Platform Pricing and Durability *

Toker Doganoglu†

November 2013

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Abstract

This paper considers pricing by a monopoly platform to two group of customers—buyers and sellers, over two periods when the subscription decision of the buyers is durable. That is, buyers can continue to enjoy their subscriptions in period two at no cost. Based on a stylized model of preferences, I derive general pricing rules which characterize optimal prices in first and second periods. I then further specify the model to show that the second period buyer prices decrease in the installed base of buyers in the first period, while the seller prices increase. In comparison to a rental policy, buyers will pay less and sellers more in the second period. In the first period the Coasian dynamics on the buyer side suggest that buyers obtain a discount, but in combination with other effects in the model buyers may end up paying more relative to the rental benchmark when sellers value buyers significantly. Similarly, sellers pay a lower price in the first period when they value buyers more, and a higher price otherwise. In the first period less buyers and sellers join the platform. I show that under durability the platform earns lower profits when compared with the rental benchmark. I demonstrate that while one side may be subsidized under the rental policy, platform may optimally offer no subsidies under durability. On the other hand, while under durability one side is subsidized, optimal rental prices may imply no subsidies. Overall the main insight from literature on platform pricing in two sided markets which suggest that consumers which value the number of consumers on the other side pays a higher price may no longer hold under durability.

Keywords: platform pricing, durable goods, Coase conjecture.

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*I would like to thank Firat Inceoglu and Norbert Schulz for illuminating discussions. All errors are mine.

†University of Würzburg, toker.doganoglu@uni-wuerzburg.de

Introduction

Since the seminal contributions of Rochet and Tirole (2002, 2003), a very large literature which studies the price structure that may arise in the so called *platform* or *two-sided* sided markets emerged. A platform facilitates interactions between two groups of users which otherwise would not be able to interact. Therefore, the services or product offered by a platform generates value. Owner of a platform will attempt to appropriate some of this value by setting participation prices to the two different groups of users in order maximize platform profits. When the members of the two different groups value the number of members of the other group, the platform is said to exhibit cross group externalities. When these externalities are important, the pricing problem of a platform may become rather complex and as a result rather interesting.

This framework has been used to analyze many different industries from credit card networks, media markets, hardware-software platforms such as game consoles and computer operating system, search engines, social networks, and online trading platforms to give a few examples. Although each of these different markets have their idiosyncratic features, the main insights generated by the two sided market models have been found convincing in explaining observed pricing patterns by academics and practitioners alike. Although I will briefly summarize the literature below, it is important highlight one issue here already: *The literature relies on a static model.* To the best of my knowledge, dynamic issues that may arise in platform markets have been largely ignored up to now. My goal in this paper make a first attempt to introduce dynamics in the optimal pricing problem of a platform.

Consider for example the market for game consoles. Naturally, a game console for which there are a large selection of available games will be more attractive. On the other hand, a game console which is expected to be adopted by a large number of gamers will be attractive for able game programmers. Thus, it is natural that there are cross group network effects. This market which is rather vibrant and dynamic tends to have a number of firms which dominated the market at one time or another, such as Nintento, Playstation and Xbox. The game consoles are not compatible with one another and hence they have their own network of gamers and game programmers. It is well known that game consoles are sold for below cost prices while game developers tend to pay a large share of their earning as royalties to the producer of the game console. Games are interesting products as after a limited number of times of play, a game will cease to provide value to a gamer. Thus, for the duration a gamer owns a console there will be several generations of games present in the market. As a result, when a gamer considers purchasing a console, she would know that she would use that console for a considerable period of time. In a sense, the decision to purchase a console is *durable*. This fact not only implies that the gamer needs to form expectations regarding the supply of games in the future, but also should also form expectations about how the producer of the game console would alter prices in the near future. On the other hand, a game programmer presumably can consider whether to make a newly developed game available for a particular console or not. The decisions of game developers are rather short term while the purchase decisions of the gamers are long term—durable, decisions. Although the immediate impact of durability will result in familiar discounts to the consumers buying game consoles, the overall effects on the price structure is not immediately apparent as there are large number of moving parts in a two sided market model.

In this paper, I will take a first step towards incorporating durability of decisions of one of the groups. In doing so I will adopt a very stylized model. For ease of exposition, I refer to one group as buyers and the other as sellers. I do not explicitly model the interaction between the two groups as in Hagiu (2009) but adopt reduced form benefit functions for buyers and sellers which account for cross group network effects, prices paid for subscriptions as well as an idiosyncratic component which generates heterogeneity within each group. Thus, I have elastic subscription demands on both sides.

I assume that the platform exists for two periods, and all buyers and sellers are present for both periods. The sellers have to make a subscription decision each period. In contrast, buyers which decide to subscribe in period one, can continue enjoy their subscription in the second period at no additional cost. In the second period, by charging a different price, the platform may be able to attract additional buyers. In summary, I assume buyer decisions are durable, while seller decisions are not. I assume that in each period the platform prices are announced first, then buyers and sellers form rational expectations for relevant magnitudes and then make purchase decisions.

I first assume general concave network benefit functions, general distributions for idiosyncratic components of buyer and seller utilities and derive pricing rules for the second period when a number of buyers have already purchased subscriptions in the first period. The presence of an installed base on the buyer side have a few notable effects on the pricing rules. First, the buyer subscription demand becomes more elastic, as the buyers left in the second period have low idiosyncratic valuations of the platform's services. Second, since the marginal increase in the willingness to pay of a seller is lower when there are more buyers, the deviation of the actual and perceived marginal cost on the buyer side is smaller. The third effect is on the seller side. The increase in marginal willingness to pay of a buyer reduces perceived marginal cost on the seller side. However, since this potential increase in price will be collected from a presumably smaller number of buyers, the cost reduction on the seller side is smaller. Although, I do not explicitly investigate in this more general framework, I conjecture that the second period buyer prices will decrease as the number of buyers which join the platform in the first period increases.

I then look at the pricing decision in the first period. In this framework, a buyer can purchase a subscription and enjoy it for two periods, or forego a purchase in the first period and subscribe to the platform in the second period only. If second period prices are expected to fall substantially, it may be wiser for some buyers to wait. This option to buy only in the second period is the reason why Coasian dynamics may arise in my model as well. By selling to more buyers in the first period, the platform will charge lower prices to buyers in the second period. Rational consumers forecast this behavior, and demand lower prices in the first period to participate. Thus, the platform in the first period is in competition with itself in the second period. In this setting, I derive optimal pricing rules in the first period. Although the seller side pricing rule resembles the standard one,[1] the pricing rule on the buyer side have two additional components. If the platform sells to a buyer in period one, it cannot sell to that buyer in period two. This lost profit discounted to period one appears as a cost for the buyer side. Although bringing on board an additional buyer reduces prices for everyone (the effect of which is taken in to account in the elasticity term), the existing buyers require an additional discount to remain on board. The reason is as follows. An additional buyer in period one reduces second period prices, and as a result increases the value of the outside option of each existing buyer. To keep each of them as well off, the platform should reduce the prices to offset the increase in the outside option. This additional effect also appears as a cost on the buyer side. The buyer price then is determined when the Lerner index computed with the marginal costs adjusted taking dynamic and network effects into account is equal to the inverse subscription elasticity of buyers.

Although the pricing principles I derive are illuminating, they unfortunately do not permit me to make sharper statements regarding the potential effects of durability on price structure and participation. Thus, I assume uniform distributions for idiosyncratic buyer and seller valuations of platform's services, and linear network benefit functions and derive dynamically optimal platform prices under durability. The specific model verifies my conjecture regarding the the optimal prices in the second period which decrease with the size of the installed base on the buyer side. In contrast, seller prices increase. I derive conditions necessary for additional buyers to always join in the second period facing the optimal prices. I then compare these optimal prices, to the prices which would emerge if the platform could enforce all buyers to make subscription decisions valid only for a single period—a policy I refer to as the rental policy due to its obvious link to the durable goods monopoly literature. The optimal rental prices coincide with prices that would emerge in the static version of my model. Depending on the strengths of network effects, more or less buyers or sellers may join in the second period relative to the rental benchmark.

I then move to the first period. Taking the outside option of the consumers which depend on the second period prices (which themselves depend on the outcome of the first period buyer subscription demands), I derive rational expectations demand function for buyers and sellers. I show that both demand functions shift downward but also become less price sensitive. For most of the permissible parameters, buyers face a lower price. However, I show that there are parameter combinations for which buyers in the first period under durability may pay a price that exceeds the price they would have paid if they rented over two periods. The sellers pay more relative to the rental benchmark if buyers value them more–a rather counterintuitive finding. I find that both the number of sellers and buyers joining the platform under durability in the

[1] See, for example, Armstrong (2006)

first period would be less than the rental benchmark. I then compare the profits of the rental policy and the profits that would arise under durability and conclude that the rental policy yields higher profits in this setting as well. I perform a few thought experiments by restricting parameters in certain extreme ways. I demonstrate possible ways that durability can affect the price structure in a fundamental fashion. Especially, I show that durability may alter when and whether a side is subsidized or not.

The paper is organized as follows. In section 1, I present the related literature. Section 2 presents the model. In section 3, I derive general pricing rules for first and second periods. I study a fully specified linear model in section 4, and derive optimal platform prices in both periods. Here, I also present a detailed comparison of prices under durability and the rental policy. Section 5 concludes.

1 Related Literature

Two sided markets have spurred a large interest in industrial organization starting with the seminal contributions of Rochet and Tirole (2002, 2003). The early literature and implications of multisidedness have been studied in Armstrong (2006) by using a series of stylized models, which assume reduced form payoffs for both groups that depend on prices and network benefits and derives pricing rules, among other market structures, for a monopoly platform. These pricing rules imply that the externalities generated by an additional user on one side result in an increase in willingness to pay of the users on the other side. This additional profit opportunity reduces the perceived marginal cost of the platform owner for the additional user in question, which in turn pushes optimal prices down. For prices to be optimal, it must be that the Lerner Index based on the marginal cost adjusted by taking in to account this externality is equal to the inverse of the subscription elasticity of the users. The main insight of these pricing rules is that the group which values the number of users on the other side will pay a higher price to enjoy the services of the platform.[2] One goal I have in this paper is to put this main insight in question, and show that it may fail to hold when contract durations on both sides differ rendering the decisions of one group of user more durable.

Variations of the this simple setting do not change this main insight and similar pricing rules arise in other models of platforms. For example, Hagiu (2009) makes the interaction between two groups explicit and assume there exists a trade between them. In his treatment, one side consists of the consumers and the other side comprises of the firms. These two groups find profitable trade opportunities only when they participate in the platform. In Hagiu (2009), the valuation of the number of sellers by consumers arise as a result of the love of variety consumer preferences

[2]Even in the stylized world of Armstrong (2006) the model can generate a wide range of pricing structures depending on the strengths of various forces. The insight that "*the side which values the other side more pays more*" is rather widely mentioned, however, it should be noted that this statement is true only when the subscription elasticities on both side are of similar magnitudes.

exhibit. For the sellers, the number of consumers participating in the platform naturally affects the demand they face and creates a positive externality. The formulation in Hagiu (2009) allows even for a negative externality on the side of the sellers, as the larger is the number of sellers, the fiercer would be the competition between them and consequently per consumer profits they would obtain would decrease. In addition to the strength of cross group network effects and subscription elasticities, the findings of Hagiu (2009) imply that how the surplus of a trade between a seller and consumer is shared and the love of variety consumer preferences exhibit will have an effect on the optimal price structure of a platform. In Hagiu (2009), the reason why game consoles are subsidized but games developers pay producers of consoles significant sums in royalties is explained by the facts that the consumers love variety and the considerable market power this awards the game developers which then can appropriate large share of the surplus generated in a trade. In contrast, computer application software is relatively homogeneous and hence software developers pay very little if at all to operating systems developers while consumers pay significant sums for the operating system. The model I study in this paper advances another possible explanation. It may very well be that the durability of software for game consoles are significantly lower than that of computer applications. If you play a game a number of times you may want to have new games, while you can use a word processing programs for many years without a desire to upgrade.

Rochet and Tirole (2006) and Rhysman (2009) present excellent surveys summarizing not only the results found in the academic literature but also characterizing which real world markets fit the two sided markets—or more generally, multi-sided platform markets, framework. My main contribution in this paper is to examine a previously ignored possibility which may affect the price structure in a platform market in a considerable fashion: durability of the decisions. I show that when one takes durability into account main insights regarding the optimal price structure of a monopoly platform may be turned upside down. While a static or non-durable standard platform monopolist subsidizes one side in order to maximize profits by earning handsome sums on the other side, a platform monopolist whose subscriptions on one side is durable, may not engage in subsidies at all. On the other hand, when the standard models suggest that there would be no subsidies, a platform monopolist who takes into account durability of decisions on one side may offer subsidies to the durable side. In general, I show that the durable side will most likely obtain a discount, which in turn may imply that even though under a static model this side needs to pay more, under durability it may end up paying less.

My paper is also closely related to the large literature on pricing of durable goods monopolists. This literature mainly attempts to identify conditions under which the famous conjecture presented in Coase (1972) holds. To rephrase, Coase (1972) conjectures that when the duration between two consecutive sales of a durable goods monopolist goes to zero, the initial price the monopolist would set will be equal to the highest of the willingness to pay of the lowest valuation consumer and his own marginal cost. A number of papers, starting with Stokey (1981) show this conjecture to hold. Bond and Samuleson (1984) incorporate depreciation in a model similar to Stokey (1981) and demonstrate that the conjecture will constitute a subgame perfect equilibrium. Karp (1996) later generalizes this result and show that a continuum of subgame perfect equilibria may arise when depreciation is taken in to account. Gul, Sonnenschein and Wilson (1986) study a discrete time version of the problem and establish that all consumers that value the good higher or equal to the monopolist's marginal cost will be served in a finite time in the subgame perfect equilibrium outcome. By allowing history dependent price expectations on the consumer side Ausubel and Deneckere (1989) provide a folk theorem which suggests that any profit level between the competitive outcome and the monopoly outcome can be supported in equilibrium. Waldman (2003), an excellent survey of literature on durable goods, presents an interesting critical view of this literature. There are a number of papers that attempt to make the basic model more realistic by incorporating inflow of new consumers such as Sobel (1991) who finds that although the folk theorem like results of Ausubel and Denecekere (1989) apply in this setting as well, restricting attention to stationary consumer strategies revive the Coase Conjecture. McAffee and Wisemann (2008) demonstrate capacity constraints may result in Coase conjecture to fail.

In an interesting contribution Deneckere and Liang (2008) show that depreciation in a discrete time model with noninfinitesimal lengths between consecutive trades may essentially result in three classes of equilibria. When depreciation rate is low, each period highest valuation consumers will leave the market giving incentives to the monopolist the reduce prices. This incentive in turn harms the firm and the Coase conjecture emerges as the unique equilibrium prediction when the durable good depreciates very slowly. On the other extreme, when depreciation is high, each period there will be sufficiently many high value consumers allowing the firm to charge monopoly prices in equilibrium. For intermediate depreciation levels a multitude of equilibria may exist. Their model considers a demand function that consists of a finite number of discrete levels and cannot deal with a continuum of types. However, they correctly identify that continuous time models may yield spurious equilibria and hence suggest that for reliable insights, a model which considers trades only at discrete time intervals should be preferred.

Mason (2000) studies pricing of a durable network good which exhibit one-sided network effects in a continuous time dynamic model. In his model consumers obtain network benefits proportional to the number users at the instant of their purchase forever. He shows that although the firm exercises marginal cost pricing, the network reaches its ultimate size in a long period of time. Laussel et al. (2013) presents another dynamic continuous time model with network effects. In addition to network effects for the durable good, they also consider a complimentary product which the monopolist can sell to those who already own the main good in an aftermarket. This aftermarket good also exhibits network effects which depend on the installed base of the durable good. They assume, in addition, that the consumers enjoy network benefits at each time instant proportional to the size of the installed base at that instant. This change in the assumption regarding how network benefits are realized imply that when marginal cost exceeds the lowest consumer valuation the Coase Conjecture applies. Their study of aftermarkets or essentially sales by a monopolist of two complimentary products resemble a two sided market setting. However, in their case network effects arise solely due to the installed base of the durable good or in other words solely due to the number of users on one side.

Obviously a realistic model should take depreciation and inflow of new consumers into account. Both of these features are forces that might allow a monopolist to sustain higher prices. Even though without depreciation continuous time dynamic models yield sensible results, with depreciation they may result in a continuum of equilibria as presented in Karp (1996). However, as argued by Denecekere and Liang (2008) many of these equilibria are likely to be spurious. However, the framework of Denecekere and Liang (2006) is also not amenable to analysis with continuous consumer types, let alone additional externalities which are the cornerstones of a two sided market model. Given these issues, I opt for a simpler model mostly following the advise of Waldmann (2003) which suggest that a two period model should suffice to demonstrate the Coasian effects on pricing. Given the two period assumption, I also do not incorporate depreciation or arrival of new consumers. These features are naturally desirable for deriving reliable insights for real world markets. I hope to develop the current framework in these directions in future work. Thus, I adopt a two period model where there are cross group network effects, the decisions of one side is durable implying that if they subscribe in the first period, they will be able to obtain benefits in the second period at no cost. The decisions of the other side is assumed to be completely perishable in that they need to make subscription decisions on both periods.

2 The Model

I will adopt a stylized model of a two sided monopoly platform. I will refer to one side as the sellers and the other side as the buyers to simplify exposition. The platform in my model is a bottleneck in that the buyers and sellers need to participate in the platform in order to interact with one another. In this sense, platform offers a necessary service. As usual, the customers of the platform on either side care about the participation levels of users on the other side, hence there are cross group network effects. I will not explicitly model the interaction between the buyers and sellers. Hence, I will allow the platform to set only participation fees.[3]

My point of departure from the earlier literature is the assumption that the participation decision of the buyers are long term, while sellers will have to decide whether to participate more frequently. I will consider the simplest possible setup to investigate the implications of this difference in the duration of participation decisions and adopt a two period model. While

[3]Clearly, in many realistic situations the buyer and seller interact with one another, and as Hagiu (2009) demonstrates the distribution of the surplus between the two plays an important role on shaping the platform's price structure. This extension is left for future research.

sellers have to decide each period on whether to subscribe[4] or not. On the other hand, those buyers who subscribe in period one will be able to enjoy the services of the platform also in period two at no additional charge. Furthermore, those buyers who have not subscribed in period one will be able to reconsider in period two. I assume that the platform lacks instruments to commit to a subscription fee for the buyers in the second period. Thus the well known commitment problem as conjectured by Coase (1972) arises. The second period incarnation of the platform presents a competitor to its first period incarnation. The buyers' outside option in the first period is to subscribe in the second period instead of not participating which by assumption would yield zero net utility. This increase in the value of the outside option of buyers in the first period then affects the pricing structure that the platform offers in the first period. Although very stylized, I aim capture the potential effects durability may have on the pricing structure of a platform with the help of this assumption.

The modelling of buyer and seller benefits follow closely the setup in Hagiu (2009). I will first put forth a relatively general model which permits one to derive general pricing principles. I will then further explore the characteristics of the optimal prices by means of commonly used functional form assumptions. Within a period, buyers obtain a net utility depending on the number of sellers expected to participate on the other side which is denoted by $\tilde{n}_S^t$, $t = 1, 2$. Although I assume that buyers' valuation of the size of the seller population is homogeneous, buyers differ from one another on their intrinsic valuation of the platform's services. If a buyer decides to join the platform in period one, she does not need to make a decision in the second period. On the other hand, a buyer which chose not to subscribe in period one, can join in period 2. The net utility of a buyer i in period 2 facing a price p_B^2 and expecting $\tilde{n}_S^2$ sellers to join on the other side is given by

$$u_i^2(p_B^2, \tilde{n}_S^2) = \alpha_B - \theta_B^i + v_B(\tilde{n}_S^2) - p_B^2. \tag{1}$$

Here $v_B(\cdot)$ with $v_B(0) = 0$, $v_B'(\cdot) > 0$ and $v_B''(\cdot) \leq 0$ represents the network benefits buyers obtain. α_B denotes a fixed benefit buyers obtain by subscribing. The idiosyncratic value consumer i attaches to participating in the platform is denoted by θ_B^i. I assume that this intrinsic value is distributed over an interval $[0, L_B]$ with a continuous and twice differentiable cumulative distribution function $F_B(\theta_B)$ and the corresponding density function, $f_B(\theta_B)$. Furthermore, I assume that L_B is sufficiently large that the buyer with this intrinsic valuation will not subscribe to the platform by paying a positive price even when all the sellers join. Thus, I focus on cases where buyer demand will be elastic.

When the buyers consider whether to subscribe or not in the first period, they are forward looking. They do take in to account the benefits they will obtain in the second period in case they subscribe in period one. That is, they reach a decision which maximizes their cumulative benefits. A buyer who has subscribed in period one can enjoy the services of the platform in

[4] I will use subscribe, purchase and participate interchangeably in the following.

period two at no cost. If buyer i decides to subscribe in period one, over the two periods she thus obtains

$$U_i(p_B^1, \tilde{n}_S^1, \tilde{n}_S^2) = \alpha_B - \theta_B^i + v_B(\tilde{n}_S^1) - p_B^1 + \delta(\alpha_B - \theta_B^i + v_B(\tilde{n}_S^2))$$

where $0 \leq \delta \leq 1$ denotes the discount factor of consumers (and firms). It is clear that in order to evaluate this net benefit, a buyer will have to form expectations regarding the number of sellers which join the platform in both periods, $\tilde{n}_S^1$ and $\tilde{n}_S^2$. The alternative faced by the buyers in the first period is to wait and purchase in period two. If the expected price in the second period is sufficiently low, waiting may become a valuable option. Thus, a consumer decides to subscribe in period one, only when $U_i(p_B^1, \tilde{n}_S^1, \tilde{n}_S^2) \geq u_i^2(p_B^2, \tilde{n}_S^2)$. It is clear from this comparison that the pricing decision of the platform to buyers must be interrelated.

The sellers also differ from one another with regards to the intrinsic value they attach to participating at the platform. The sellers also derive network benefits as a function of the number of buyers, $\tilde{n}_B^t$, they expect to participate at the platform. These network benefits are described by an increasing concave function $v_S(\tilde{n}_B^t)$, with $v_S(0) = 0$, $v_S'(\cdot) > 0$ and $v_S''(\cdot) < 0$. As mentioned earlier, the sellers have to renew their subscription decision in each period. Furthermore, as will become clear below, the participation decision of sellers have no dynamic implications, thus seller j subscribes to the platform by paying a subscription fee of p_S^t in period t whenever her net benefit

$$\pi_j^t(p_s^t, \tilde{n}_B^t) = \alpha_S - \theta_S^j + v_S(\tilde{n}_B^t) - p_S^t \tag{2}$$

exceeds zero—the benefit of not subscribing at all. In this net benefit expression, α_S corresponds to a fixed subscription benefit and θ_S^j represents the idiosyncratic valuation of seller j. I assume that the idiosyncratic valuations of sellers are distributed over an interval $[0, L_S]$ according to a cumulative distribution function $F_S(\theta_S)$ and a corresponding density function $f_S(\theta_S)$.

The monopoly platform announces prices p_B^t and p_S^t in each period. I assume that the platform has a constant marginal cost of serving a buyer given by c_B and a constant marginal cost of serving a seller that is given by c_S. Buyers and sellers after observing the prices form rational expectations regarding participation levels on the opposite side and make their subscription choices. As will become clear below, if x buyers join the platform in the first period, the platform, in most cases, will have an incentive to sell to further buyers in the second period at a lower price. The second period prices turn out depend on the number of buyers who join the platform in the first period, x. Thus, when choosing its first period prices, the monopoly platform considers the effect of these prices on second period profits. I will assume that the platform discounts the future with the discount rate δ as well.

Before proceeding with the analysis, I would like to point out an intriguing possibility. If the sellers value the number of buyers more than the other way around, it is easy to think of examples where buyers are subsidized. It may in fact be that the buyers receive a payment to join, which in turn implies that the buyers are offered a price below marginal cost in the first period. Note that the standard mechanism Coase conjectured relies on prices that exceed marginal costs. In case, the platform charges below cost prices to buyers in the first period, inducing more buyers to join in the second period may require charging prices which are even lower. In this case, the monopoly platform may not face a commitment problem at all. In a possible equilibrium, the monopoly platform will sell to a certain amount of buyers in the first period at a price below marginal cost as well as sellers and then leverage the buyer base carried over from the first period in making sales to the sellers only in the second period. Although I was able to construct specific examples where such an equilibrium arises, I have not succeeded in a general formulation treating such a case. In the remainder of the paper, I will consider only those cases, where at the optimal prices some buyers always join in period two.

3 Pricing Decisions

Before starting to discuss how a platform will set its prices when buyer decisions are durable, it is useful to explore a benchmark policy which does not suffer from the Coasian dynamics. As customary in the durable goods literature, the potential issues arising from durability can be offset if the platform is simply renting or able to charge a per period subscription price to the buyers. This requires an ability to limit the second period participation of a buyer which joins the platform in the first period. Supposing that the platform has access to such a technology, and imposing limitations are feasible, the buyers subscription decisions in the first period are no longer affected by subscription prices in the future. In addition, all buyers need to make a subscription decision in the second period regardless of their first period decisions. As a result, buyers and sellers will make their subscription decisions in a myopic fashion in each period. The resulting pricing rules corresponding to the model I presented above will be the same as those derived in Armstrong (2006).[5] The prices in both periods will be determined by the same inverse elasticity rules which are given by

$$\frac{p_k^t - (c_k - v'_{-k}(n_k^t)n_{-k}^t)}{p_k^t} = \frac{1}{\epsilon_k(\alpha_k + v_k(n_{-k}^t) - p_k^t)} \qquad \text{with } t = 1, 2 \text{ and} k = B, S$$

with $-k$ denoting the opposite side. The subscription elasticity $\epsilon_k^t(\cdot)$ is defined as in Hagiu (2009) and given by

$$\epsilon_k^t(\alpha_k + v_k(n_{-k}^t) - p_k^t) = \frac{F_k(\alpha_k + v_k(n_{-k}^t) - p_k^t)}{p_k^t f_k(\alpha_k + v_k(n_{-k}^t) - p_k^t)}.$$

In calculating this elasticity the network sizes, n_{-k}^t, are held constant.

These, by now standard, pricing rules suggest that the platform takes in to account the potential increase of its profits from one side when an additional user is brought on board on

[5] I do not provide an explicit proof of this claim as the second period pricing rules which I derive below with the number of buyers which join in period one set to zero replicate the expressions derived in Armstrong (2006).

the other side. The additional user on side k, increases the network benefits of users on the other side by $v'_{-k}(n_k^t)$. The platform can increase prices on side $-k$ by exactly this amount and keep the subscription level on side $-k$ the same. These effects imply that when externalities are positive, the perceived marginal costs of the platform on both sides are lower. The optimal prices equate the Lerner index on each side computed with the perceived marginal costs to subscription elasticity of users on that side.

The typical insight from these pricing rules is that the side which values the number of users on the other side higher tends to receive a lower price offer.[6] When the strength of externalities on the two sides are significantly different, the side which is valued more can in fact receive a price offer that is below cost. In the subsections below, I will investigate the first and second period prices when the platform cannot impose rental prices on the buyers. I will demonstrate that the basic insight which I just mentioned may fail to be true when durability is taken in to account. That is, even though the platform may prefer to charge higher rental prices to buyers, absent a rental technology, it may end up charging lower prices to buyers when buyers decisions are *durable*.

3.1 Optimal Pricing Rules in the Second Period

Suppose that x buyers have joined the platform in the first period. Thus, the seller i in period 2 knows for sure that she will be able to interact with at least x buyers. In addition, given the prices of the platform, further m_B buyers may be expected join the platform in second period. Thus, the network size of buyers the sellers use in evaluating their net benefit is $\tilde{n}_B^2 = x + \tilde{m}_B$. Suppose given the prices of the platform, p_B^2 and p_S^2, and the expected number of buyers that join the platform $\tilde{n}_B^2$, a seller with an intrinsic value $\hat{\theta}_S$ is indifferent between joining the platform and staying out. Using (2), the value of $\hat{\theta}_S$ is given by $\hat{\theta}_S = \alpha_S + v_S(x + \tilde{m}_B)p_S^2$. All those sellers with $\theta_S \geq \hat{\theta}_S$ will join the platform. Facing a subscription fee of p_S^2 and expecting m_B additional buyers to join the platform, the number of sellers which subscribe in period two is given by $d_S^2(p_S^2, x + \tilde{m}_B) = F_S(\alpha_S + w(x + \tilde{m}_B) - p_S^2)$.

Let θ_B^1 denote the intrinsic value of the marginal consumer that decided to subscribe in period 1. That is $x = F_B(\theta_B^1)$. If the platform will sell to further buyers, it will need to attract buyers which have intrinsic values that are lower, or more formally, those buyers with $\theta_B > \theta_B^1$. When $\tilde{n}_S^2$ sellers are expected to join and facing a subscription price of p_B^2, there will be a buyer with an intrinsic value of $\hat{\theta}_B$ who is just indifferent between joining the platform in period two or not. Note that as the second period is the last period, the outside option of an unsubscribed buyer is remaining unattached to the platform and obtaining a benefit of zero. Using (1), it is easy to verify $\hat{\theta}_B = \alpha_B + v_B(\tilde{n}_S^2) - p_B^2$. I implicitly assume here that $\hat{\theta}_B > \theta_B^1$. Thus, the number

[6] Of course, the subscription elasticities play an important role in determining the optimal prices. Therefore, this main insight should be amended with the phrase "when subscription elasticities on both sides are similar".

of additional buyers which will join the platform in the second period is given by

$$m_B(p_B^2, n_S^2, x) = F_B(\alpha_B + v(\tilde{n}_S^2) - p_B^2) - F_B(\theta_B^1) = F_B(\alpha_B + v(\tilde{n}_S^2) - p_B^2) - x$$

Rational expectations require that $\tilde{m}_B = m_B(p_B^2, n_S^2, x)$ and $\tilde{n}_S^2 = n_S^2(p_B^2, n_S^2, x)$ and the resulting demand functions need to simultaneously solve

$$m_B(p_B^2, n_S^2, x) = F_B(\alpha_B + v_B(n_S^2(p_B^2, n_S^2, x)) - p_B^2) - x. \tag{3}$$

$$n_S^2(p_B^2, n_S^2, x) = F_S(\alpha_S + v_S(x + m_B(p_B^2, n_S^2, x)) - p_S^2) \tag{4}$$

Suppose $n_S^2(p_B^2, p_S^2, x)$ and $m_B(p_B^2, p_S^2, x)$ solve these two equations uniquely to describe the rational expectations buyer and seller demand functions respectively.

Assuming that for relevant prices these demand functions imply positive demands from both sides, the second period platform profits are simply given by

$$\Pi_P^2(p_B^2, p_S^2, x) = (p_B^2 - c_B)m_B(p_B^2, p_S^2, x) + (p_S^2 - c_S)n_S^2(p_B^2, p_S^2, x) \tag{5}$$

The optimal second period prices maximize this profit function. Assuming that the first order conditions characterize the optimal prices, they need to solve:

$$m_B(p_B^2, p_S^2, x) + (p_B^2 - c_B)\frac{\partial m_B(p_B^2, p_S^2, x)}{\partial p_B^2} + (p_S^2 - c_S)\frac{\partial n_S^2(p_B^2, p_S^2, x)}{\partial p_B^2} = 0 \tag{6}$$

$$(p_B^2 - c_B)\frac{\partial m_B(p_B^2, p_S^2, x)}{\partial p_S^2} + n_S^2(p_B^2, p_S^2, x) + (p_S^2 - c_S)\frac{\partial n_S^2(p_B^2, p_S^2, x)}{\partial p_S^2} = 0 \tag{7}$$

The required partial derivatives can be obtained via totally differentiating equations (3) and (4) with respect to p_B^2 and p_S^2 and solving the resulting system of four equations. Substituting these partial derivatives in equations (6) and (7), results in two pricing equations characterizing the profit maximizing prices in period two as follows:[7]

$$\frac{p_B^2 - (c_B - v_S'(x + m_B^*)n_S^{2*})}{p_B^2} = \frac{1}{\eta_B^2(\alpha_B + v_B(n_S^{2*}) - p_B^2)} - \frac{x}{p_B^2 f_B(\alpha_B + v_B(n_S^{2*}) - p_B^2)} \tag{8}$$

$$\frac{p_S^2 - (c_S - v_B'(n_S^{2*})m_B^*)}{p_S^2} = \frac{1}{\eta_S^2(\alpha_S + v_S(x + m_B^*) - p_S^2)}. \tag{9}$$

where

$$\eta_i(z) = \frac{F_i(z)}{f_i(z)p_i^2}, \qquad i = B, S$$

represents the subscription elasticity of buyers and sellers as defined in Hagiu (2009).

The elasticity rules presented in equation (8) and (9) seem similar to those presented in Armstrong (2006) and Hagiu (2009) however there are a few important distinctions. First note that when there are no buyers already on the platform at the start of period two, these expressions coincide with those of Armstrong (2006). The markups on both sides are inversely

[7] I present the derivation of these expressions in the Appendix.

related to the subscription elasticities. The pricing on each side also takes in to account the additional profits generated on the other side by inducing one more user join. Namely, when a new buyer joins, the network benefits of sellers increase by $v'_S(\cdot)$, which then allows the platform to increase the price to the sellers by the same amount without changing seller participation. This gain aggregated over all the participating sellers is $v'_S(\cdot)n_S^{2*}$ and the marginal cost the platform incurs per buyer is reduced by this amount. When there is a positive customer base of buyers, x, this reduces marginal network benefits generated on the seller side for an additional user, which in turn implies a smaller gain which arises by attracting one more buyer. The likely effect of this change is an increase in buyer prices as the perceived marginal cost of a buyer increases. On the other hand, given x customers with higher idiosyncratic valuations on the buyer side do not make subscription decisions in the second period, the remaining buyers have a higher subscription elasticity, which in turn is a force toward lowering buyer prices. Finally, the additional gain which arises by attracting one seller from the buyer side, $v'_B(n_S^{2*})$, is potentially collected from a smaller number of trading buyers, m_B^*. All in all, it is not straightforward to derive the overall effect on the pricing structure in the second period of a user base of size x on the buyer side acquired in the first period. Below, in the context of a further stylized specification, I will pin down the effect of x on both the buyer and seller prices. To preview the results, which I conjecture would hold more generally, are such that the presence of a user base on the buyer side yields a reduction of the second period buyer prices and an increase of the seller prices. This reduction in buyer prices is the very reason that an option to wait until second period arises for the buyers in the first period which in turn results in a framework where the Coasian price dynamics may arise.

The optimal prices the platform sets in the second period, naturally, depend on the number of buyers who have already subscribed in period one, x. The sales of the platform also then depends on x. Thus, the second period platform profits can be written as a function of x after substituting the optimal buyer and seller prices and the corresponding sales amounts in equation (5). Namely what we have is $\bar{\Pi}_P^2(x) = \Pi_P^2(p_B^2(x), p_S^2(x), x)$. It is interesting to note here that any effect the customer base x can have on this profit via its effect on the optimal buyer and sellers prices, $p_B^2(x)$ and $p_S^2(x)$, respectively, are internalized by the platform.[8] Therefore, the only effect of x on second period profits arises due to its direct effect on sales. Namely, an additional buyer attracted in the first stage reduces number of sales to buyers by one in the second period. As a result we have

$$\frac{d\bar{\Pi}_P^2(x)}{dx} = \frac{\partial \Pi_P^2(p_B^2(x), p_S^2(x), x)}{\partial x} = -(p_B^{2*}(x) - c_B)$$

since $m_B(p_B^2, p_S^2) = F_B(\alpha_B + v_B(n_S^2) - p_B^2) - x$. This expression will play a significant role in the next section when we derive the optimal pricing rules of the platform in the first period.

[8]This is so as $\frac{\partial \Pi_P^2(p_B^2, p_S^2, x)}{\partial p_B^2} = 0$ and $\frac{\partial \Pi_P^2(p_B^2, p_S^2, x)}{\partial p_S^2} = 0$ when prices are optimally selected.

3.2 Optimal Pricing Rules in the First Period

In beginning of the first period, I assume that no buyer or seller is attached to the platform. As a result, all buyers and sellers contemplate joining the platform. Although all the sellers will get a chance to revise their decisions in the second period, those buyers which decide to subscribe in period one, can enjoy the benefits of the platform at no additional cost in the second period. Therefore buyer i, after observing the first period price offer of the platform, must form expectations regarding not only about the number of sellers which will subscribe in period one, but also the number of sellers which will participate in the second period in order evaluate the net benefit of having a subscription in period 2. On the other hand, this buyer may opt for waiting until period two to subscribe to the platform. Evaluating the benefit of this option requires a buyer to also form an expectation regarding the second period subscription prices as well. Formally, buyer i will only join the platform in period one when

$$\alpha_B - \theta_B^i + v_B(\tilde{n}_S^1) - p_B^1 + \delta(\alpha_B - \theta_B^i + v_B(\tilde{n}_S^2)) \geq \delta(\alpha_B - \theta_B^i + v_B(\tilde{n}_S^2) - p_B^2)$$

or, equivalently, whenever

$$\theta_B^i \leq \alpha_B + v(\tilde{n}_S^1) - p_B^1 + \delta p_B^2$$

as the benefits of a subscription in the second period is the same regardless of when subscription takes place. However, a buyer subscribing in period two incurs an additional cost of p_B^2 which in turn appears with discount factor in the choice problem faced by a buyer in period one.

Given that the subscription price in the second period will depend on the number of buyers subscribing in period one, $\tilde{n}_B^1$, this introduces an intergroup externality between the buyers in the first period. If the second period price, $p_B^2(n_B^1)$, decreases with the size of subscriber base of the platform in period one, which most likely is the case, there would be negative externalities between the first period buyers. That is, when one more buyer decides to join in the first period, all other buyers expect a higher utility from waiting until the second period and then joining the platform. This negative externality in the buyer decisions is present in addition to the regular positive externality that arises due to the number of sellers joining the platform.

Given the distribution of the idiosyncratic valuations of the buyers, we can calculate the actual number of buyers who will join the platform facing a subscription price of p_B^1, and holding expectations $\tilde{n}_S^1$ and $\tilde{n}_B^1$ as

$$n_B^1(p_B^1, \tilde{n}_B^1, \tilde{n}_S^1) = F_B(\alpha_B + v(\tilde{n}_S^1) - p_B^1 + \delta p_B^2(\tilde{n}_B^1)). \tag{10}$$

The first period decision of sellers is much like their second period decision. Furthermore, since the sellers would obtain exactly the same benefit in the second period regardless of their first period decision, sellers need not engage in any dynamic considerations. Observing the seller price in the first period, and expecting $\tilde{n}_B^1$ buyers to subscribe, seller j will join the platform whenever

$$\alpha_S - \theta_S^i + v_S(\tilde{n}_B^1) - p_S^1 \geq 0$$

or equivalently whenever

$$\theta_S^i \leq \alpha_S + v_S(\tilde{n}_B^1) - p_S^1.$$

As a result, the actual number of sellers which would subscribe is simply given by

$$n_S^1(p_B^1, n_B^1) = F_S(\alpha_S + v_S(\tilde{n}_B^1) - p_S^1). \tag{11}$$

The buyers and sellers form rational expectations in the first period as well implying that $\tilde{n}_B^1 = n_B^1(p_B^1, n_B^1(p_B^1, \tilde{n}_B^1, \tilde{n}_S^1), n_S^1(p_S^1, \tilde{n}_B^1)) = n_B^1(p_B^1, p_S^1)$ and $\tilde{n}_S^1 = n_S^1(p_S^1, n_B^1(p_B^1, \tilde{n}_B^1, \tilde{n}_S^1)) = n_S^1(p_B^1, p_S^1)$. Given these rational expectations demands, the profit function of the platform in the beginning of period one is

$$\Pi_P(p_B^1, p_S^1) = (p_B^1 - c_B)n_B^1(p_B^1, p_S^1) + (p_S^1 - c_S)n_S^1(p_B^1, p_S^1) + \delta\bar{\Pi}_P^2(n_B^1(p_B^1, p_S^1)).$$

Thus, the platform takes in to account the effect of its pricing decision on the profits from the second period. One again assuming that the first order conditions characterize the solution to the profit maximization problem if the platform, the first period prices have to satisfy

$$\begin{aligned} n_B^1(p_B^2, p_S^2) + (p_B^1 - c_B)\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} + (p_S^1 - c_S)\frac{\partial n_S^1(p_B^1, p_S^1)}{\partial p_B^1} & \\ -\delta({p_B^2}^*(n_B^1) - c_B)\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} &= 0 \quad (12) \\ (p_B^1 - c_B)\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_S^1} + n_S^1(p_B^1, p_S^1) + (p_S^1 - c_S)\frac{\partial n_S^1(p_B^1, p_S^1)}{\partial p_S^1} &= 0 \quad (13) \end{aligned}$$

where I use the fact that $\frac{d\bar{\Pi}_P^2(x)}{dx} = -({p_B^2}^*(x) - c_B)$ in equation (12).

In order to further characterize the optimal first period prices, I derive the necessary partial derivatives of the demand functions using total derivation of equations (10) and (11) in the appendix. Substituting these partial derivatives in (12) and (13) and simplifying yields two elasticity rules that the first period prices have to satisfy which are given by

$$\frac{p_B^1 - (c_B + \delta({p_B^2}^*({n_B^1}^*) - c_B) - v_S'({n_B^1}^*){n_S^1}^* - \delta\rho {n_B^1}^*)}{p_B^1} = \frac{1}{\eta_B^1(\alpha_B + v_B({n_S^1}^*) - p_B^1 + \delta p_B^2({n_B^1}^*))} \tag{14}$$

$$\frac{p_S^1 - (c_S - v_B'({n_S^1}^*){n_B^1}^*)}{p_S^1} = \frac{1}{\eta_S^1(\alpha_S + v_S({n_B^1}^*) - p_S^2)} \tag{15}$$

where ρ denotes marginal change in the second period price when the number of buyers subscribing in the first period slightly increases. Formally, $\rho = \frac{\partial {p_B^2}^*(x)}{\partial x}$.

Although the seller side pricing rule given in equation (15) is exactly as the one found in Armstrong (2006), since the buyer side pricing incentives change quite a bit, the optimal first period price to sellers is likely to be substantially different. In addition to taking into account the gains an additional buyer will bring on the seller side, the platform needs to take into account a number of other effects. Each buyer who joins the platform in period one, will not need to purchase in the second period, and hence results in a loss (viewed in the first period) equal to

$\delta({p_B^2}^*({n_B^1}^*) - c_B)$. Moreover, an additional buyer in the first period results in a change in the second period prices measured by ρ. This change in the second period prices changes the outside option of all the buyers which consider to join the platform in the first period. In order to keep the overall number of buyers subscribing constant, the platform needs to adjust its first period with $\delta\rho$. This results in a gain or loss from all the participating buyers which is given by $\delta\rho {n_B^1}^*$. As argued before, the most likely case is that the second period price will decrease when one more buyer joins in the first period, i.e. $\rho < 0$. Thus bringing one more buyer on board implies a perceived marginal cost that is adjusted downwards due to the potential gains from the seller side in the first period, upwards due to the loss in profits from the second period and due to additional compensation required by all the buyers to stay on board in the first period.

The first period price to the buyers can be thought of the sum of two prices, one for the service for the first period and another component for the second period. Interpreted in this fashion, the payment collected from first period buyers in the first period for the second period service is equal to $\delta {p_B^2}^*$. The first period buyers and second period buyers pay the same price for the service in the second period. However, the very fact that the platform cannot commit to the second period prices implies that platform must offer an additional discount $\delta\rho$ to each of the buyers in the first period to insure their participation. This discount lowers the first period component. Given the increased subscription elasticity, the second period component is also likely to lower be when compared with prices absent dynamic effects. As a result, the buyers joining the platform in the first period are likely to pay a lower price.

These pricing rules unfortunately do not allow me to derive sharper results on the impact of the *durability* of the buyers' and sellers' subscription decisions. In order to further investigate the optimal prices, I adopt assumptions on the distribution of intrinsic values as well as the shape of the network benefit function in the next section and derive optimal platform prices in closed form.

4 A more stylized model

In this section, I assume that the intrinsic values of buyers and sellers are uniformly distributed. Namely, $f_k(\theta_k) = 1$ for $0 \leq \theta_k \leq L_k$ and $k \in \{B, S\}$. This implicitly assumes that the total buyer (seller) population is given by $L_B(L_S)$. Moreover, I assume that both buyers and sellers enjoy network benefits which are proportional to the number of users on the other side. Namely, I set $v_B(x) = w_B x$ and $v_S(x) = w_S x$. These assumptions then suggest that for well-behaved rational expectations demand we need $Q_2 = 1 - w_B w_S > 0$, or equivalently, it is necessary that the product of the marginal network benefits of buyers and sellers be not too large. Furthermore, given the linearity of implied demand functions, one can set marginal costs of serving buyers and sellers to zero without loss of generality.[9]

In subsection 4.1, I will derive the rational expectations demand in the second period and the corresponding prices as a function of the number of buyers which subscribe in period one. Subsequently, in subsection 4.2, I derive the first period rational expectations demands and derive the subgame perfect platform prices. I then compare these to the benchmark prices which would obtain under a rental (or per period subscription) policy.

4.1 The second period

The linearity of the cumulative distribution functions for θ_B and θ_S along with the linearity of the network benefit functions greatly simplifies the calculation of the rational expectations demand system. Namely, the rational expectations demand in period two satisfy,

$$\begin{aligned} m_B(p_B^2, n_S^2, x) &= \alpha_B + w_B n_S^2(p_B^2, n_S^2, x) - p_B^2 - x. & (16)\\ n_S^2(p_B^2, n_S^2, x) &= \alpha_S + w_S(x + m_B(p_B^2, n_S^2, x)) - p_S^2 & (17)\\ & & (18)\end{aligned}$$

These two expressions implicitly assume that $m_B(p_B^2, n_S^2, x) > 0$. If for some prices and x values no buyers in the second period join, then the sellers demand is simply given by (17) when $m_B(p_B^2, n_S^2, x) = 0$. I will first focus on the case where some buyers are expected to join in a rational expectations equilibrium given prices and x.

Let $Q_2 = 1 - w_B w_S$. Then, the second period rational expectations demand function for the buyers is given by

$$m_B(p_B^2, n_S^2, x) = \frac{1}{Q_2}\big(\alpha_B + \alpha_S w_B - p_B^2 - w_B p_S^2\big) - x$$

and the rational expectations demand function for the sellers is

$$n_S^2(p_B^2, n_S^2, x) = \frac{1}{Q_2}\big(\alpha_S + \alpha_B w_S - p_S^2 - w_S p_B^2\big).$$

The immediate effect of a customer base for the platform on the buyer side, x, is to reduce the buyer side demand.[10] As usual, the demands from both sides are complementary to one another. An increase of a price on one side results in a decrease of subscriptions on both sides. When price increases on one side, it results in a decline of subscriptions on that side. In addition, the users on the other side rationally forecast the decline in the number of users where the price increase occurs, and hence less of them end up subscribing as well. The second period demands are well defined only when the product of marginal network benefits, $w_B w_S$, is not too large. Although both of these marginal network benefits cannot be too large simultaneously,

[9]In a model with constant marginal costs c_B and c_S, setting α_k to $\alpha_k - c_k$ and setting p_k^t to $p_k^t - c_k$ for $k \in \{B, S\}$ transforms the model to one with zero marginal costs on both sides.

[10]I, of course implicitly, assume in this formulation that at the relevant prices, there would be new buyers subscribing to the platform so that $m_B(p_B^2, p_S^2, x) > 0$.

the formulation permits situations where network effects are extremely small on one side, and large on the other. Nevertheless, I will refrain from placing further conditions on the parameters at this point, as the optimal prices, as well as sensibility of first period demands place more stringent restrictions.

In order to simplify exposition, define $w = \frac{w_B + w_S}{2}$, $\tau_B = \alpha_B(1 - w_S w) + \alpha_S(w - w_S)$ and $\tau_S = \alpha_S(1 - w_B w) + \alpha_B(w - w_B)$. Given the demand functions derived above, solving the profit maximization problem of the platform is straightforward and the resulting prices are given by

$$\begin{aligned} p_B^{2\,*}(x) &= \frac{1}{2(1-w^2)}\big(\tau_B - Q_2 x\big) \\ p_S^{2\,*}(x) &= \frac{1}{2(1-w^2)}\big(\tau_s + Q_2 w x\big). \end{aligned}$$

At these prices, the corresponding sales of the platform to buyers and sellers are

$$m_B{}^*(x) = \frac{1}{2(1-w^2)}\big(\alpha_B + \alpha_S w - (1 - w_S w)x\big) \tag{19}$$

$$n_S^{2\,*}(x) = \frac{1}{2(1-w^2)}\big(\alpha_S + \alpha_B w - (w - w_S)x\big). \tag{20}$$

When there are no buyers carried over from the first period, i.e. whenever $x = 0$, the corresponding prices and sales represent the outcome which would emerge if the platform were able to rent or restrict subscriptions to a single period. That is $p_B^R = p_B^{2\,*}(0)$ and $p_S^R = p_S^{2\,*}(0)$ as well as $n_B^R = m_B{}^*(0)$ and $n_S^R = n_S^{2\,*}(0)$. In this case, since $\alpha_B + \alpha_S w > 0$ and $\alpha_S + \alpha_B w > 0$, for the demands to be positive it is necessary that $w < 1$ or equivalently $w_B + w_S < 2$. When this condition is not satisfied, the platform will choose to sell to all customers at least on one side. I will maintain this assumption in the following not to deal with such situations. Thus, with this restriction there will always be some buyers and some sellers which do not participate in the platform facing the optimal prices.

It is clear from (19), that for larger values of x and when $(1 - w_S w)$ is positive, there may be no buyers joining the platform facing the optimal prices. This naturally will contradict with the initial hypothesis that some buyers join. Namely, the solutions I derived are valid only when $1 - w_S w > 0$ and

$$x \le \frac{\alpha_B + \alpha_S w}{1 - w_S w} = x_{CR}.$$

In case, $1 - w_S w > 0$ but $x > x_{CR}$, then the platform will optimally not make any sales to buyers in the second period. In this case the seller demand only depends on x and p_S^2 and is given by $\hat{n}_S^2 = \alpha_S + w_S x - p_S^2$ and the second period profit of the platform is maximized when $p_S^{2\,o} = \frac{1}{2}(\alpha_S + w_S x)$, and the corresponding platform profits is given by $\Pi_P^{2\,o} = \frac{1}{4}(\alpha_S + w_S x)^2$. To implement such an outcome the platform simply needs to set $p_B^{2\,o}$ very high that no buyer considers purchasing a subscription.

Whenever $1 - w_S w > 0$, but $w_S > w_B$, we have $w < w_S$ and hence the equilibrium number of sellers subscribing in the second period is always positive. On the other hand, whenever

$w_B > w_S$, for large values x the expression in (20) can become negative. It turns out the critical value of x which drive the seller demand at the optimal prices to zero exceeds x_{CR}, the critical value of x above which no buyer will join the platform in the second period. Hence, the relevant condition on the x to keep track is whether it is smaller than x_{CR} implying some buyers will join in the second period, or whether the number of buyers which join in period one are so large, i.e. $x > x_{CR}$, that no new buyer joins in the second period, and the platform only sells to the sellers in the second period. Below I will rule out this possibility, and consider those parameter values where the sales to buyers at the optimal prices in the first period does not exceed x_{CR}.

Whenever $1 - w_S w < 0$ new buyers will always join. This is the case whenever $w_S > \frac{1}{2}[\sqrt{w_B^2 + 8} - w_B] = w_S^{cr}$. However, it turns out that $1 - w_S w < 0$ and $1 - w > 0$ can only be satisfied together whenever $w_B < 1$. When $w_B < 1$ and $w_S > w_S^{cr}$, however, we have that $w_S > w_B$ and hence the seller demand will always be positive. Therefore the model's results will also hold whenever $1 - w > 0$ and $1 - w_S w < 0$.

A comparison of the optimal buyer price with the rental price charged to the buyers suggests that when buyer decisions are durable, buyers will receive a discount with durability. That is, since

$$p_B^{2\,*}(x) - p_B^R = p_B^{2\,*}(x) - p_B^{2\,*}(0) = -\frac{Q_2 x}{2(1-w^2)} < 0,$$

the second period buyers receive a discount that grows with the size of the user base which subscribe to the platform in period 1. As I will use it later on when I study the first period optimal pricing problem, it is worthwhile to compute response of the buyer price to the number of buyers which join in the first period. It is easy to verify that

$$\hat{\rho} = \frac{\partial p_B^{2\,*}(x)}{\partial x} = -\frac{Q_2}{2(1-w^2)} < 0.$$

Comparing the sales to buyers in the second period under the rental policy and durability, namely $m_B^*(x)$ and $m_B^*(0)$, suggests that a smaller number of buyers will purchase with durability whenever $1 - w_S w > 0$. This would be not surprising as the high value buyers leave the market in period one, platform may find to attract less of the low value buyers in the second period. Indeed, when sufficiently many buyers purchased subscriptions in period one, namely x_{CR}, the platform will attract no new buyers. What is surprising however is when the marginal network benefits of the sellers exceeds w_S^{cr}, it may be that more buyers may join the platform with durability if sellers value buyers sufficiently strongly. Note that in this case the sellers will have access to a buyer population of size $x + m_B^*(x)$.

The sellers on the other hand end up paying a larger price when the decisions of x buyers which join the platform in period one is durable. It is easy to verify that

$$p_S^{2\,*}(x) - p_S^R = p_S^{2\,*}(x) - p_S^{2\,*}(0) = \frac{Q_2 w x}{2(1-w^2)} > 0.$$

Similarly evaluating

$$n_S^{2\,*}(x) - n_S^R = n_S^{2\,*}(x) - n_S^{2\,*}(0) = -\frac{(w - w_S)x}{2(1-w^2)}$$

reveals that although they face a higher price, more sellers might join whenever average marginal network benefits w is less than the marginal network benefits of sellers. This is possible only when sellers value buyers more. Otherwise, the number of sellers participating in the second period would be below the rental benchmark. I summarize these findings in the next proposition.

Proposition 1 *Whenever $w < 1$, $Q_2 > 0$ and $x > 0$ buyers joined the platform in period one, the platform charges a higher price to sellers and a lower price to buyers in the second period. For sufficiently small values of the marginal network benefits of sellers, namely, for $w_S \leq w_S^{cr}$, a smaller number of additional buyers join the platform relative to the rental benchmark. In fact, when $w_S \leq w_S^{cr}$ but $x > x_{CR}$ no new buyers will join in the second period. In this case, the platform sells only to the sellers. On the other hand, whenever $w_S > w_S^{cr}$, a larger number of additional buyers will join. Whenever $w_S \leq w_B$, a smaller number of sellers join in the second period relative to the rental benchmark, while whenever $w_S > w_B$ more sellers will join. Surprisingly, for $w_S > w_S^{CR}$ and $w_B < 1$, more sellers and buyers join in the second period relative to the rental benchmark.*

Proof. See the preceding arguments. ■

4.2 The first period

I now proceed to the analysis of the first period. In the first period, the platform announces prices and subsequently buyers and sellers form the necessary expectations to reach their subscription decisions. I will proceed with the assumption that in the second period some buyers will join the platform. This implicitly assumes that the platform attracts a buyer population that is not too large that the second period buyer demand is driven to zero. Namely, even when $1 - w_S w > 0$, $x < x_{CR}$ in period two. I will later verify when this assumption indeed holds.

Sellers' problem is relatively straightforward. Given p_S^1 and an expectation regarding the number of buyers that would join the platform, $\tilde{n}_B^1$, they make their subscription decisions. Since the distribution of the intrinsic values of the sellers θ_S is uniform, the actual number of sellers that would join the platform under these conditions is given by

$$n_S^1(p_S^1, \tilde{n}_B^1) = \alpha_S + w_S \tilde{n}_B^1 - p_S^1.$$

On the other hand, buyers take into account the possibility of waiting and purchasing in the second period. This requires them to take into account the future subscription price which turns out to be a function of the number of buyers which participate in period one. Thus they also need to form an expectation about the contemporaneous participation on the buyers' side. Given the uniformly distributed θ_B, translating equation (10), implies that actual number of buyers subscribing to the platform facing a price of p_B^1 and holding expectations $\tilde{n}_B^1$ and $\tilde{n}_S^1$ is given by

$$n_B^1(p_B^1, \tilde{n}_B^1, \tilde{n}_S^1) = \alpha_B + w_B \tilde{n}_S^1 - p_B^1 + \frac{\delta}{2(1 - w^2)} \left(\tau_B - Q_2 \tilde{n}_B^1\right).$$

Once again rational expectations require, $\tilde{n}_B^1 = n_B^1(p_B^1, n_B^1(p_B^1, \tilde{n}_B^1, \tilde{n}_S^1), n_S^1(p_S^1, \tilde{n}_B^1))$ and $\tilde{n}_S^1 = n_S^1(p_S^1, n_B^1(p_B^1, \tilde{n}_B^1, \tilde{n}_S^1))$. Define $Q_1 = \big(2(1-w^2)+\delta\big)Q_2$. The solution of these two equations for $\tilde{n}_B^1$ and $\tilde{n}_S^1$ yields the rational expectations demands in period one which are given by

$$
\begin{aligned}
n_B^1(p_B^1, p_S^1) &= \frac{1}{Q_1}\Big(2(1-w^2)(\alpha_B + w_B\alpha_S) + \delta\tau_b - 2(1-w^2)(p_B^1 + w_B p_S^1)\Big) && (21)\\
n_S^1(p_B^1, p_S^1) &= \frac{1}{Q_1}\Big(2(1-w^2)(\alpha_B + w_B\alpha_S) + \delta w_S\tau_b + \delta\alpha_S Q_2 \\
&\quad -2(1-w^2)(w_S p_B^1 + p_S^1) - \delta Q_2 p_S^1\Big) && (22)
\end{aligned}
$$

Relative to the demand that would arise on the buyer side, under durability buyer demand is less price sensitive with respect to both prices as

$$\frac{2(1-w^2)}{Q_1} - \frac{1}{Q_2} = -\frac{\delta}{Q_2(2(1-w^2)+\delta)} < 0.$$

This is to be expected as a price decrease in the first period will induce more buyers to join. However, the more buyers join in the first period, the lower would be the second period price and hence the outside option of the buyers becomes more valuable. Under the rental policy, a price cut has no impact on the outside option of buyers. Thus the buyers become less price sensitive in period one. The demand function from buyers in the first period also shifts down since

$$\frac{1}{Q_1}\Big(2(1-w^2)(\alpha_B + w_B\alpha_S) + \delta\tau_b\Big) - \frac{1}{Q_2}\Big(\alpha_B + s w_B\Big) = -\frac{\delta w(\alpha_B w_S + \alpha_S)}{Q_1} < 0.$$

The impact of durability of buyer decisions on the demand from sellers seems to be more elaborate. The demand of the sellers also shift down as

$$\frac{1}{Q_1}\Big(2(1-w^2)(\alpha_B + w_B\alpha_S) + \delta w_S\tau_b + \delta\alpha_S Q_2\Big) - \frac{1}{Q_2}\Big(\alpha_S + b w_S\Big) = -\frac{\delta w w_S(\alpha_B w_S + \alpha_S)}{Q_1} < 0.$$

The seller demand becomes less responsive to the buyer price for the same reason as buyer demand becomes less price sensitive. It turns out that although there is an additional effect on the seller side of the seller price, overall the seller demand is less price sensitive to the seller price relative to the demand that would arise under rental policy since

$$\frac{2(1-w^2)+\delta Q_2}{Q_1} - \frac{1}{Q_2} = -\frac{\delta(1-Q_2)}{Q_1} < 0.$$

As both demands shift down but become less price sensitive the effects on the prices will be ambiguous.

Given the rational expectations demands, the platform maximizes the discounted sum of its profits in periods one and two. In computing the expected profits from period 2, the platform evaluates the second period profit $\Pi_P^{2^*}(x) = \Pi_P^2(p_B^{2^*}(x), p_S^{2^*}(x), x)$ with $x = n_B^1(p_B^1, p_S^1)$. Namely, the platform maximizes that following profit function by choosing p_B^1 and p_S^1:

$$\Pi_P = p_B^1 n_B^1(p_B^1, p_s^1) + p_S^1 n_S^1(p_B^1, p_s^1) + \delta\Pi_P^{2^*}(n_B^1(p_B^1, p_s^1)).$$

Define $Q_0 = 16(1-w^2)^2 + 4\delta Q_2$. Solving the first order conditions in both first period prices, it is easy to verify that the optimal first period prices are given by

$$p_B^{1\,*} = \frac{8(1-w^2)\tau_B - 2\delta\alpha_S w_S Q_2}{Q_0} - \frac{\delta\tau_B}{2(1-w^2)} \tag{23}$$

$$p_S^{1\,*} = \frac{8(1-w^2)\tau_S - 2\delta\alpha_S Q_2}{Q_0} \tag{24}$$

The corresponding demands from buyers and the sellers in the first period are given by

$$n_B^{1\,*} = \frac{8(1-w^2)(\alpha_b + \alpha_S w)}{Q_0} \tag{25}$$

$$n_S^{1\,*} = \frac{8(1-w^2)(\alpha_S + \alpha_B w) + 2\delta\alpha_S Q_2}{Q_0} \tag{26}$$

For these equilibrium prices and corresponding sales values to be the optimal choices, it is needed that the platform does not sell to too many buyers in the first period when $1 - w_S w > 0$. If this were to be the case, then no additional buyers will join in the second period, and hence the decision problem of the buyers will differ from the one I constructed above. As a consequence the optimization problem faced by the platform will also be different. Thus, the results apply only for those parameter values where $n_B^{1\,*} < x_{CR}$ whenever $1 - w_S w > 0$.[11] For cases, where $1 - w_S w \leq 0$, in the second period some buyers will always join, thus the first period decision problems faced by the buyers and the platform are consistent with the underlying assumptions. Thus, the first period prices I derive above apply in this case.

The seller prices in the rental strategy and under durability of buyer decisions are directly comparable. Computing the difference, I obtain

$$p_S^{1\,*} - p_S^R = \frac{\delta Q_2(\alpha_B + \alpha_S w)(wb - ws)}{Q_0(1-w^2)}.$$

The seller prices with durability is higher (lower) when sellers value buyers less (more) relative to the rental prices. The comparison between the price buyers pay under durability and under rental strategy are not directly comparable. The buyers under durability pay a price and obtain in return a subscription that is valid for two periods. Under the rental regime, the same buyers will have to pay the platform in each period the rental price. Given that in the model the rental prices remain constant over time, I compare $p_B^{1\,*}$ and $(1+\delta)p_B^R$. Computing the difference yields

$$p_B^{1\,*} - (1+\delta)p_B^R = -\frac{\delta Q_2(\alpha_b + \alpha_S w)(1 - w_S w)}{2Q_0(1-w^2)}.$$

Provided that $1 - w_S w > 0$, the buyers pay a lower price in the first period relative to the rental benchmark. This is to be expected, as the Coasian dynamics yield competition between the incarnations of the platform in the two periods. However, if $1 - w_S w < 0$ buyers end up paying

[11] There are parameter values for which $1 - w_S w > 0$, $w < 1$, and the implied number of buyers who would join in period one exceed x_{CR}. Although the complete analysis of these cases are interesting, it requires a reformulation of the first period decision problems. Thus, the pursuit of optimal prices in such cases are left for future research.

more under durability. This is surprising. However, recall my model is valid in this situation only when $w_S > w_S^{CR}$ and $w_B < 1$ implying $w_S > w_B$. For such parameters, whenever α_B and α_S are not too different, the buyers obtain a subsidy with the rental policy. The increase in the buyer price need not imply that buyers are not being subsidized. It may be that the size of the subsidy becomes smaller. However, it is possible to find parameter values where buyers may indeed end up paying positive prices. The implication is that the price structure may be substantially altered when durability of the buyers' decisions is taken in to account.

Comparing the sales to buyers in the first period under durability and with the rental policy implies that under durability less buyers will join in the first period. Formally

$$n_B^R - n_B^{1*} = \frac{2\delta Q_2(\alpha_B + \alpha_S w)}{(1-w^2)Q_0} > 0.$$

A similar comparison on the seller side reveals that the number of sellers joining in the first period will also be lower since

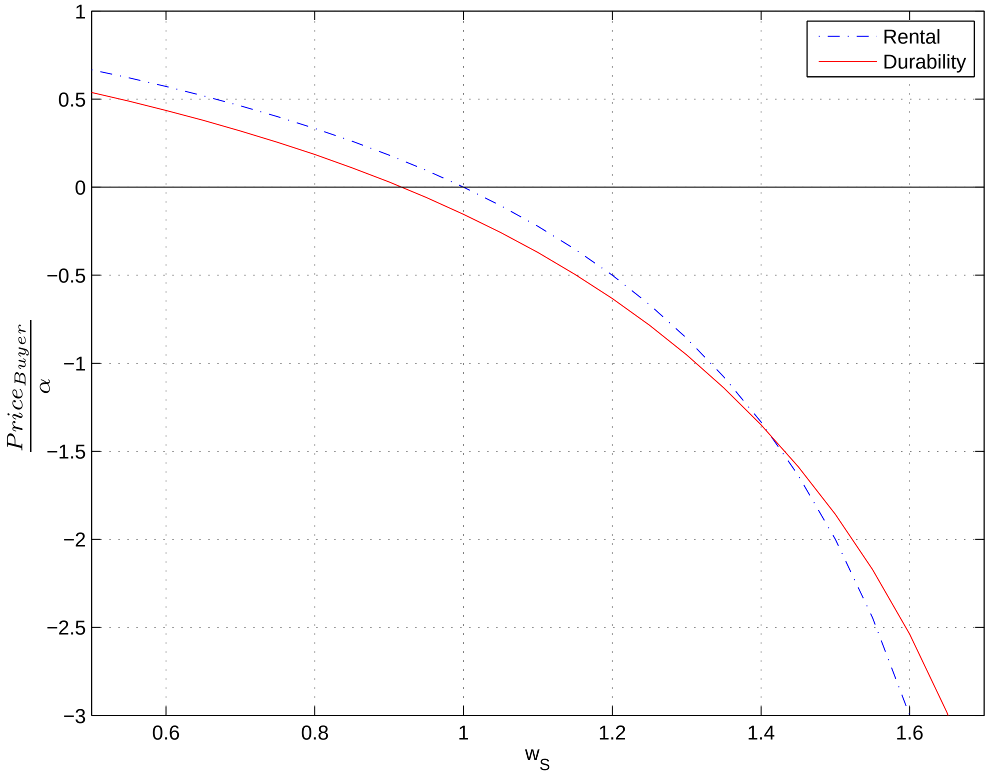
$$n_S^R - n_S^{1*} = \frac{2\delta w Q_2(\alpha_B + \alpha_S w)}{(1-w^2)Q_0} > 0.$$

Proposition 2 *Assume $w < 1$ and $w_B w_S < 1$. Then, under durability sellers pay more relative to the rental prices if $w_S < w_B$, and pay less whenever $w_S > w_B$. Buyers face a lower price under durability relative to the rental benchmark provided that $1 - w_S w > 0$. If, on the other hand, $1 - w_S w < 0$ buyers face a higher price. Under durability the number of buyers and sellers which subscribe in the first period is lower.*

In order to better illustrate the changes in pricing which arise as a result of the durability of buyers' decisions, it is useful to investigate a few more specific cases. Consider first an extreme setting where $w_B = 0$. Assume furthermore that $\alpha_B = \alpha_S = \alpha$. Under these conditions, $\bar{p}_B^R = \frac{\alpha(1-w_S)}{2-w_S}$ and $\bar{p}_S^R = \frac{\alpha}{2-w_S} > 0$. Therefore, the buyers will receive a subsidy whenever $1 < w_S < 2$ under the rental policy. Under these parameter restrictions, $n_B^{1*} < x_{CR}$ whenever $1 - w_S w > 0$ as well. Therefore, the solutions provided above for the durability case are valid for all $0 \leq w_S < 2$. Furthermore, both p_B^{1*} and p_S^{1*} will be proportional to α similar to their counterparts in the rental case. Since $w_S > w_B = 0$, sellers pay a lower price under durability for all values of w_S. A comparison of buyer prices however is interesting as it exhibits quite some changes in the price structure. Figure 1 presents this comparison. As can be seen buyers are subsidized whenever w_S exceeds unity with the rental policy. But for all $w_S < \sqrt{2}$, under durability first period buyers pay less. This implies that for an interval of w_S values, namely for $w_S \in [0.918, 1]$ if $\delta = 1$, buyers are subsidized under durability although they pay positive prices with the rental policy.

A second exercise worth performing is the polar opposite case. Assume $w_S = 0$ and continue assuming that $\alpha_B = \alpha_S = \alpha$. The rental prices in this case become $\bar{p}_B^R = \frac{\alpha}{2-w_B}$ and $\bar{p}_S^R = \frac{\alpha(1-w_B)}{2-w_B}$. With $w_S = 0$, we have always $1 - w_S w = 1 > 0$. If in addition δ is sufficiently close

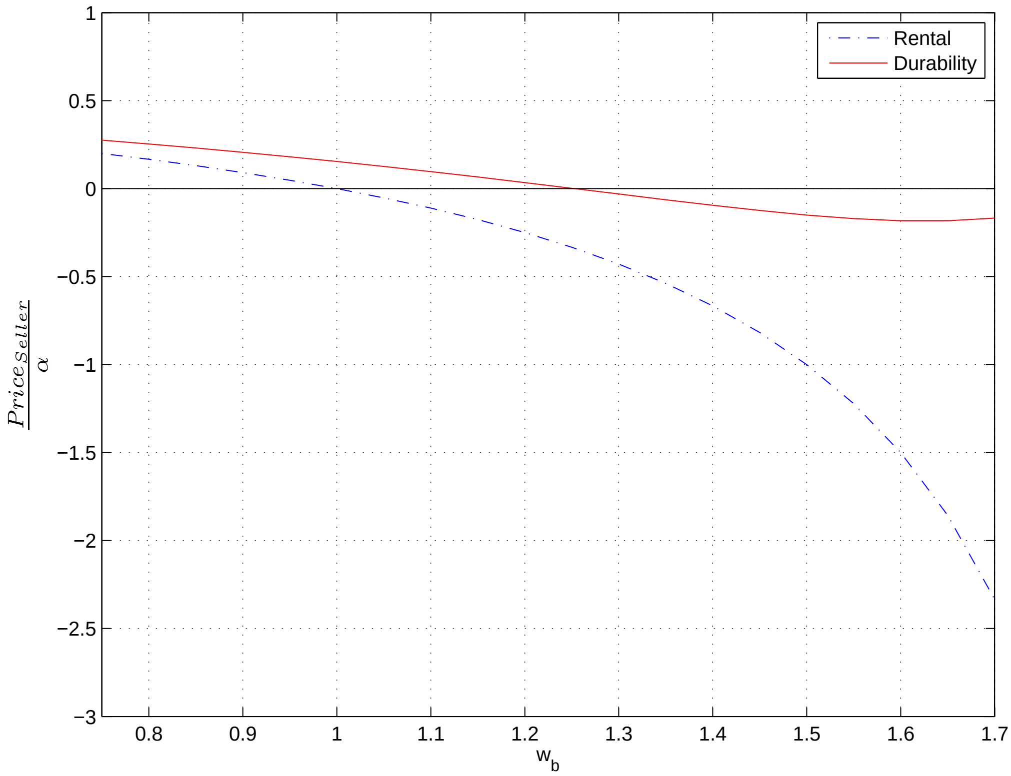
Figure 1: A comparison of buyer prices under durability and rental policies: $w_B = 0$, $\delta = 1$



to unity, for all $0 < w_B < 2$, ${n_B^1}^* < x_{CR}$, therefore the prices derived above are valid. I have already established that seller prices will be higher relative to the rental benchmark as sellers value buyers less, i.e. $w_S = 0 < w_B$. But since $1 - w_S w > 0$, buyers pay a lower price in the first period. Note that under the rental policy the sellers will be subsidized whenever $1 \leq w_B < 2$. It turns out that under durability the sellers will be subsidized only when w_B further increases. In other words, there is a range of buyer marginal network effects, for which under rental policy sellers are subsidized, and under durability they pay positive prices. Figure 2 presents seller prices under durability and the rental policy when in addition to other parameter restriction δ is set to unity as well. In this case for $w_B \in [1, 1.253]$, the sellers are subsidized under the rental policy, while they pay a positive price under durability.

A third though experiment which is useful to consider is when $w_S = \kappa < 1$ and $w_B = \kappa + \epsilon$ where $\epsilon > 0$ is a small number. I maintain the assumption that $\alpha_B = \alpha_S = \alpha$. The validity of the results require that $\frac{w_B + w_S}{2} = \kappa + \frac{\epsilon}{2} < 1$ which in turn implies $1 - w_B w_S = 1 - \kappa^2 - \kappa\epsilon > \frac{\epsilon^2}{4} > 0$ and $1 - w_S w = 1 - \kappa^2 - \frac{\kappa\epsilon}{2} > 0$. Moreover, as ϵ goes to zero, for all $\kappa < 1$, we have that ${n_B^1}^* < x_{CR}$. Therefore, the solution provided above are valid for values of κ. In this case, the optimal rental policy entails charging the buyers a price slightly larger than the price faced by the sellers. Formally, $p_B^R = \frac{1-\kappa}{2(1-\kappa)+\epsilon}$ and $p_S^R = \frac{1-\kappa-\epsilon}{2(1-\kappa)+\epsilon}$, so that $p_B^R > p_S^R$. I will compare the life time payments of buyers and sellers which would subscribe the platform under both settings.

Figure 2: A comparison of seller prices under durability and rental policies: $w_S = 0$, $\delta = 1$

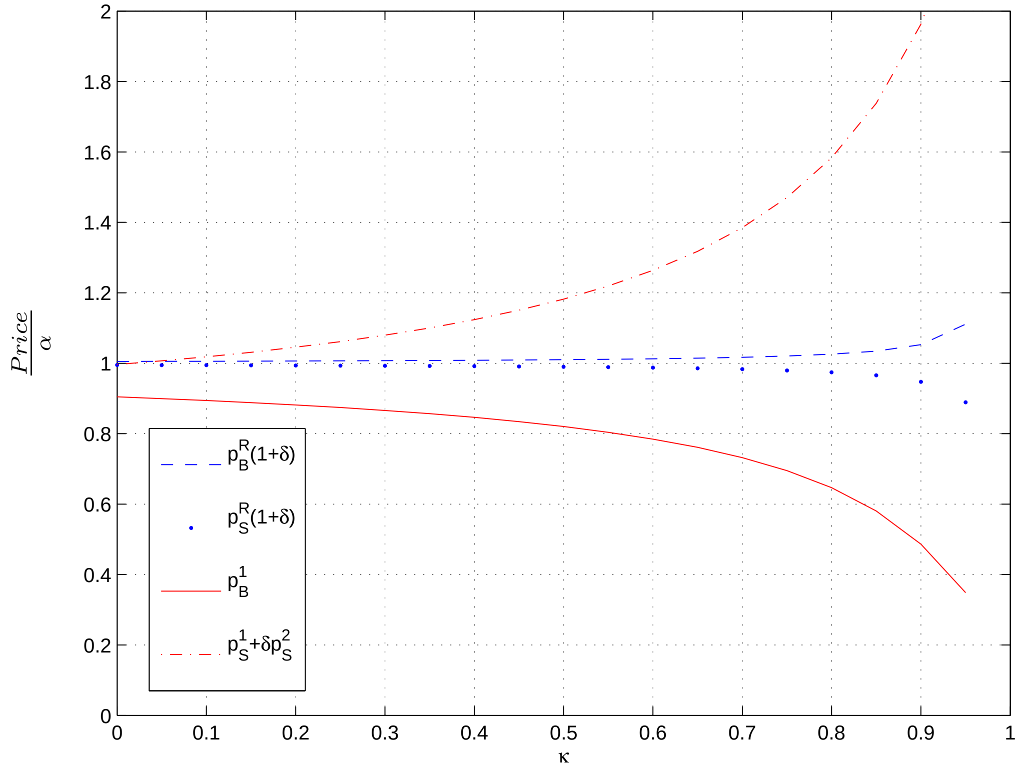


The prices are presented in Figure 3 when the discount factor is set to unity. As can be seen from the figure, the pricing behavior may turn upside down if durability is taken into account. Under the rental policy, buyers face higher prices relative to sellers as they value them slightly more. Moreover, the as κ increases buyer price increases and the seller price decreases under the rental policy. When durability is taken into account, the buyers end up receiving a large discount and this discount increases with the value of κ. Thus overall buyers face a lower price when the marginal network benefits κ increases. This in turn increases the willingness to pay from the sellers in both periods. Hence, the sellers end up paying more under durability. Moreover, the price they face increases as κ increases.

Up to now, I have used the rental policy as a benchmark without showing that indeed it is the more profitable alternative. After deriving the optimal prices under durability, I now can also formally investigate whether rental policy would provide a more profitable option for the platform. Under the rental policy, the platform charges constant prices in both periods given by $p_k^{2*}(0)$, for $k \in \{B, S\}$. As a result the number of buyers and sellers joining in each period will also be the same. One can construct the total profits with the rental policy then as

$$\begin{aligned}\Pi_P^R &= (1+\delta)\Big(\sum_{k\in\{B,S\}} p_B^{2\,*}(0) n_B^{2\,*}(0)\Big) \\ &= \frac{(1+\delta)(\alpha_B^2 + \alpha_S^2 + 2\alpha_B\alpha_S w)}{4(1-w^2)}\end{aligned}$$

Figure 3: A comparison of seller prices under durability and rental policies: $w_S = \kappa$, $w_B = \kappa + \epsilon$, $\delta = 1$ and $\epsilon = 0.01$



Using the optimal prices and sales values in both periods under durability, the cumulative profits of the platform, Π_P^*, can be written after simplifications as

$$\Pi_P^* = \Pi_P^R - \frac{\delta Q_2(\alpha_B + \alpha_S w)}{(1 - w^2)Q_0}.$$

Thus, the the rental policy yields always higher profits relative to the optimal outcome under durability, as $\Pi_P^R - \Pi_P^* > 0$. This result formally establishes the rental policy as the appropriate benchmark and I summarize it in the next proposition.

Proposition 3 *Rental policy yields a higher cumulative profit when compared with the cumulative profit the platform earns under durability.*

5 Conclusion

In this paper I investigated the effects of durability of choices made by one group of consumers on the price structure of a monopoly platform. The model is admittedly a stylized two period model but directly comparable to the standard models used in the literature for studying two sided markets in static settings. I show that durability introduces Coasian dynamics and as a result it is likely that the users on the durable side pay low prices in the second period, and consequently the users on that side also receive discounts in the first period. I establish that the platform may eliminate these problems by for example following a rental policy which restricts subscriptions to a single period. The price structure that emerges with the rental policy is identical to that arises in a static model. I then demonstrate that when the rental policy is not feasible, the effects of durability may turn price structure upside down.

It is possible that a platform which subsidizes one side under the rental policy does not offer any subsidies under durability. In contrast, it possible to find situations where the optimal rental policy does not require any subsidies, while optimal pricing under durability subsidizes the durable side. More generally the main insight that the users who value the other side more should pay a higher price may no longer hold when durability is taken into account.

The model in its current form is rather stylized and ignores a number of relevant dimensions. I do not consider arrival of new consumers. I also do not take into account depreciation in time possibly at different rates on both sides. A desirable extension to the current model would be one where the platform offers its services over a long time horizon, new consumers arrive at each period while some old consumers leave the market. In addition, the decisions of users on both sides may be durable, but they depreciate at varying rates. Of course, it would be very interesting to investigate competitive consequences of durability as well. The analysis of such issues with more elaborate models are left for future research.

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Appendix

A.1 Derivation of the second period pricing rules

The second period rational expectations demand functions are determined by the two functions $m_B(p_B^2, p_S^2, x)$ and $n_S^2(p_B^2, p_S^2, x)$ which solve the following two equations:

$$\begin{aligned} m_B(p_B^2, p_S^2, x) &= F_B(\alpha_B + v_B(n_S^2(p_B^2, p_S^2, x))) - p_B^2) - x \\ n_S^2(p_B^2, p_S^2, x) &= F_S(\alpha_S + v_S(x + m_B(p_B^2, p_S^2, x))) - p_S^2). \end{aligned}$$

Given the identities provided by these two equations, the derivatives of the left and right hand sides has to be equal with respect to any variable. Thus, taking the derivative of each equation first with respect to p_B^2 and then with respect to p_S^2 yields four linear equations in the partial derivatives of interest: $\frac{\partial m_B(p_B^2,p_S^2,x)}{\partial p_B^2}$, $\frac{\partial m_B(p_B^2,p_S^2,x)}{\partial p_S^2}$, $\frac{\partial n_S^2(p_B^2,p_S^2,x)}{\partial p_B^2}$ and $\frac{\partial n_S^2(p_B^2,p_S^2,x)}{\partial p_S^2}$. In order to simplify the exposition, let $\hat{v}'_B = v'_B(n_S^2(p_B^2, p_S^2, x))$, $\hat{v}'_S = v'_S(x + m_B(p_B^2, p_S^2, x))$, $\hat{f}_B = f_B(\alpha_B + v_B(n_S^2(p_B^2, p_S^2, x) - p_B^2)$, $\hat{f}_S = f_S(\alpha_S + v_S(x + m_B(p_B^2, p_S^2, x) - p_S^2)$, $\hat{F}_B = F_B(\alpha_B + v_B(n_S^2(p_B^2, p_S^2, x) - p_B^2)$ and $\hat{F}_S = F_S(\alpha_S + v_S(x + m_B(p_B^2, p_S^2, x) - p_S^2)$. Furthermore define $\hat{Q} = 1 - \hat{v}'_B \hat{v}'_S \hat{f}_B \hat{f}_S$. Using these definitions, the system of equations that the partial demand derivatives have to satisfy is given by

$$\begin{aligned} \frac{\partial m_B(p_B^2, p_S^2, x)}{\partial p_B^2} &= \hat{f}_B\big(\hat{v}'_B \frac{\partial n_S^2(p_B^2, p_S^2, x)}{\partial p_B^2} - 1\big) \\ \frac{\partial m_B(p_B^2, p_S^2, x)}{\partial p_S^2} &= \hat{f}_B \hat{v}'_B \frac{\partial n_S^2(p_B^2, p_S^2, x)}{\partial p_S^2} \\ \frac{\partial n_S^2(p_B^2, p_S^2, x)}{\partial p_B^2} &= \hat{f}_S \hat{v}'_S \frac{\partial m_B(p_B^2, p_S^2, x)}{\partial p_B^2} \\ \frac{\partial n_S^2(p_B^2, p_S^2, x)}{\partial p_S^2} &= \hat{f}_S\big(\hat{v}'_S \frac{\partial m_B(p_B^2, p_S^2, x)}{\partial p_S^2} - 1\big) \end{aligned} \tag{27}$$

Given that the system of equations in (27) is linear in the partial derivatives of interest, solution of the system yields the desired demand derivatives. These are given by

$$\begin{aligned} \frac{\partial m_B(p_B^2, p_S^2, x)}{\partial p_B^2} &= -\frac{\hat{f}_B}{\hat{Q}} \\ \frac{\partial m_B(p_B^2, p_S^2, x)}{\partial p_B^2} &= -\frac{\hat{v}'_B \hat{f}_B \hat{f}_S}{\hat{Q}} \\ \frac{\partial n_B^2(p_B^2, p_S^2, x)}{\partial p_B^2} &= -\frac{\hat{v}'_S \hat{f}_B \hat{f}_S}{\hat{Q}} \\ \frac{\partial n_B^2(p_B^2, p_S^2, x)}{\partial p_B^2} &= -\frac{\hat{f}_S}{\hat{Q}} \end{aligned} \tag{28}$$

For the system of rational expectation demands to be sensible, it is necessary that $\hat{Q} > 0$, or equivalently $\hat{v}'_B \hat{v}'_S \hat{f}_B \hat{f}_S < 1$.

Solving (7) for $p_S^2 - c_S$, substituting in (6) along with the demand derivatives demand derivatives presented in equation (28) and solving for p_B^2 yields

$$p_B^2 = c_B - \hat{v}_S'\hat{F}_S + \frac{\hat{F}_B - x}{\hat{f}_B}$$

which after rearranging yields the elasticity rule presented in equation (8). Similarly, solving (6) for $p_B^2 - c_B$, substituting in (7) along with the demand derivatives demand derivatives presented in equation (28) and solving for p_S^2 yields

$$p_S^2 = c_S - \hat{v}_B'(\hat{F}_B - x) + \frac{\hat{F}_S}{\hat{f}_S}$$

which results in the elasticity rule for the seller price as given in equation (9).

A.2 Derivation of the first period pricing rules

The first period rational expectations demand functions are determined by the two functions $n_B^1(p_B^1, p_S^1)$ and $n_S^1(p_B^1, p_S^1, x)$ solve

$$\begin{aligned}
n_B^1(p_B^2, p_S^2) &= F_B(\alpha_B + v_B(n_S^1(p_B^2, p_S^2, x))) - p_B^2 + \delta p_B^{2\,*}(n_B^1(p_B^1, p_S^1))) \\
n_S^1(p_B^1, p_S^1) &= F_S(\alpha_S + v_S(n_B^1(p_B^1, p_S^1))) - p_S^1).
\end{aligned}$$

The demand derivatives required to solve the optimization problem of the platform can be found with the help of totally differentiating these two expressions with respect to the buyer and seller prices. Once again it is useful to introduce some notation to simplify the exposition. In particular, let $\bar{v}_B' = v_B'(n_S^1(p_B^1, p_S^1))$, $\bar{v}_S' = v_S'(n_B^1(p_B^1, p_S^1))$, $\bar{f}_B = f_B(\alpha_B + v_B(n_S^1(p_B^1, p_S^1) - p_B^1 + deltap_B^{2\,*}(n_B^1(p_B^1, p_S^1)))$, $\bar{f}_S = f_S(\alpha_S + v_S(n_B^1(p_B^1, p_S^1) - p_S^1)$, $\bar{F}_B = F_B(\alpha_B + v_B(n_S^1(p_B^1, p_S^1) - p_B^1 + \delta p_B^{2\,*}(n_B^1(p_B^1, p_S^1)))$ and $\bar{F}_S = F_S(\alpha_S + v_S(n_B^1(p_B^1, p_S^1) - p_S^1)$. Also let $\rho(x) = \frac{\partial p_B^{2\,*}(x)}{\partial x}$ and define $\bar{\rho} = \rho(n_B^1)$. Furthermore, define $\bar{Q} = 1 - \delta \bar{r} ho \bar{f}_B - \bar{v}_B' \bar{v}_S' \bar{f}_B \bar{f}_S$. Using these definitions, the system of equations that the partial demand derivatives have to satisfy is given by

$$\begin{aligned}
\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} &= \bar{f}_B\big(\delta\bar{\rho}\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} + \bar{v}_B'\frac{\partial n_S^1(p_B^1, p_S^1)}{\partial p_B^1} - 1\big) \\
\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_S^1} &= \bar{f}_B\left(\delta\bar{\rho}\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_S^1} + \bar{v}_B'\frac{\partial n_S^1(p_B^1, p_S^1)}{\partial p_S^1}\right) \\
\frac{\partial n_S^1(p_B^1, p_S^1)}{\partial p_B^1} &= \bar{f}_S\bar{v}_S'\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} \\
\frac{\partial n_S^2(p_B^1, p_S^1)}{\partial p_S^1} &= \bar{f}_S\big(\bar{v}_S'\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_S^1} - 1\big)
\end{aligned} \tag{29}$$

Solving the linear system of equations in (29) for the demand derivatives of interest yields

$$\begin{aligned}
\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} &= -\frac{\bar{f}_B}{\bar{Q}} \\
\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} &= -\frac{\bar{v}'_B \bar{f}_B \bar{f}_S}{\bar{Q}} \\
\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} &= -\frac{\bar{v}'_S \bar{f}_B \bar{f}_S}{\bar{Q}} \\
\frac{\partial n_B^1(p_B^1, p_S^1)}{\partial p_B^1} &= -\frac{\bar{f}_S(1 - \delta\bar{\rho}\bar{f}_B)}{\bar{Q}}
\end{aligned} \tag{30}$$

In order that the demands from both sides are complementary and decreasing in both prices, it is necessary that $\bar{Q} > 0$, or equivalently $\delta \bar{r} ho \bar{f}_B + \bar{v}'_B \bar{v}'_S \bar{f}_B \bar{f}_S < 1$.

Similar to the solution for the second period pricing problem, I first solve (13) for $p_S^1 - c_S$, and substitute the result in (12). Then, substituting the demand derivatives given in equation (30) and solving for p_B^1 yields

$$p_B^1 = c_B + \delta(p_B^{2\,*}(n_B^1) - c_B) - \delta\bar{\rho}\bar{F}_B - \bar{v}'_S \bar{F}_S + \frac{\bar{F}_B}{\bar{f}_B}.$$

Rearranging this expression results in the pricing rule presented in equation (14). Similarly, I then solve (12) for $p_B^1 - c_B$ and substitute the result in (13). As a next step, I substitute the demand derivatives from equation (30) and solve for the first period seller price, p_S^2, which is given by

$$p_S^1 = c_S - \bar{v}'_B \bar{F}_B + \frac{\bar{F}_S}{\bar{f}_S}.$$

Rearranging this expression results in the elasticity rule for the seller price as given in equation (15).