.

For moderate degrees of competition between sellers the equilibrium features mixing by platforms. In this equilibrium, agglomeration and segmentation occur with positive probability. The price distribution generically features at least one mass point and its support is, under some condition, non-convex. Overall this paper informs us whether multiple platforms can co-exist as a function of the intensity of competition among sellers.

We have also shown that the possibility of firms to multi-home may break the segmentation equilibrium. The agglomeration equilibrium then occurs on a larger parameter range. Since platforms obtain zero profits in the agglomeration equilibrium, multi-homing of sellers hurts platforms. This insight contrasts with results from two-sided markets with differentiated platforms, which finds that multi-homing agents can be exploited because platforms do not compete for them (e.g., Armstrong, 2006).

In our model, we assumed that platforms do not incur fixed costs independent of the number of participants the platform is catering to. If platforms had (arbitrarily small) fixed cost, instead of an agglomeration equilibrium with zero profits for both platforms,[22] with endogenous entry, only one platform would be present in the market and the market would be a natural monopoly. Thus, with fixed costs only one platform is present and obtains a large profit when there is little competition between sellers. In such a setting fierce competition between sellers is needed to avoid monopolization of the platform market.

[22]Zero profits obtain, since, at the participation stage of buyers and sellers, we selected the equilibrium which is most favorable to sellers.

7 Appendix

Proof of Proposition 1

Consumer behavior at the second stage. Suppose that for a fraction $\beta \in [0,1)$ of product categories, both firms are listed on platform i, for a fraction $\gamma < \beta$, with $\gamma + \beta \leq 1$ both firms are listed on platform $-i$ and for the remaining fraction $1 - \beta - \gamma$ one firm is listed on platform A and the other firm on platform B. Because consumers do not know in the second stage which product category they will be interested in, they strictly prefer to join platform i due to $\beta > \gamma$. Therefore, their best response gives rise to $x_i = 1$ and $x_{-i} = 0$, where x_i denotes the mass of consumers on platform i. Given such consumer behavior, the best response of firms is then also to be active on platform i (or not to be active) because no consumer is active on platform $-i$.

By contrast, if $\beta = \gamma = 0$ (that is, in all categories, firms are listed on different platforms), consumers are indifferent between both platforms and, by assumption, they choose $x_A = x_B = 1/2$. With this consumer choice, the firms best response indeed gives rise to $\beta = \gamma = 0$. A firm then obtains a profit of $\pi^m/2$. A firm that deviates and decides to be active on the platform where its rival is active only obtains a profit of $\pi^d/2$.

From the argument above, it follows that there can be only two types of equilibria: An agglomeration equilibrium and a segmentation equilibrium. In an agglomeration equilibrium, all consumers and all firms go to a single platform. In a segmentation equilibrium, in each product category one firm lists on one platform and the other firm on the other platform. Similarly, half of all consumers are active on platform A and the other half on platform B.

Agglomeration equilibrium. We now turn to the first stage (i.e. the listing fee decisions). Suppose first that the agglomeration equilibrium is played in the second stage. A firm's profit in this equilibrium is $\pi^d - f_i$, if it is listed on platform i. It follows that a firm is willing to participate as long as $f_i \leq \pi^d$. Therefore, an agglomeration equilibrium can be obtained with fees $(f_A, f_B) \in [0, \pi^d] \times [0, \pi^d]$. We focus on equilibria, which are preferred by the firms at stage 2. Firms and consumers will therefore coordinate on the equilibrium at stage 2 such that they list on the platform with the lower fee. As a consequence, all agglomeration equilibria with strictly positive listing fees do not survive our selection criteron. It follows that there is a unique equilibrium within the set of all agglomeration equilibria that survives our selection criterion with listing fees $(f_A^*, f_B^*) = (0, 0)$.

We can now determine under which conditions the agglomeration equilibrium with listing fees $(f_A^*, f_B^*) = (0, 0)$ exists. If firms and consumers in the second stage play the agglomeration equilibrium, a firm's profit is π^d. Instead, if the segmentation equilibrium is played, a firm's profit equals $\pi^m/2$. Hence, given listing fees $(f_A^*, f_B^*) = (0, 0)$, the agglomeration equilibrium is preferred from the firm's perspective if

$$\pi^d \geq \frac{\pi^m}{2}.$$

Now consider listing fees $(f_A, f_B) \neq (0,0)$ but $f_A, f_B \leq \pi^d/2$. It is evident, that, as long as $\pi^d \geq \pi^m/2$, firms prefer the agglomeration equilibrium on the platform with the lower listing fee to the segmentation equilibrium. Therefore, in the region $\pi^d \geq \pi^m/2$, a segmentation equilibrium does not exist. It follows that in this region, the unique equilibrium is an agglomeration equilibrium with listing fees $(f_A^*, f_B^*) = (0,0)$.

Segmentation equilibrium. Let us now turn to the region $\pi^d < \pi^m/2$. In a segmentation equilibrium, a firm active on platform i obtains profits of $\pi^m/2 - f_i$. Therefore, the highest possible fees that platform can charge equals $\pi^m/2$, leaving firms with zero profits. Let us first determine under which conditions an equilibrium with listing fees $\pi^m/2$ exist. If both platforms charge $f_i = \pi^m/2$, the only possible configuration in the second stage is the separating equilibrium. This follows because the profit that a firm obtains in the agglomeration configuration equals π^d, which is below the listing fee. Therefore, we can focus on deviations in the listing fees.

Suppose that platform i deviates to induce an agglomeration equilibrium in the second stage such that all participate on platform i. To do so, it needs to charge a lower fee $f_i^d = \pi^d - \epsilon$, where $\epsilon > 0$ can be arbitrarily small. Since all consumers will agglomerate on platform i if all firms do, firms earn then a small positive profit when agglomerating on platform i but zero in the segmentation equilibrium. The deviation profit of platform i is then (letting $\epsilon \to 0$) $\Pi_i^d = 2\pi^d$. A deviation is therefore not profitable if $\pi^m/2 > 2\pi^d$ or

$$\pi^d < \frac{\pi^m}{4}.$$

It follows that in the region $\pi^d < 1/4\pi^m$, a segmentation equilibrium with listing fees $(f_A^\star, f_B^\star) = (\pi^m/2, \pi^m/2)$ is the unique equilibrium. Platforms' equilibrium profits are $\pi^m/2$.

Non-existence of a pure-strategy equilibrium. Finally, we turn to the region $\pi^m/4 \leq \pi^d < \pi^m/2$. We know that, in this region, a segmentation equilibrium will be played in the second stage if both platforms charge the same listing fees (conditional on these fees being lower than $\pi^m/2$, which will always be fulfilled in equilibrium). However, platforms cannot extract the full profits from firms because then each platform will have an incentive to deviate and induce firms and consumers to play an agglomeration equilibrium. We proceed by first determining the highest fee that platforms can charge to make such a deviation unprofitable. Suppose that both platforms charge a fee of $\pi^m/2 - x$. The platforms' resulting profit is then $\pi^m/2 - x$, whereas the profit of a firm is x. If platform i deviates to attract all firms and consumers in the second stage, it must offer firms at least a profit of x. This implies that its fee must be such that $\pi^d - f_i^{dev} > x$. The highest possible deviation listing fee is therefore $f_i^{dev} = \pi^d - x - \epsilon$, leading to a deviation profit of (letting $\epsilon \to 0$) $2\pi^d - 2x$. Such a deviation is unprofitable if $\pi^m/2 - x \geq 2\pi^d - 2x$ or $x \geq 2\pi^d - \pi^m/2$. Hence, with an x equal to $2\pi^d - \pi^m/2$, platforms prevent such a deviation. The resulting listing fee is then

$$f_i = \pi^m/2 - x = \pi^m - 2\pi^d$$

and the platforms profit is also $\pi^m - 2\pi^d$.

To determine if listing fees $f_i = f_j = \pi^m - 2\pi^d$ can constitute an equilibrium, we need to check if a platform has an incentive to deviate by charging a higher listing fee. Suppose that platform i charges $f_i = \pi^m - 2\pi^d$ and platform j charges a deviation fee $f_j^{dev} > f_i$ such that segmentation is still the continuation equilibrium in the second stage. To induce a segmentation equilibrium, we must have $\pi^m/2 - f_j^{dev} > 3\pi^d - \pi^m$. The right-hand side is the profit that firms obtain when agglomerating on platform i. The inequality therefore states that a firm's profit when listing on platform j in a segmentation equilibrium is higher than in an agglomeration equilibrium. The highest possible listing fee is therefore $f_j^{dev} = 3\pi^m/2 - 3\pi^d - \epsilon = 3(\pi^m/2 - \pi^d) - \epsilon > 2(\pi^m/2 - \pi^d) = f_i$. As a consequence, a profitable deviation exists and both platforms charging listing fees of $\pi^m - 2\pi^d$ cannot constitute an equilibrium.

It follows that in the range $\pi^m/4 \leq \pi^d < \pi^m/2$ no equilibrium in pure strategies exists. The only candidate equilibrium, which prevents downward deviations was $f_i = f_j = \pi^m - 2\pi^d$ but then an upward deviation is profitable. In turn, for all listing fees above $\pi^m - 2\pi^d$, a downward deviation is profitable. We will now characterize the mixed-strategy equilibrium.

Randomization domain. The optimal deviation from $f_i = f_j = \pi^m - 2\pi^d$ is (letting $\epsilon \to 0$) $3(\pi^m/2 - \pi^d)$. We start with the case in which $3(\pi^m/2 - \pi^d) \leq \pi^m/2$. We will check if there can be a mixed-strategy equilibrium, in which the fee $3(\pi^m/2 - \pi^d)$ is the highest one in the randomization domain. Suppose firm j charges $f_j = 3(\pi^m/2 - \pi^d)$. To induce agglomeration on its platform, platform i needs to set a fee such that $\pi^d - f_i > 3\pi^d - \pi^m$. platform i then optimally sets $f_i = \pi^m - 2\pi^d - \epsilon$ to induce agglomeration. This leads to a profit of (letting $\epsilon \to 0$) $2(\pi^m - 2\pi^d)$.

With a fee combination of $f_i = \pi^m - 2\pi^d - \epsilon$ and $f_j = 3(\pi^m/2 - \pi^d)$, the profit of platform j is zero because firms and consumers agglomerate on platform i. The best response of platform j to $f_i = \pi^m - 2\pi^d - \epsilon$ is to reduce its fee marginally (i.e., from $3(\pi^m/2 - \pi^d)$ to $3(\pi^m/2 - \pi^d) - \epsilon$), thereby inducing again segmentation of firms and consumers.[23] We can now determine the best response of platform i to $f_j = 3(\pi^m/2 - \pi^d) - \epsilon$. By the same argument as above, the best response is to marginally reduce its fee to $\pi^m - 2\pi^d - \epsilon$ to induce agglomeration again, and so on.

We can now determine the lowest fee in the randomization domain, given that $3(\pi^m/2 - \pi^d)$ is the highest one. The lowest fee is characterized by the fact that the profit when charging this lowest fee (and induce agglomeration) must be equal to the profit when charging the highest fee (and induce segmentation). Then, marginally lowering the fee to still induce segmentation does not pay off. Denote the lowest fee in the randomization domain by $\pi^m - 2\pi^d - y$, leading to an agglomeration profit of $2(\pi^m - 2\pi^d - y)$. Setting the profit with the lowest fee and the highest fee equal to each other yields

$$2\pi^m - 4\pi^d - 2y = \frac{3\pi^m}{2} - 3\pi^d$$

or $y = \pi^m/4 - \pi^d/2$. Inserting this into $\pi^m - 2\pi^d - y$, we obtain $3\pi^m/4 - 3\pi^d/2$, which is the lower boundary of the mixing domain.

We can now check if a mixed-strategy equilibrium in which the highest fee is $3(\pi^m/2 - \pi^d)$ and the lowest fee is $3\pi^m/4 - 3\pi^d/2$exists. For such an equilibrium to exist, there must be a *unique* fee to which the best response is not to marginally go beyond the lowest fee but to set the highest fee. In other words, if platform j charges this fee, platform i just prefers to set the highest fee instead of the lowest fee.

To determine if such a unique fee exists, let us first derive the fee of platform j such that $f_i = 3\pi^m/4 - 3\pi^d/2$ makes firms indifferent between plaing agglomeration on platform i and playing segmentation. From the calculations above, this fee is given by $3\pi^m/2 - 3\pi^d - y = 5/2(\pi^m/2 - \pi^d)$. Therefore, in a potential mixed-strategy equilibrium, in which the highest listing fee is $3(\pi^m/2 - \pi^d)$, the listing fee that renders a downward deviation unprofitable must $5/2(\pi^m/2 - \pi^d)$. We now need to check if platform i indeed wants to set a listing fee of $f_i = 3(\pi^m/2 - \pi^d)$ if platform j charges $f_j = 5/2(\pi^m/2 - \pi^d)$ or if platform i benefits by setting an even higher listing fee. The highest listing fee that platform i can set to induce segmentation is

$$\frac{\pi^m}{2} - f_i > \pi^d - \frac{5\pi^m}{4} + \frac{5\pi^d}{2}$$

or $f_i < 7/2(\pi^m/2 - \pi^d)$. Since $7/2(\pi^m/2 - \pi^d) > 3(\pi^m/2 - \pi^d)$ platform i optimally reacts to $f_j = 5/2(\pi^m/2 - \pi^d)$ with a listing fee that is above $3(\pi^m/2 - \pi^d)$. As a consequence, there does not exist a mixed-strategy equilibrium in which the highest

[23]Note that the best response of firm j is not to slightly undercut firm i's listing fee because, due to the fact that $\pi^m/2 > \pi^d$, firms will then still play the segmentation equilibrium. This implies that a platform cannot induce the agglomeration equilibrium by slightly undercutting.

listing fee is $3(\pi^m/2 - \pi^d)$.

Iterative procedure We proceed in the same way by checking if there is a mixed-strategy equilibrium, in which the highest fixed fee is $7/2(\pi^m/2 - \pi^d)$. Doing so yields that the lowest listing fee in such a candidate equilibrium is $3/4(\pi^m/2 - \pi^d)$. If the rival platform charges a listing fee of $11/4(\pi^m/2 - \pi^d)$, undercutting is no longer profitable and instead a platform wants to raise is price. However, the highest price that a platform can charge to still induce segmentation is not $7/2(\pi^m/2 - \pi^d)$ but $15/4(\pi^m/2 - \pi^d) > 7/2(\pi^m/2 - \pi^d)$. Therefore, a mixed-strategy equilibrium in which the highest listing fee is $7/2(\pi^m/2 - \pi^d)$ does not exist.

We now iteratively follow this procedure to check if it converges. As we have seen, the optimal deviation from a candidate equilibrium with a highest fee of $2(\pi^m/2 - \pi^d)$ was $3(\pi^m/2 - \pi^d)$. The deviation from the candidate equilibrium with a highest fee of $3(\pi^m/2 - \pi^d)$ was $7/2(\pi^m/2 - \pi^d)$, and the deviation from the candidate equilibrium with a highest fee of $7/2(\pi^m/2 - \pi^d)$ was $15/4(\pi^m/2 - \pi^d)$. If we iteratively apply this method, we obtain

$$\left(2 + \sum_{k=0}^{\infty} \frac{1}{2^k}\right)\left(\frac{\pi^m}{2} - \pi^d\right) = \left(2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right)\left(\frac{\pi^m}{2} - \pi^d\right) = 4\left(\frac{\pi^m}{2} - \pi^d\right).$$

Let us therefore determine if there is a mixed-strategy equilibrium in which the highest fee in the mixing domain is $4(\pi^m/2 - \pi^d)$.

Equilibrium randomization domain. First, we determine the best response to $f_j = 4(\pi^m/2 - \pi^d)$. If platform i responds by a higher fee, the one that still induces segmentation is such that the inequality $\pi^m/2 - f_i > \pi^d - 2\pi^m + 4\pi^d$ is fulfilled. This gives $f_i < 5(\pi^m/2 - \pi^d)$ and a profit of $\Pi_i < 5(\pi^m/2 - \pi^d)$. Instead, if the platform responds by a lower listing fee to induce agglomeration, it optimally does so by charging a fee such that $\pi^d - f_i > \pi^m/2 - 2\pi^m + 4\pi^d$, leading to a fee of $3(\pi^m/2 - \pi^d) - \epsilon$ and a profit of $\Pi_i = 6(\pi^m/2 - \pi^d) - \epsilon > 5(\pi^m/2 - \pi^d)$. Therefore, the platform wants to charge a lower listing fee.

The best response of platform j to $f_i = 3(\pi^m/2 - \pi^d)$ is to marginally lower its fee to $f_j = 4(\pi^m/2 - \pi^d) - \epsilon$ to still induce segmentation. platform i then lowers its fee to $3(\pi^m/2 - \pi^d) - \epsilon$ as a best response, and so on. Let us denote the lowest listing fee in the randomization domain by $3(\pi^m/2 - \pi^d) - y$, which leads to a profit of $6(\pi^m/2 - \pi^d) - 2y$. We will now show that a mixed-strategy equilibrium with a highest fee of $4(\pi^m/2 - \pi^d)$ indeed exists. A platform prefers to set the highest listing fee instead of marginally lowering the listing fee $6(\pi^m/2 - \pi^d) - 2y$ if

$$6(\pi^m/2 - \pi^d) - 2y < 4(\pi^m/2 - \pi^d),$$

or $y = \pi^m/2 - \pi^d - \epsilon$. Therefore, the lowest listing fee (letting $\epsilon \to 0$) is $3(\pi^m/2 - \pi^d) - y = 2(\pi^m/2 - \pi^d)$. We now show that there exists indeed a *unique* fee to which the best response is not to marginally go beyond $2(\pi^m/2 - \pi^d)$ but to set the highest fee. When setting the lowest listing fee, a platform makes firms exactly indifferent between agglomeration and segmentation if the rival platform sets $4(\pi^m/2 - \pi^d) - y = 3(\pi^m/2 - \pi^d)$. It remains to show that the best response to $3(\pi^m/2 - \pi^d)$ is indeed the highest fee $f_i = 4(\pi^m/2 - \pi^d)$. The highest fee that the platform can charge to still induce segmentation is a fee that satisfies $\pi^m/2 - f_i \geq \pi^d - 3\pi^m/2 + 3\pi^d$, which implies $f_i = 4(\pi^m/2 - \pi^d)$.[24]

As a consequence, there is mixed-strategy equilibrium in which $f_i \in [2(\pi^m/2 - \pi^d), 4(\pi^m/2 - \pi^d)]$. By the same token as above, one can show that there is no mixed-strategy equilibrium in which platforms charge a higher price than $4(\pi^m/2 - \pi^d)$. As a consequence, the equilibrium just determined is the unique one. The expected profit in this equilibrium is $3(\pi^m/2 - \pi^d)$. This is because when charging this fee, a platform induces the segmentation with probability 1.

In the mixed-strategy equilibrium just determined, the highest listing fee is $4(\pi^m/2 - \pi^d)$. To ensure participation of firms, the highest fee a platform can charge (in a segmentation equilibrium) is $\pi^m/2$. Therefore, the equilibrium characterized above is only valid if $4(\pi^m/2 - \pi^d) \leq \pi^m/2$ or $\pi^d \geq 3\pi^m/8$.

Non-convex randomization domain. For $\pi^d > 3\pi^m/8$ the highest fee in any mixed strategy equilibrium is $\pi^m/2$. Suppose platform j sets this fee. The best response of platform i is then to set its fee such that it attracts all firms and consumers (i.e., induces an agglomeration equilibrium on its platform). To do so, it needs to set $f_i = \pi^d - \epsilon$. As a best response, platform j wants to marginally reduce its fee to $\pi^m/2 - \epsilon$ and induce a segmentation equilibrium, and so on.

Denote the lowest fee in the mixing domain (i.e., the fee at which a platform prefers to raise its price to $\pi^m/2$ instead of marginally reducing it) by $\pi^d - y$. We have that y is given by $2(\pi^d - y) = \pi^m/2$ or $y = \pi^d - \pi^m/4$. The resulting fee is therefore

$$\frac{\pi^m}{4}.$$

This fee makes firms exactly indifferent between agglomeration and separation if the rival platform charges a fee of $\pi^m/2 - y = 3\pi^m/4 - \pi^d$. It is easy to check that with prices $f_i = \pi^m/2$ and $f_j = 3\pi^m/4 - \pi^d$, we in fact have a segmentation equilibrium in the second stage.[25]

[24]Note that we use a tie-breaking rule in favor of segmentation in this paper. Otherwise, for example, $f_i = 2(\pi^m/2 - \pi^d)$ could trigger agglomeration if the rival platform set $f_j = 3(\pi^m/2 - \pi^d)$.

[25]This is because in an agglomeration equilibrium, firms profits are $\pi^d - f_j = 2\pi^d - 3\pi^m/4$, which is

Finally, note that $\pi^d - \epsilon$ (i.e., the fee that induces agglomeration if the rival platform charges the highest fee) is strictly lower than $3\pi^m/4 - \pi^d$ (i.e., the fee which induces a platform to stop undercutting and instead raise the fee to the highest one) since we are in the range $\pi^m > 3\pi^d/8$. This implies that in the mixed-strategy equilibrium, there are two disjoint sets of mixing ranges. The upper one $[3\pi^m/4 - \pi^d, \pi^m/2]$ is a best response to a fee in the lower range $[\pi^m/4, \pi^d)$, that induces segmentation, whereas a fee in the lower range is intended to induce agglomeration.

Therefore, in the range $3/8\pi^m > \pi^d > \pi^m/4$, there is a symmetric mixed-strategy equilibrium with fees $f_i \in [\pi^m/4, \pi^d) \cup [3\pi^m/4 - \pi^d, \pi^m/2]$. The expected profit in this range is given by $3\pi^m/4 - \pi^d$. As above, this is because setting this fee induces segmentation with probability 1. ■

Proof of Proposition 2:
Let us first look at the range $3/8\pi^m \leq \pi^d < 1/2\pi^m$. In this region, platforms set fees in the domain $f_i \in [\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$ and the expected profit is $\Pi^\star_A = \Pi^\star_B = 3\pi^m/2 - 3\pi^d$.

Let $\delta \equiv (\pi^m/2 - \pi^d)$ and $\epsilon > 0$ but infinitesimally small. Denote $\underline{f} \equiv 2\delta$, $\tilde{f} \equiv 3\delta$, and $\overline{f} \equiv 4\delta$ such that the domain of interest can be expressed as $f_i \in [\underline{f}, \overline{f}] = [2\delta, 4\delta]$. For $i \neq j$ and $i, j \in \{A, B\}$, the corresponding best response function is given by

$$\hat{f}_i(f_j) = \begin{cases} f_j - \delta - \epsilon, & \text{if } f_j \in (\tilde{f}, \overline{f}]; \\ f_j + \delta, & \text{if } f_j \in [\underline{f}, \tilde{f}]. \end{cases}$$

We know that all fees in the mixing domain should give an expected profit of 3δ because setting a fee of 3δ is triggering a segmentation equilibrium with probability 1 yielding a profit of 3δ. In this mixing domain we need to distinguish between two cases, a lower and an upper range. The lower range is $f_i \in [2\delta, 3\delta]$ and the lower range from $f_i \in [3\delta, 4\delta]$. The reason for this distinction is that in the lower range, firms may agglomerate on platform i (i.e., this happens if $f_j > f_i + \delta$) but will never agglomerate on platform j. That is, if f_i is in this lower range, platform i will always obtain a positive profit. By contrast, if f_i is an element of the upper range, with some probability firms will choose to agglomerate on platform j (i.e., this occurs if platform j charges $f_j < f_i - \delta$) and platform i obtains no profit then. This can be expressed as follows

$$\Pi_i(f_i, f_j) = \begin{cases} 0, & \text{if } f_i \in (f_j + \delta, 4\delta] \wedge f_j \in [2\delta, 3\delta); \\ f_i, & \text{if } f_i \in [\max\{2\delta, f_j - \delta\}, \min\{f_j + \delta, 4\delta\}] \wedge f_j \in [2\delta, 4\delta]; \\ 2f_i, & \text{if } f_i \in [2\delta, f_j - \delta) \wedge f_j \in (3\delta, 4\delta]. \end{cases}$$

Let us start with the case in which platform i charges a fee in the lower range, that

negative because $3\pi^m/8 > \pi^d$.

is, $f_i \in [2\delta, 3\delta]$. Denote the cumulative density function with which platform j mixes by $G(f_j)$. The profit of platform i when setting fees in this lower range is then given by (abbreviating f_i by f)

$$G(f+\delta)f + (1 - G(f+\delta))\,2f.$$

In equilibrium, this must be equal to 3δ, yielding a first equation of

$$G(f+\delta)f + (1 - G(f+\delta))\,2f = 3\delta. \tag{1}$$

This equation determines the mixing probabilities of platform j in its upper range. This is because only if platform j sets a fee above $f+\delta$ (which happens with probability $1 - G(f+\delta)$), firms will agglomerate on platform i. Such a fee must necessarily be in the upper range.

In case platform i charges a fee from the upper range, that is, $f_i \in [3\delta, 4\delta]$, the equation is

$$G(f-\delta)0 + (1 - G(f-\delta))\,f = 3\delta. \tag{2}$$

This equation determines the mixing probability in the lower range.

Let us first look at (1). We can substitute $h \equiv f + \delta$ to get

$$G(h)\,(h-\delta) + (1 - G(h))\,2\,(h-\delta) = 3\delta. \tag{3}$$

Therefore, h is the fee in the upper range. Remember that (1) was relevant for f in the lower range and since $h = f + \delta$, these are exactly the fees in the upper range. Solving (3) for $G(h)$ gives

$$G(h) = \frac{2h - 5\delta}{h - \delta}. \tag{4}$$

It is easy to check that $G(4\delta) = 1$.

Now we turn to (2). Here we can substitute $h \equiv f - \delta$ representing that h is now in the lower range. We obtain

$$(1 - G(h))\,(h+\delta) = 3\delta. \tag{5}$$

Solving (5) for $G(h)$ gives

$$G(h) = \frac{h - 2\delta}{h + \delta}. \tag{6}$$

It is easy to check that $G(2\delta) = 0$. Using (4) and (6), we obtain $\lim_{h \searrow 3\delta} = 1/2$ and $\lim_{h \nearrow 3\delta} = 1/4$. This implies the existence of a mass point with mass $1/4$ at $h = 3\delta$. Intuitively, equation (2) requires a sufficiently low probability of $f - \delta$ being close to 3δ because otherwise setting f close to 4δ would lead to zero profit too often due to an agglomeration equilibrium in the lower range. A profitable deviation because of the mass point at 3δ is ruled out by equation (1) and (2).

The resulting mixing probability is characterized by a cdf of

$$G(f) = \begin{cases} \frac{f-2\delta}{f+\delta}, & \text{if } f \in [2\delta, 3\delta); \\ \frac{2f-5\delta}{f-\delta}, & \text{if } f \in [3\delta, 4\delta]. \end{cases}$$

Because the distribution is not absolutely continuous with respect to the Lebesgue measure, it fails to have a density. Nevertheless, we define a generalized density, which is a generalized function (since it will be comprised of a dirac delta function) such that integration against this generalized function yields the correct desired probabilities. The corresponding probability density function is given by

$$g(f) = G'(f) + \frac{1}{4}\delta^D(f - 3\delta),$$

where

$$G'(f) = \begin{cases} \frac{3\delta}{(f+\delta)^2}, & \text{if } f \in [2\delta, 3\delta); \\ \frac{3\delta}{(f-\delta)^2}, & \text{if } f \in [3\delta, 4\delta], \end{cases}$$

and $\delta^D(f - f_0)$ denotes Dirac's delta function which is 0 everywhere except for f_0 where it is ∞. Furthermore, $\int \delta^D(f - f_0)df = 1$. Inserting $\delta = \pi^m/2 - \pi^d$ yields the result stated in the Proposition.

We now turn to the range $1/4\pi^m \leq \pi^d < 3/8\pi^m$, where platforms set fees in the domain $f_i \in [\pi^m/4, \pi^d) \cup [3/4\pi^m - \pi^d, \pi^m/2]$ and the expected profit is $\Pi_A^\star = \Pi_B^\star = 3/4\pi^m - \pi^d$.

Let $\delta \equiv (\pi^d - \pi^m/4)$, $\eta \equiv (\pi^m/2 - \pi^d)$ and $\epsilon > 0$ but infinitesimally small. Denote $\underline{f} \equiv \pi^m/4$, $\overline{f} \equiv \pi^d$, $\underline{f'} \equiv 3/4\pi^m - \pi^d$, and $\overline{f'} \equiv \pi^m/2$ such that the domain of interest can be expressed as $f_i \in [\underline{f}, \overline{f}) \cup [\underline{f'}, \overline{f'}]$. In addition, it holds that $\overline{f} - \underline{f} = \overline{f'} - \underline{f'} = \delta$ and $\overline{f'} - \overline{f} = \underline{f'} - \underline{f} = \eta$. Using that $2\underline{f} = \overline{f'}$ and $\underline{f} + \delta + \eta = \overline{f'}$ yields $\underline{f} = \delta + \eta$. This implies that $f_i \in [\delta + \eta, 2\delta + \eta) \cup [\delta + 2\eta, 2\delta + 2\eta]$. For $i \neq j$ and $i, j \in \{A, B\}$, the corresponding best response function is given by

$$\hat{f}_i(f_j) = \begin{cases} f_j - \eta - \epsilon, & \text{if } f_j \in (\underline{f'}, \overline{f'}]; \\ f_j + \delta, & \text{if } f_j = \underline{f'}; \\ f_j + \eta, & \text{if } f_j \in [\underline{f}, \overline{f}). \end{cases}$$

We know that all fees in the mixing domain should give an expected profit of $3/4\pi^m - \pi^d = \underline{f'} = \delta + 2\eta$.

We now proceed analogously to above. Let us start with the case in which platform i charges a fee in the lower range, that is, $f_i \in [\underline{f}, \overline{f})$. Denote the cumulative density function with which platform j mixes by $G(f_j)$. The profit of platform i when setting fees in this lower range is then given by (again abbreviating f_i by f)

$$G(f+\eta)f + (1 - G(f+\eta))\, 2f.$$

In equilibrium, this must be equal to $\delta + 2\eta$, yielding a first equation of

$$G(f+\eta)f + (1 - G(f+\eta))\, 2f = \delta + 2\eta. \tag{7}$$

This equation determines the mixing probabilities of platform j in its upper range.

In case platform i charges a fee from the upper range, that is, $f_i \in [\underline{f'}, \overline{f'}]$, the equation is

$$G(f-\eta)0 + (1 - G(f-\eta))\, f = \delta + 2\eta. \tag{8}$$

This equation determines the mixing probability in the lower range.

Let us first look at (7). We can substitute $h \equiv f + \eta$ to get

$$G(h)\,(h-\eta) + (1 - G(h))\, 2\,(h-\eta) = \delta + 2\eta. \tag{9}$$

Therefore, h is the fee in the upper range. Remember that (7) was relevant for f in the lower range and since $h = f + \eta$, these are exactly the fees in the upper range. Solving (9) for $G(h)$ gives

$$G(h) = \frac{2h - 4\eta - d}{h - \eta}. \tag{10}$$

It is easy to check that $\lim\limits_{f \searrow \underline{f'}} G(f) = \lim\limits_{f \searrow \delta + 2\eta} G(f) = \delta/(\delta+\eta) < 1/2$ because $\eta > \delta$. Moreover, it holds that $\lim\limits_{f \nearrow \overline{f'}} G(f) = \lim\limits_{f \nearrow 2\delta + 2\eta} G(f) = 3\delta/(2\delta+\eta) < 1$, which implies the existence of a mass point with mass $1 - 3\delta/(2\delta+\eta) = (\eta-\delta)/(2\delta+\eta)$ at $h = 2\delta + 2\eta$. The intuition for this result is that equation (7) is barely satisfied for f close to $\overline{f} = 2\delta + \eta$ because the support of G is a non-convex set and $\overline{f} = 2\delta + \eta < \delta + 2\eta$, the expected profit. In order to satisfy this equation, there must be a positive probability of triggering an agglomeration equilibrium and receiving $2f$ in the lower range even for $f = \overline{f}$. This is achieved by a mass point at $h = 2\delta + 2\eta = \overline{f'}$. Note that in the lower range, a profitable deviation from the mass point at $2\delta + 2\eta$ is ruled out by equation (7). We show next that $2\delta + 2\eta$ also satisfies the equilibrium condition.

Consider (8). Here we can substitute $h \equiv f - \eta$ representing that h is now in the lower range. We obtain

$$(1 - G(h))\,(h+\eta) = \delta + 2\eta. \tag{11}$$

Solving (11) for $G(h)$ gives

$$G(h) = \frac{h - \delta - \eta}{h + \eta}. \tag{12}$$

It is easy to check that $G(\underline{f}) = G(\delta + \eta) = 0$, whereas $\lim_{f \nearrow \overline{f}} G(f) = \lim_{f \nearrow 2\delta+\eta} G(f) = \delta/(2(\delta+\eta))$. Note that $\lim_{f \nearrow \overline{f}} G(f) = \delta/(2(\delta+\eta)) < \lim_{f \searrow \underline{f'}} G(f) = \lim_{f \searrow \delta+2\eta} G(f) = \delta/(\delta+\eta)$, which implies the existence of a second mass point with mass $\delta/(2(\delta+\eta))$ at $h = \delta + 2\eta$. Intuitively, equation (2) requires a sufficiently low probability of $f - \eta$ being close to $\overline{f} = 2\delta + \eta$ because otherwise setting f close to $\overline{f'} = 2\delta + 2\eta$ would lead to zero profit too often due to an agglomeration equilibrium in the lower range. A profitable deviation because of the mass point at $\delta + 2\eta$ is ruled out by equation (7) and (8).

The resulting mixing probability is characterized by a cdf of

$$G(f) = \begin{cases} \frac{f-\delta-\eta}{f+\eta}, & \text{if } f \in [\delta+\eta, 2\delta+\eta); \\ \frac{2f-\delta-4\eta}{f-\eta}, & \text{if } f \in [\delta+2\eta, 2\delta+2\eta); \\ 1, & \text{if } f = \delta+2\eta. \end{cases}$$

Because the distribution is not absolutely continuous with respect to the Lebesgue measure, we define a generalized density. The corresponding generalized density is given by

$$g(f) = G'(f) + \frac{\delta}{2(\delta+\eta)}\delta^D(f - (\delta+2\eta)) + \frac{\eta-\delta}{(2\delta+\eta)}\delta^D(f - (2\delta+2\eta)),$$

where

$$G'(f) = \begin{cases} \frac{\delta+2\eta}{(f+\eta)^2}, & \text{if } f \in [\delta+\eta, 2\delta+\eta); \\ \frac{\delta+2\eta}{(f-\eta)^2}, & \text{if } f \in [\delta+2\eta, 2\delta+2\eta), \end{cases}$$

and $\delta^D(f - f_0)$ denotes Dirac's delta function. Replacing δ and η by their respective definitions yields the result stated in the proposition. ■

Proof of Proposition 3

From the main text we know that if firms segment themselves, then half of the single-homing consumers are active on platform A and the other half on platform B. This implies that the total number of consumers of a firm is $(1+\alpha)/2$. Instead, if firms agglomerate, all single-homing consumers also choose the platform where firms agglomerate and the total number of consumers per firm is 1.

In the latter case, the profit of a firm is π^d. If a firm in a category deviates and is active on the other platform, it offers its products only to the mass α of multi-homing consumers who have seen both offers. The firm's profit is then $\alpha\pi^d < \pi^d$. Therefore, a deviation is not profitable, implying that an agglomeration equilibrium exists, given that a platform's fee is lower (or equal) than π^d. If firms segment (and single-homing consumers follow suit), the profit of each firm is $\pi(\alpha)(1+\alpha)/2$. If a firm in a category deviates, it obtains a profit of $\pi^d(1+\alpha)/2 < \pi(\alpha)(1+\alpha)/2$. As a consequence, an agglomeration equilibrium exists, if platforms charge fees lower (or equal) to $\pi(\alpha)(1+\alpha)/2$.

Again, only these two types of equilibria that can exist in equilibrium. There can never be an equilibrium in which firms in some categories choose segmentation whereas in others they choose agglomeration. The reason is that in this case all single-homing consumers are active on the platform with a larger number of firms (say, platform i). Therefore, the total number of consumers on the two platforms is $x_i = 1$ and $x_{-i} = \alpha$. But then in all categories in which firms segment, the firm on platform $-i$ wants to deviate and go to platform i because it obtains a profit of π^d per consumer on both platforms but platform i has a larger number of consumers.

Having established that thre are only two types of equilibria in the second stage, we can now move to the first-stage choices of the platforms. Agglomeration is preferred from the firms' perspective if

$$\pi^d \geq \pi(\alpha)\frac{1+\alpha}{2}.$$

Following the same argument as in the proof of Proposition 1, we obtain that in this range an agglomeration equilibrium with fees $f_i = f_{-i} = 0$ is the unique equilibrium.

Similarly, if both platforms charge a fee of $\pi(\alpha)(1+\alpha)/2$, the only equilibrium is that firms segment, and a platform's profit equals $\pi(\alpha)(1+\alpha)/4$. A platform has no incentive to deviate from this fee combination, if

$$\pi^d < \pi(\alpha)\frac{1+\alpha}{4}.$$

Hence, in this range, the unique equilibrium involves $f_i = f_{-i} = \pi(\alpha)(1+\alpha)/2$ and firms segment.

It is evident, that the regions are the same as in case of $\alpha = 0$ but $\pi^m/2$ is replaced by $\pi(\alpha)(1+\alpha)/2$. The same logic applies for the region

$$\pi(\alpha)\frac{1+\alpha}{2} > \pi^d \geq \pi(\alpha)\frac{1+\alpha}{4}.$$

By following the same steps as in the proof of Proposition 1, we obtain the same results as the ones in Propositions 1 and 2. ■

Proof of Proposition 4

As before, we start with the potential equilibrium configurations at stage 2. For any set of listing fees, there can be three potential configurations. First, an agglomeration equilibrium, in which in each category both firms and all consumers are active on only one platform, and firms obtain a gross profit of π^d. Second, a segmentation equilibrium, in which in each category firms and consumers segment and firms obtain a gross profit of $\pi^m/2$. Third, all firms multi-home in equilibrium, consumers split equally on both platforms and firm earn π^d. There can never be an equilibrium with partial multi-homing, that is, in some categories only one firm multi-homes, because then all consumers would join the platform on which both firms are active. The multi-homing firm's best response is then to single-home on the platform where all consumers are active.

We now move to the first stage and examine the equilibrium in the fee-setting game of platforms. First, note that in the agglomeration and in the multi-homing equilibrium, firms' profits are equal to π^d. From the proof of Proposition 1, we know that the agglomeration equilibrium involves listing fees of zero. If one firm sets a positive listing fee, it is better for all firms to choose agglomeration on the other platform instead of coordinating on the multi-homing equilibrium. This is because the gross profit in both equilibria is π^d and firms can save listing fees by agglomerating. As a consequence, the multi-homing equilibrium can only exist if both platforms charge a listing fee of zero. The outcome of the multi-homing and the agglomeration equilibrium is then equivalent. As stated in the man text, without loss of generality, we assume that firms will play the agglomeration equilibrium in this case. We can therefore focus on the conditions under which the agglomeration and the segmentation equilibrium exist.

First, we know that in region (i), given that $f_A = f_B = 0$, firms obtain higher profits in the segmentation than in the agglomeration equilibrium. Therefore, in this region the agglomeration equilibrium is the unique equilibrium.

If $\pi^m/2 > \pi^d$, firms obtain higher profits when agglomerating instead of separating, given that $f_A = f_B = 0$. However, in contrast to the previous analysis, separation is not necessarily an equilibrium of the second stage even if $\pi^m/2 > \pi^d$ and $f_A = f_B = 0$. This is because a firm can multi-home. In particular, suppose that platforms set $f_A = f_B = 0$, and firms and consumers in the second stage play the separation equilibrium. The profit of each firm is then $\pi^m/2$. By deviating to multi-homing, a firms obtains a profit of π^{MH}.[26] Therefore, if

$$\pi^{MH} > \frac{\pi^m}{2},$$

the separating equilibrium does not exist for $f_A = f_B = 0$.

We will now show that an equilibrium, in which both listing fees are equal to zero and firms play the agglomeration equilibrium exists for $\pi^{MH} > \pi^m/2$. First, note that neither a single firm nor a single consumer have an incentive to deviate from this equilib-

[26] Remember that π^{MH} is the profit a firms receives when it multi-homes, the competitor single-homes, and consumers are split equally on the platforms.

rium because no agent is active on the other platform. Second, since the segmentation equilibrium fails to exist, there is no other equilibrium but agglomeration on which firms can coordinate on in the second stage. Finally, if platform $-i$ sets a listing fee of $f_{-i} = 0$ and $\pi^{MH} > \pi^m/2$, the best response of platform i is to set a listing fee of zero as well. This reasoning holds regardless of the exact parameter constellation (i.e., it also holds for $\pi^d < \pi^m/2$).[27]

It follows that platforms setting $f_A = f_B = 0$ and firms playing the agglomeration equilibrium, always constitutes an equilibrium of the full game. The question is if there are constellations such that another equilibrium exists, which is profit-dominant for platforms. As in the proof of Proposition 1, let us start with the case $\pi^d < \pi^m/4$ (i.e., region (iv)). If both platforms charge a listing fee of $\pi^m/2$, no firm has an incentive to multi-home. A firm's payment is then equal to π^m, which is below the profit the firms earns on the product market. Firms will therefore play the segmentation equilibrium. In addition, a platform cannot profitably set a different listing fee, because its profit in an agglomeration equilibrium is lower. Therefore, setting $f_A = f_B = \pi^m/2$ and firms segmenting constitutes an equilibrium in region (iv). This equilibrium dominates the zero-profit equilibrium for the platforms and will therefore by played.

Finally, we turn to the range $\pi^m/2 > \pi^d \geq \pi^m/4$. We know from above that for $\pi^{MH} > \pi^m/2$ an agglomeration equilibrium with zero listing fees exist. From the proof of Proposition 1, we also know that if a segmentation equilibrium exist, it must be the one described in Propositions 1 and 2. We will now check under which conditions the possibility to multi-home breaks the segmentation equilibrium. This equilibrium exist if the circle of best responses described in the proof of Proposition 1 works in the same way if firms can multi-home. However, this circle does no longer exist if one of the fees in the mixing range is below $\pi^{MH} - \pi^m/2$. The reason is as follows: Suppose platform i sets a fee below $\pi^{MH} - \pi^m/2$. Platform $-i$'s best response in case of single-homing firms was to set a higher fee to induce agglomeration. However, inducing agglomeration is no longer possible with multi-homing firms. Specifically, if a firm decides to single-home on platform $-i$ it obtains a profit of $\pi^m/2 - f_{-i}$. Instead if it multi-homes, its profit is

$$\pi^{MH} - f_i - f_{-i} > \pi^{MH} - (\pi^{MH} - \pi^m/2) - f_{-i} = \pi^m/2 - f_{-i},$$

where the inequality follows from the fact that f_i is lower than $\pi^{MH} - \pi^m/2$. As a consequence, the best response of platform $-i$ to a listing fee of f_i below $\pi^{MH} - \pi^m/2$

[27]Note that $\pi^{MH} > \pi^d$ because if the multi-homing firm sets the duopoly price p^d, its rival firm will optimally react with p^d as well. The multi-homing firm then gets weakly higher profits than π^d because for half of the consumers it obtains π^d whereas for the other half it has set a price of π^d but the firm faces no competitor. By a revealed-preference argument, if the firms sets a different price than p^d, it must earn even higher profits than π^d.

is to undercut this fee slightly to induce an agglomeration equilibrium on platform $-i$ in the second stage. The lowering of prices then leads to the agglomeration equilibrium with $f_A = f_B = 0$.

It remains to check, under which conditions the lowest fee in the mixing range is below $\pi^{MH} - \pi^m/2$. Starting with region (ii) we obtain that this is true if $\pi^{MH} - \pi^m/2 > \pi^m - 2\pi^d$ or

$$\pi^{MH} > \frac{3\pi^m}{2} - 2\pi^d.$$

If this is fulfilled, then π^{MH} is also larger than $\pi^m/2$, implying that the unique equilibrium is $f_A = f_B = 0$ and agglomeration. Instead, if $\pi^{MH} \leq 3\pi^m/2 - 2\pi^d$, the mixed-strategy equilibrium derived above exists and gives higher profits to platforms.

Proceeding in the same way for region (iii), we obtain that for

$$\pi^{MH} > \frac{3\pi^m}{4}$$

the unique equilibrium involves $f_A = f_B = 0$ and agglomeration, whereas for $\pi^{MH} \leq 3\pi^m/4$, platforms coordinate on the mixed-strategy equilibrium. ■

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