Either or Both Competition: A "Two-sided" Theory of Advertising with Overlapping Viewerships*

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Abstract

This paper develops a fairly general model of platform competition in media markets allowing viewers to use multiple platforms. This leads to a new form of competition between platforms, in which they do not steal viewers from each other, but affect the viewer composition and thereby the resulting value of a viewer for the other platform. We label this form of competition "either or both." A central result is that platform ownership does not affect advertising levels, despite nontrivial strategic interaction between platforms. This result holds for general viewer demand functions and is robust to allowing for viewer fees. We show that the equilibrium advertising level is inefficiently high. We also demonstrate that entry of a platform leads to an increase in the advertising level if viewers' preferences for the platforms are negatively correlated, which contrasts with predictions of standard models with either/or competition. We validate this result in an empirical analysis using panel data for the U.S. cable television industry.

Keywords: Platform Competition, Two-Sided Markets, Market Entry, Multi-Homing, Viewer Preference Correlation.

JEL-Classification: D43, L13, L82, M37

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1 Introduction

The traditional frame in media economics posits that viewers have idiosyncratic tastes about media platforms, for instance TV stations, and stick to those they like best.[1] This is an appropriate assumption in some domains. For example, a recurrent theme in the market for news is that viewers and readers hold beliefs that they like to be confirmed (Mullainathan and Shleifer, 2005). News providers cater to these preferences by slanting stories towards these beliefs. Competition for viewers in this market segment is likely to take place in what we call an *either/or* fashion, that is, viewers watch either one or the other channel. Broadcasters fight for an *exclusive* turf of viewers and for the stream of advertising dollars that comes with them.

In other domains consumers exhibit a different kind of taste diversity. Viewers may want to watch different networks at different times expressing a preference for variety. For example, viewers may like a particular category of programming, e.g., TV shows or sports events, and choose to follow these programmes on whichever network produces or broadcasts them. Competition for viewers in this world is likely to take place in what we call an *either/both* fashion, that is, viewers watch either one or both channels (or abstain from viewing at all). Here, broadcasters try to get viewers who are also watching the other channel, i.e., channels compete for shared viewers.

The distinction between either/or or either/both competition arises partly from consumers' preferences, but partly from advertising practices. For instance for short enough periods of time, it is a good approximation that every viewer watches just one channel. So for those advertisers who only want to broadcast commercials between say 8pm and 9pm on Fridays, for all practical purposes any viewer is an exclusive viewer of some broadcaster, implying that channels engage in either/or competition. However, consider advertisers that want to place commercials of various sports events during the course of a week. Then it is likely that a lot of viewers will watch many of these broadcasts, implying that TV channels broadcasting the events engage in either/both type competition.

Given that the economics literature, both on media markets and more generally, primarily focused on pure either/or competition, in this paper we investigate the opposite end of the spectrum: pure either/both competition. In particular, we assume that consumer demand for one channel (in jargon: platform) does not affect the demand for another platform. So instead of assuming mutual exclusivity, we assume mutual independence.

A question that naturally comes to mind is if there is competition at all in such a framework. The answer is yes, because a change in the viewership of one platform changes the composition of viewerships on the other platform, in particular the fraction of the other platform's viewers who watch both channels. An important component of our model is that these "multi-homing" viewers are less valuable for competing platforms than exclusive ones, as an overlapping viewer can be reached by an advertiser through both platforms. Hence, there is a positive probability that the viewer has become aware of an advertiser's product on the other platform, and so platforms can only charge the incremental value of reaching these viewers via a second platform. By contrast, platforms are monopolists with respect to selling advertising opportunities reaching their exclusive viewers, and can extract the full

[1]See for example Anderson and Coate (2005) and several follow-up papers. We provide a detailed literature review in the next section.

surplus for these transactions from advertisers.[2] Because of this, changes in the ratio of single-homing versus multi-homing viewers of a channel can change the trade-off that a platform faces when marginally increasing advertising level, between gains in the extensive margin and losses in the intensive margin. This in general changes the optimal advertising level decision of the platform.

This paper provides a conceptual framework to analyze either/both competition. The framework allows us to analyze questions about viewer composition and its competitive effects, which is by definition not possible in previous papers. We are then able to draw conclusions on how competition changes the viewer composition of a channel and what are the implications for social welfare. In particular, we address a series of questions in this new framework: *Will market provision lead to excessive advertising levels in the either/both framework? How does the ownership structure of broadcasting impact market outcomes? How does entry affect the incentives of incumbent firms? Can viewer charges improve the market outcome?*

An additional motivation for conducting this analysis is that the traditional either/or framework exhibits problems in answering some of the above questions in a way that matches empirical regularities. For example, the wave of channel entry at the end of the 1990s in the cable TV industry came with an *increase* of advertising levels per hour of programming in some channels but with a *decrease* in others. However, in the either/or framework, competition unambiguously decreases ad levels as networks try to woo viewers back from their rivals by increasing the quality of programming. Similarly, most industry observers state that there is excessive broadcasting of commercials relative to the welfare optimal level. However, if there is fierce competition between channels, the either/or framework predicts that there is too little advertising relative to the socially optimal amount.

To answer the questions raised above and to resolve the puzzles posed by the traditional framework we present a theory of market provision of broadcasting when competition is of the either/both fashion with general viewer demands and advertising technologies. Specifically, we deploy a model with two channels, and a continuum of viewers and advertisers. We assume that consumers can choose whether to watch one of the channels, or both, or neither. Consumption choices are driven by preferences over channels summarized by a bivariate joint probability distribution. In particular, and contrary to existing models on the traditional framework, we allow viewer preferences to be correlated any way between channels. This allows us to capture many different situations with regard to channel content. In particular, observing that a viewer watches one channel is likely to be informative of whether the same viewer watches the other channel.

Our framework of either/both competition yields the following results. First, competition works differently in an either/both framework and does not affect advertising levels, i.e., the equilibrium advertising level is the same if two channels compete and when they are owned by the same company. This occurs although there is nontrivial strategic interaction between platforms. The intuition is as follows:

[2] That multi-homing viewers are worth less to advertisers is consistent with the empirically well-documented fact that the per-viewer fee of an advertisement on programmes with more viewers is larger. In the U.S., e.g., Fisher, McGowan and Evans (1980) find this regularity. In the U.K. television market, ITV, the largest commercial network, enjoys a price premium on its commercials, which, despite entry of several competitors, increased steadily in the 1990s. This is commonly referred to as the ITV premium puzzle. Our model is consistent with this regularity since reaching the same number of eyeball pairs through broadcasting a commercial to a large audience implies reaching more viewers than reaching the same number of eyeball pairs through a series of of commercials to smaller audiences, because the latter audiences might have some viewers in common. See Ozga (1960) for an early observation of this fact.

A monopolist can extract more rents from advertisers than competing channels can. Hence, the monopolist has an incentive to set a larger amount of advertising. However, the lower rent that a channel in competition receives is due to the fact that this channel can only charge a low price for the overlapping viewers. But this implies that a channel in competition loses less when increasing its advertising level because some overlapping viewers switch off. Overall, these two effects balance out, leading to the same amount of advertising in both scenarios.

It is important to note that this result holds for general viewer demand displaying either/both competition and general advertising technologies. We also demonstrate that the same result arises with either/or competition given that advertisers can coordinate their decisions. Therefore, the result obtained in previous literature depends on the hidden assumption that advertisers cannot coordinate their decisions. The result is important both for theory and policy discussion on changes in the media landscape, i.e., how to evaluate mergers of television companies. In particular, mergers in these markets can be neutral with respect to social welfare.

Second, as long as advertisers are homogeneous enough in how much surplus they can generate by reaching consumers, the amount of advertising in the market equilibrium is always inefficiently high. This is because stations do not compete directly for viewers in the either/both framework. By contrast, in the either/or framework, if competition for viewers is fierce, e.g., because channels are very alike, the equilibrium amount of advertising is very small, leading to insufficient advertising. In the either/both framework this effect is not present. The effect that remains and is therefore responsible for our result is that, when choosing their advertising levels, channels do not consider viewer utility but only how viewer behavior affects their advertising revenue. This leads to excessive advertising.

Third, due to the generality of our viewer demand function we are able to analyze how correlation of viewer preferences affects advertising levels. This is not possible in previous models of either/or competition which either use Hotelling-style preferences implying perfectly negative correlation, or consider a representative viewer. In our framework we obtain that the more positive the correlation between viewer preferences, the lower the advertising level. This is because with a positive correlation the viewer composition consists of many overlapping viewers. By lowering the advertising level, a channel can obtain new exclusive viewers, which are of larger value than its existing ones, implying that the channel has a strong incentive to reduce its advertising level. Therefore, our result demonstrates that using Hotelling preferences in the either/both competition puts an upper bound on advertising levels.

Fourth, we analyze the effect of entry on advertising. As mentioned, in the either/or framework, entry unambiguously lowers advertising levels, which does not match empirical regularities. In the either/both framework, we show that both an increase and a decrease in advertising levels are possible depending on the viewer preference correlation and the advertising technology. In particular, we show that the more negative the viewer preference correlation for the channels, the more likely it is that entry leads to increased advertising. For example, this implies that CNN increases its advertising level after entry of FOX News. By contrast, if the viewer preference correlation between two channels is positive, as is the case for sports and leisure programs, entry leads to lower advertising.

Fifth, we consider the case of viewer charges. There we first show that the neutrality result carries over. Therefore, even if viewer pricing is possible, competition does not help change advertising levels. Furthermore, advertising in equilibrium still tends to be inefficiently high, although channels now can also charge viewers. The reason is that channels cannot obtain the full viewer surplus and therefore evaluate advertising is a more important source of revenue relative to a social planner. Overall, channels charge viewers a higher aggregate price than with only advertising. As a consequence, viewer demand and advertiser revenue fall.

Finally, to validate our result on market entry, we use panel data for the U.S. cable television industry from 1989-2002. As our dataset is limited, this exercise is primarily suggestive, calling attention to the importance of a careful empirical investigation in future research.

In the above time period, a large number of entries occurred, which allows us to test by a simple empirical analysis how advertising levels of incumbent channels changed after these entry events. In general, we find that entry is associated with an increase in the advertising level. However, when controlling for content type by looking at different categories, namely news, sports, and info-tainment, a more refined picture emerges. Specifically, in the sports category, where viewer preferences are likely to be positively correlated, advertising levels fell after entry, while in the info-tainment segment, in which casual evidence would suggest that the viewer preference correlation is close to independent, advertising levels stayed roughly constant. Only in the news category, in which correlation is arguably negative, advertising levels significantly increased after entry. These results are consistent with the predictions of our theory.

The rest of the paper is organized as follows: Section 2 discusses the relationship with existing works. Section 3 introduces the model and Section 4 presents the equilibrium analysis. Section 5 explores in detail the effects of viewer preference correlation. Section 6 considers market entry. Section 7 contains the empirical evidence and Section 8 concludes.

2 Related Literature

The traditional framework in media economics makes the assumption that viewers do not switch between channels, but rather select the program they like most, see e.g., Spence and Owen (1977) or Wildman and Owen (1985). These early works usually do not allow for endogenous advertising levels or two-sided externalities between viewers and advertisers.

The seminal paper modelling the television market as a two-sided market with competition between platforms for viewers and advertisers is Anderson and Coate (2005).[3] In their model, viewers are distributed on a Hotelling line where platforms are located at the ends of the line. In line with early works, viewers watch only one channel while advertisers can buy commercials on both channels.[4] In this framework, Anderson and Coate (2005) predict that the number of entering stations can either be too high or too low compared to the socially optimal number, or that the advertising level can also be higher or lower than the efficient one.

The basic model of Anderson and Coate has been extended and modified in several ways. For example, Gabszewicz, Laussel and Sonnac (2004) allow viewers to mix their time between channels, Peitz and Valletti (2008) analyze optimal locations of stations, and Reisinger (2012) considers single-homing of advertisers. Dukes and Gal-Or (2003) explicitly consider product market competition between advertisers and allow for price negotiations between platforms and advertisers, while Choi (2006) or Crampes, Haritchabalet and Jullien (2009) consider the effects of free entry of platforms.

[3]For different applications of two-sided market models, see Rochet and Tirole (2003) or Armstrong (2006).

[4]In Section 5 of their paper Anderson and Coate (2005) extend the model by allowing a fraction of viewers to switch between channels, that is, to multi-home.

These papers do not allow viewers to watch more than one station, i.e., they assume either/or competition, and consider a spatial framework for viewer demand. By contrast, our paper allows viewers to watch more than one channel and analyze a much more general viewer demand system. In addition, we allow for a general advertising technology.[5]

The paper that is closest to ours is Anderson, Foros and Kind (2012b).[6] They also consider the case of multi-homing viewers and, in addition, allow for endogenous platform quality. They show that with multi-homing viewers, advertising levels increase after entry and generate different equilibrium configurations in which either one or both sides multi-home. However, the modelling structure is very different from ours. For example, to focus on quality choice they consider an adapted Hotelling framework developed by Anderson, Foros and Kind (2012a), suppose that the value of overlapping viewers equals zero, and consider linear pricing to advertisers by platforms. By contrast, we suppose that quality is fixed, but allow for general viewer demand functions, advertising technology, and payments.

A paper that also allows for multi-homing viewers is Athey, Calvano and Gans (2011). In their model, the effectiveness of advertising can differ for users who switch between platforms and those who stick to one platform. This is because of imperfect tracking of users. In contrast to our model, they are mainly concerned with different tracking technologies and do not allow for advertisements generating (negative) externalities on viewers, which is at the core of our model.

3 The Model

The model features a mass one of heterogeneous viewers, a mass one of homogeneous advertisers, and two platforms (or channels), indexed by $i \in \{1, 2\}$.

Viewer Demand

Viewers in our model are parametrized by their reservation values for channel 1 and channel 2. We assume that a viewer of (q_1, q_2)-type joins channel i if and only if $q_i - \gamma n_i \geq 0$ where n_i is the amount of ads on platform i, $\gamma > 0$ is a nuisance parameter and q_i is the viewer type's valuation for channel i when the latter has an advertising level of 0. In the baseline case we assume $\mathbf{q} := (q_1, q_2)$ has a joint distribution exhibiting density function $h(q_1, q_2)$. Given the amount of advertising on each platform, we can back out the demand schedules:

$$\begin{aligned}
\text{Multi-homers:} \quad & D_{12} \equiv \text{Prob}\{q_1 - \gamma n_1 \geq 0\ ;\ q_2 - \gamma n_2 \geq 0\}, \\
\text{Single-homers}_1: \quad & D_1 \equiv \text{Prob}\{q_1 - \gamma n_1 \geq 0\ ;\ q_2 - \gamma n_2 \leq 0\}, \\
\text{Single-homers}_2: \quad & D_2 \equiv \text{Prob}\{q_1 - \gamma n_1 \leq 0\ ;\ q_2 - \gamma n_2 \geq 0\}, \\
\text{Zero-homers:} \quad & D_0 \equiv 1 - D_1 - D_2 - D_{12}.
\end{aligned}$$

To ensure uniqueness of the equilibrium and interior solutions we assume that for each $i = 1, 2$ and

[5] A different framework to model competition in media markets is to use a representative viewer who watches more than one program. This approach is developed by Kind, Nilssen and Sørgard (2007) and is used by Godes, Ofek and Savary (2009) and Kind, Nilssen and Sørgard (2009). These papers analyze the efficiency of the market equilibrium with respect to the advertising level and allow for user payments. Due to the representative viewer framework, they are not concerned with overlapping viewers or viewer preference correlation.

[6] See also the survey by Anderson, Foros, Kind and Peitz (2012).

$j = 3 - i$,

$$\frac{\partial^2 D_i}{\partial(n_i)^2} \leq 0, \quad \frac{\partial^2 D_{12}}{\partial(n_i)^2} \leq 0 \quad \text{and} \quad \left|\frac{\partial^2 D_i}{\partial(n_i)^2}\right| \geq \left|\frac{\partial^2 D_i}{\partial n_i \partial n_j}\right|.$$

These assumptions are stricter than necessary. If instead each of the three inequalities were violated but only slightly so, we still have interior solutions. For a detailed discussion of why the above conditions ensure concavity of the profit function, see e.g., Vives (2000).

Platforms and Timing

Platforms (or channels) compete for viewers and for advertisers. In the basic model, platforms receive payments only from advertisers but not from viewers.[7] The timing of the game is as follows. In the first stage, each platform i announces its total advertising level n_i. Afterwards, viewers decide which platform to watch. Given these decisions, platforms sell the advertising levels, i.e., they announce the charge to an advertiser in exchange for broadcasting its commercials, that is, for devoting a fraction of the total advertising level to this advertiser. Finally, advertisers decide which platform to join.

In case of monopoly, where one firm owns both platforms, we consider the same timing. First, the monopolist announces the total advertising levels on both platforms, then viewers decide which channel to watch and afterwards the monopolist sells the advertising levels to advertisers, by announcing a transfer and a pair of advertising intensities.[8] Finally, advertisers decide which platform to join.

The solution concept we use throughout the paper is subgame perfect Nash equilibrium.

Advertising technology

Advertising in our model is informative. Let $\omega \geq 0$ denote the expected return of informing a viewer about a product. In line with the literature, see e.g., Anderson and Coate (2005) or Crampes, Haritchabalet and Jullien (2009), we assume that viewers are fully expropriated of the value of being informed.[9] So advertising is only a nuisance for them.

The mass of informed viewers is determined by the number of ads that channels broadcast, $\mathbf{n} \equiv (n_1, n_2)$. We denote the probability with which a single-homing viewer on channel i becomes informed of a firm's good by $\phi_i(n_i)$. We assume that ϕ_i is smooth, nondecreasing, concave and equal to zero at $n_i = 0$. That is, an additional ad is always valuable but less so with the number of messages already sent. Likewise, the probability that a multi-homing viewer becomes informed depends on the number of ads he is exposed to. We assume $\phi_{12}(n_1, n_2)$ is smooth with $\partial\phi_{12}/\partial n_i \geq 0$ and $\phi_{12}(n_i, n_j) = \phi_i(n_i)$ whenever $n_j = 0$. We also impose that ϕ_{12} is strictly concave in each argument, and that $\partial^2\phi_{12}/\partial n_1 \partial n_2 \leq 0$.[10]

Payoffs

[7] We allow for viewer pricing in Section 7.

[8] We will show that in the case of homogenous advertisers, given the concavity of the advertising technology, the monopolist never wants to offer a nontrivial menu of contracts, for example to indice different advertisers to single-home on its platforms.

[9] The motivation for this simplifying assumption, adopted from the above-referenced papers, is that each advertiser is the monopolist seller of a unique good. Then if the reservation price of all consumers who have a strictly positive evaluation of the good is ω, the monopolist sells the good at price ω, appropriating all surplus from consumers who became informed of the good.

[10] A natural class of functions fulfilling these conditions is $\phi_{12}(n_1, n_2) = \phi_{12}(n_1 + n_2)$, with $\phi'_{12} \leq 0$.

A platform's payoff is equal to the total amount of transfers it receives (for simplicity we assume that the cost of programming is 0). An advertiser's payoff, in case he is active on both platforms, is $u(n_1, n_2) - t_1 - t_2$, where

$$u(n_1, n_2) := \omega D_1(n_1, n_2)\phi_1(n_1) + \omega D_2(n_1, n_2)\phi_2(n_2) + \omega D_{12}(n_1, n_2)\phi_{12}(n_1, n_2) \tag{1}$$

and t_1 and t_2 are the payments to platforms 1 and 2, respectively. If he only joins platform i, the payoff is $u(n_i) - t_i = \omega\phi_i(n_i)\left(D_i(n_i, n_j) + D_{12}(n_i, n_j)\right) - t_i$. Reservation utilities are set to zero for all players.

Discussion of Modeling Assumptions

The ϕ_1, ϕ_2 and ϕ_{12} functions capture, in a very parsimonious way, several relevant aspects of viewer behavior, platform asymmetry, and advertising technology. For example, if one platform is more effective at reaching viewers for all nonzero levels, this could be captured by the following restriction: $\phi_i(n) > \phi_j(n)$ for all $n > 0$.

Individual preferences are not necessarily independent across platforms. The model thus nests those specifications which add structure to preferences by positing a positive or negative relationship between valuations of different platforms. One extreme class in the framework we consider are Hotelling-type spatial models with the two platforms at the opposite ends of a unit interval and viewers distributed along the interval. Specifically Hotelling is captured by the above setup via the following restriction: $q_1 = 1 - q_2$.[11]

An important property of the demand schedules, following directly from the way we defined them, is that if n_i changes but n_j is unchanged, the choice of whether to watch j remains unaffected. This restriction is in stark contrast with either/or formulations where individuals choose one channel *over* the other. For example, if n_i increases then channel i loses some of its single-homing and some of its multi-homing viewers. The former single-homing viewers now become zero homers while the former multi-homers become single-homers on channel j. The latter implies that $\partial D_{12}/\partial n_i = -\partial D_j/\partial n_i$.

Our assumptions on the timing are meant to capture in a simple way contracting in the US and Canadian broadcast markets. On a seasonal basis, broadcasters and advertisers meet at an "upfront" event to sell commercials for the prime-time programs of the networks. There, many series and movies are already produces and so the total number of commercials that networks are able to air is mainly fixed. Due to the Nielsen rating system, which measures the audience viewership for the different programs, channels (and advertisers) have a very precise estimate about viewerships when signing the contracts. At the upfront event, contracts that specify the number of the aired ads (so called "avails") in exchange for a payment are then signed between broadcasters and advertisers.

[11] Transportation costs and intercepts should be encoded in the distribution function. That is, if $k - \tau\lambda$ and $k - \tau(1 - \lambda)$ are the utility (gross of nuisance) of watching channel one and channel two, respectively, with λ uniformly distributed on $[0, 1]$, then one can compute the implied distribution on $q_1 = k - \tau\lambda$ (and similarly for q_2) which will depend on τ.

4 Equilibrium Advertising Levels

4.1 Market Provision

We start with the case of two competing platforms. In the last stage each advertiser either joins both, one, or no platforms, depending on the utilities it can obtain in the different scenarios.

We first argue that in each subgame after viewerships got determined, for any $(D_1(n_1, n_2), D_2(n_1, n_2), D_{12}(n_1, n_2))$, there is no continuation equilibrium in which some of the advertisers single-home. To see this, first note that $t_1 \leq u(n_1, n_2) - u(0, n_2)$ and $t_2 \leq u(n_1, n_2) - u(n_1, 0)$ imply that all advertisers join both platforms. And if $t_1 > u(n_1, n_2) - u(0, n_2)$ then platform 2's only possible best response induces all advertisers single-home on platform 2, by offering t_2 that makes advertisers indifferent between single-homing on platform 1 versus 2. But this would yield a profit of 0 for platform 1, while $t_1 = u(n_1, n_2) - u(0, n_2)$ would guarantee a strictly positive payoff. Hence, there cannot be an equilibrium as above. A symmetric argument establishes that there cannot be an equilibrium with $t_2 = u(n_1, n_2) - u(n_1, 0)$. Finally, note that for any $t_1 \leq u(n_1, n_2) - u(0, n_2)$, the best response of platform 2 can only be $t_2 = u(n_1, n_2) - u(n_1, 0)$, since all advertisers multi-home for any $t_2 < u(n_1, n_2) - u(n_1, 0)$. Similarly, for any $t_2 \leq u(n_1, n_2) - u(n_1, 0)$, the best response of platform 1 can only be $t_1 = u(n_1, n_2) - u(0, n_2)$. This concludes that the unique continuation equilibrium of the subgame starting with viewerships $(D_1(n_1, n_2), D_2(n_1, n_2), D_{12}(n_1, n_2))$ is $t_1 = u(n_1, n_2) - u(0, n_2)$ and $t_2 = u(n_1, n_2) - u(n_1, 0)$, and all advertisers multi-homing.

This is anticipated by the viewers in their decision which channel to watch. Hence, if channels in the first stage announce advertising levels of (n_1^d, n_2^d), the resulting viewers demands are $D_i(n_1^d, n_2^d)$, $i = 1, 2$, and $D_{12}(n_1^d, n_2^d)$.

Now we turn to the first stage, in which total advertising levels asre chosen. First observe that, given a candidate equilibrium allocation (n_1^d, n_2^d), each platform extracts the incremental value it brings over its competitor's offer. That is

$$t_1^d = u(n_1^d, n_2^d) - u(0, n_2^d) \quad \text{and} \quad t_2^d = u(n_1^d, n_2^d) - u(n_1^d, 0). \tag{2}$$

Since advertisers are multi-homing in equilibrium, higher transfers would make it a dominant strategy for advertisers to reject the offer. Lower transfers would simply leave money on the table.

Note that competing platforms cannot extract the full rent of the advertisers, i.e., advertisers receive positive profits $u(n_1^d, n_2^d) - t_1^d - t_2^d \geq 0$.[12] Platform i's incremental value is given by the value of delivering ads to single-homing viewers (who exclusively watch platform i) plus the incremental value for the multi-homing viewers: $\omega(\phi_{12}(n_1, n_2) - \phi_j(n_j))$. The profit of platform i is therefore

$$\Pi_i^d = \omega \left[D_i(n_i, n_j)\phi_i(n_i) + D_{12}(n_i, n_j)(\phi_{12}(n_i, n_j) - \phi_j(n_j)) \right]. \tag{3}$$

The candidate equilibrium allocation is characterized by the following system of first-order conditions

[12] To see this note that our assumptions on ϕ_{12} ensure $\phi_{12}(n_1, n_2) \leq \phi_1(n_1) + \phi_2(n_2)$, which implies $t_1^d + t_2^d \leq u(n_1^d, n_2^d)$.

(arguments omitted for ease of exposition):[13]

$$\frac{\partial \Pi^d}{\partial n_i} = \omega\left(\frac{\partial D_i}{\partial n_i}\phi_i + D_i\phi_i' + \frac{\partial D_{12}}{\partial n_i}(\phi_{12}-\phi_j) + D_{12}\frac{\partial \phi_{12}}{\partial n_i}\right) = 0. \tag{4}$$

The above argument establishes that there can be at most one equilibrium in which all advertisers multi-home, characterized by the above first-order conditions. In particular, in any other candidate profile in which advertisers multi-home, at least one of the platforms can profitably deviate to another total advertising level that also leads to all advertisers multi-homing.

Let us note here that if we change the model such that the platforms are engaged in a Cournot style competition choosing only advertising intensities n_1 and n_2, and assume that the resulting transfers are the market-clearing prices given by (2), then the unique equilibrium advertising levels are also characterized by (4). Despite the fact that the Cournot duopoly model is more conventional then the model in which platforms compete with first specifying an advertising intensity and then a transfer, we stick with the latter formulation as it is more symmetric to the contracting environment that is natural to assume in case of a monopolist provider.

We also note that if we consider a model in which platforms first offer contracts of the form (t_i, n_i) to advertisers, and afterwards viewers and advertisers simultaneously decide which platform to join, under some technical conditions, there exists an outcome-equivalent subgame perfect Nash equilibrium. In Proposition 0 in the Appendix we provide these conditions and a sketch of the proof.[14] The game with posted contract offers is however much more difficult to analyze since a deviation by one platform leads to a change in the viewership and in the advertiser acceptance decision at the same time, and these decisions are influenced by each other. For that reason and due to the outcome-equivalence, we stick to the easier formulation.

We now switch to the problem of a monopolist that owns both platforms and announces advertising intensities (n_1, n_2) and a fixed transfer t after viewers' decisions are made. Since advertisers are homogeneous, their surplus is fully extracted through the fixed transfer. Therefore, the profit of the monopolist is larger than the sum of the profits in duopoly. By a similar argument as in the duopoly case, the monopolist can never do better by inducing only partial participation of advertisers. In particular, if some advertisers single-home, then strict concavity of ϕ_1, ϕ_2 and ϕ_{12} imply that the monopolist can strictly do better by inducing full participation and charging a unique fee to each advertiser that makes him indifferent between accepting or rejecting. For that reason, the monopolist can also not do better by charging different payments.

The profit function of a monopolist is therefore given by

$$\Pi^m(\mathbf{n}) = \omega D_1\phi_1 + \omega D_2\phi_2 + \omega D_{12}\phi_{12}. \tag{5}$$

[13] Our assumptions on the demand and advertising technology functions guarantee that the second-order conditions are satisfied.

[14] The conditions are rather technical, e.g., that viewer demands and advertising technologies for the two platforms are not too asymmetric, and that aggregate viewership of a channel is sensitive enough, relative to the sensitivity of the probability that a viewer gets informed, to increasing advertising intensity.

Taking the first-order condition of (5) and using $\partial D_{12}/\partial n_i = -\partial D_j/\partial n_i$ in (4) we obtain

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\phi_i' + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_j) + D_{12}\frac{\partial \phi_{12}}{\partial n_i} = 0. \tag{6}$$

(6) is equivalent to (4) which implies $\mathbf{n}^m = \mathbf{n}^d$. We therefore obtain the following simple yet powerful result.

Proposition 1 (Neutrality). *Equilibrium advertising levels do not depend on the competitive structure, that is,* $\mathbf{n}^m = \mathbf{n}^d$.

The following reformulation of Π_i^d aids intuition.

$$\Pi_i^d = \Pi^m - \omega\phi_j(D_j + D_{12}). \tag{7}$$

The above profit is reminiscent of the payoff induced by Clarke-type mechanisms. Each agent's payoff equals the entire surplus minus a constant term equal to what the other agents would jointly get in his absence. Clarke mechanisms implement socially efficient choices, here represented by the joint monopoly solution. An alternate way to build intuition is to inspect the first-order conditions for an optimum. When marginally increasing n_i^m, a monopolistic platform trades off that it loses some multi-homing viewers but increases the single-homing viewers on platform j. With the first kind of viewers the monopolist loses ϕ_{12} while with second he gains ϕ_2. Now in duopoly, when a platform increases n_i^d it loses multi-homing viewers and the gain that it receives from these viewers is $\phi_{12} - \phi_2$. But this implies the trade-offs in both market structures are the same.

It is important to note that the result does not obtain due to the absence of contracting externalities. Ceteris paribus, a "more aggressive" choice by a platform, i.e., a higher advertising quantity, lowers the payoff of the other platform and shifts its marginal revenue function. This occurs because overlapping viewers can be reached through either platform, i.e., platforms are imperfect substitutes from the advertisers' perspective. As a consequence of this, the best reply functions are not flat in the rival's quantity choice. Yet, despite these strategic externalities, competition does not affect the equilibrium levels of advertising.

To understand in more general terms the driving mechanism, consider the broader context of multi-principal / single agent contracting environments with perfect information. Principals—the platforms in our case—propose simultaneously and non-cooperatively an allocation in exchange for a fixed transfer to the agent—the advertiser. The equilibrium transfers are equal to the incremental surplus. It follows that if the principals do not have conflicting preferences over the allocation, then a neutrality result obtains regardless of the preferences of the agent. In our context, this condition is satisfied as the platforms' payoffs do not depend directly on (n_i, n_j) but only indirectly through the advertisers' payoffs. In other words, the platforms do not care directly about the impact of advertising levels on viewerships, but only indirectly since changes in viewerships induced by changes in the advertising level affect the advertisers' willingness to pay. Platform i's equilibrium transfer is $u(n_i, n_j) - u(0, n_j)$. Since the latter term reflects what an advertiser would get if he were to reject i's offer, it cannot depend on n_i. Since both players independently maximize the entire payoff $u(n_i, n_j)$ minus a constant, the neutrality result follows.

The above argument is very general and, as we shall see, extends to the either/or framework although with one important caveat. There, platforms do also not have conflicting preferences over the allocation for the same reason. The only difference to the either/both framework are the advertisers' preferences over the allocation. This is since viewers either watch platform i or j, implying that $D_{12} = 0$ and $D_i' = -D_j'$. To establish neutrality, consider a slight variation of the either/or framework in which there is only one advertiser (as opposed to a mass 1 of them). This can be seen as a shortcut for a setting in which all advertisers coordinate their choices. The transfer that platform i can charge to make the advertiser accept is still the incremental value of the advertiser. Therefore, the profit of platform i is $\Pi_i^d = u(n_1^d, n_2^d) - u(0, n_j^d)$, which in the either/or framework can be written as

$$\Pi_i^d = D_1(n_1^d, n_2^d)\phi_1(n_1) + D_2(n_1^d, n_2^d)\phi_2(n_2) - D_j(0, n_j^d)\phi_j(n_j). \tag{8}$$

The first two terms are equivalent to the profit of a monopoly firm controlling both stations while the last term is independent of n_i^d. Therefore, the first-order conditions for monopoly and duopoly coincide and neutrality obtains. By contrast, consider the case in which (the mass 1 of) advertisers do not coordinate their choices. Then the last term in (8) is $D_j(n_i^d, n_j^d)\phi_j(n_j)$, that is, what an advertiser would get if he were to reject the offer of platform i conditional on all other advertisers accepting. But this implies that the profit of platform i is just $\Pi_i^d = D_i(n_i^d, n_j^d)\phi_i(n_i)$, which is maximized at a level n_i^d that is below n_i^m since $\partial D_j/\partial n_i^d > 0$. Hence, competition results in lower equilibrium advertising levels than those that would be implemented by a joint monopoly owner.

Observe that in the either/both framework neutrality obtains regardless of whether advertisers are able to coordinate. It is thus the *combination* of either/or competition for viewers and uncoordinated choices by advertisers that breaks down the result, creating scope for competition.[15]

We note that the neutrality result also applies if the monopolist can only set payments for each platform plus a participation fee, that is, before advertisers decide which platform to join, it can set only payments t_i, $i = 1, 2$, plus a fixed fee for each advertiser who accepts at least one offer. This strips the monopolist from the ability to bundle, as was the case in the (general) payment case, where the monopolist can announce a payment t, that every advertiser who is active on one or both platforms must pay. Such a payment is sometimes impossible e.g., because payments charged by a platform are not allowed to be conditioned on the ones offered by the other platform. However, in our case non-bundling payments are sufficient because the monopolist can extract the incremental surpluses via the two payments depending on the advertising intensities, which act as marginal prices, while the rest surplus can be extracted by the participation fee.

We conclude this subsection by discussing how the neutrality result extends to advertisers with heterogeneous product values, as in Anderson and Coate (2005). First, it is evident that the result also holds if platforms can offer a menu of advertising intensities and payments and can perfectly discriminate between advertisers. In that case, the result is similar to the one for the case of homogeneous advertisers.

Matters are more nuanced if advertisers are heterogeneous and platforms cannot perfectly discriminate, in particular when ω is private information to each advertiser. The main additional difficulty of the analysis is that one needs to consider a menu of contracts offered by platforms, instead of a single contract. In the Appendix we show that the neutrality result prevails if one restricts attention to the

[15] Ambrus and Argenziano (2009) addresses the question of consumer coordination in a different context of platform competition with positive externalities.

simple class of contracts discussed above, that is, when each platform owner can charge an participation fee plus marginal prices for the platform(s) owned. We note that the neutrality result does not necessarily hold when a joint monopolist has the possibility to offer bundling contracts specifying a single transfer in exchange for an advertising intensity on each platform. The reason is that in this case an advertiser cannot report different types to the two platforms. This may change the advertiser's outside option and affects the optimal allocation induced by the monopolist. From this argument it follows that it is not competition per se that changes the allocation but rather the limited possibility of advertisers to report their types, which is responsible for the different outcomes in monopoly and duopoly. If this possibility is the same in monopoly and duopoly—as is the case when each platform owner can offer a contract depending on the advertising intensities on this platform—the neutrality result obtains again, i.e., competition has no bite in reducing advertising levels.

4.2 Socially optimal provision

A common opinion of most industry observers is that advertising levels are inefficiently high. To validate this concern we proceed to characterize the socially optimal allocation. As mentioned, $q_i - \gamma n_i$ is the utility of a single-homing viewer of platform i and by $q_1 - \gamma n_1 + q_2 - \gamma n_2$ the utility of a multi-homing viewer. Social welfare equals:

$$W = \int_{\gamma n_1}^{\infty} \int_{0}^{\gamma n_2} q_1 - \gamma n_1 h(q_1, q_2) dq_2 dq_1 + \int_{0}^{\gamma n_1} \int_{\gamma n_2}^{\infty} q_2 - \gamma n_2 h(q_1, q_2) dq_2 dq_1$$

$$+ \int_{\gamma n_1}^{\infty} \int_{\gamma n_2}^{\infty} q_1 - \gamma n_1 + q_2 - \gamma n_2 h(q_1, q_2) dq_2 dq_1 + \omega D_1 \phi_1 + \omega D_2 \phi_2 + \omega D_{12} \phi_{12}.$$

Comparing the equilibrium advertising level denoted by n_i^d with the socially efficient advertising level we obtain the following:

Proposition 2. *The equilibrium advertising levels are inefficiently high.*

Proof: See the Appendix.

To see why this is the case, it is useful to go back considering the incentives of a joint monopoly platform. Note that under our assumptions such a platform fully internalizes the advertisers' welfare. On the contrary, it does not internalize the viewers' welfare. More precisely, it only cares about viewers' utilities inasmuch as they contribute to the advertising revenue. The nuisance costs to viewers of an increase in advertising levels are not taken into account. This leads to over-provision. By proposition 1 competing platforms implement the same allocation. Equilibrium advertising levels are therefore inefficiently high.

Proposition 2 should be interpreted with caution. The overprovision result hinges on the assumption that advertisers are homogenous. Otherwise, much as in previous works, a total surplus maximizing platform would have to trade off the social benefits of having an extra advertiser on board with the social nuisance costs. A discussion of what lesson should be drawn from proposition 2 is thus warranted. The result shows that platform competition does not alleviate the upward distortion in advertising levels. Such result is important insofar as it cannot be obtained when competition for viewers is not of the either/both type. For instance, in Anderson and Coate (2005) competition for (exclusive) viewers can lead to under-provision even with homogeneous advertisers. The assumption of homogeneous advertisers simply allows to focus on the viewers' side of the market by shutting off screening considerations. As we indicated, the neutrality result—in a qualified form—extends to the case of heterogeneous advertisers. Hence, competition fails to reduce ad levels in this case as well. However, the extent of this failure depends on whether there is overprovision to begin with. Competition authorities sometimes use consumer surplus as the basis for regulation. Clearly, welfare measures that underplay the loss of surplus on the advertisers side of the market would add to the case of inefficient overprovision. Nevertheless, the mere existence of regulatory "caps" or ceilings on the number of commercials per hour in many countries is suggestive of concerns of over provision and hence make the above failure particularly relevant.

5 Viewer Preference Correlation

Due to the generality of the demand specification, our framework allows us to draw conclusions on how the correlation between viewers' preferences for the two stations affects the equilibrium advertising levels. Such an analysis cannot be conducted in previous models of platform competition. These models draw either on Hotelling competition or assume a representative viewer. In the first case the correlation between viewer preferences is perfectly negative, in particular the viewer who likes station i most likes station j least, while in the second case viewers are all the same per assumption.

We pursue this analysis by ways of a simple example that puts more structure on viewers' tastes. We do so to present the results in a simple way. However, as will become clear from the explanations, the main insights extend to a more general tastes. Let us suppose that viewer types are distributed on a unit square, that is q_1 and q_2 are distributed between 0 and 1. A fraction $1-\lambda$ of viewers is uniformly distributed on this square. The remaining fraction λ is uniformly located on the 45-degree line from $(0,0)$ to $(1,1)$. This is illustrated in the left-hand side of Figure 1. By varying λ we can express different degrees of correlation ranging from independent preferences if $\lambda = 0$ to perfect positive correlation if $\lambda = 1$. For simplicity, assume $\gamma = 1$, implying that a viewer watches station i if $q_i - n_i \geq 0$. Finally assume $\phi_i(n_i) = 1 - e^{-n_i}$ and $\phi_{12}(n_1, n_2) = 1 - e^{-(n_1+n_2)}$, which implies that $\phi(\cdot)$ is strictly concave.

As can be seen from the right-hand side of Figure 1, the demand functions for the types that are uniformly distributed on the unit square are given by $D_1 = (1-n_1)n_2$, $D_2 = (1-n_2)n_1$ and $D_{12} = (1-n_1)(1-n_2)$. For the types located on the 45-degree line the demands, are given by $D_1 = \max\{n_2 - n_1, 0\}$, $D_2 = \max\{n_1 - n_2, 0\}$ and $D_{12} = 1 - \max\{n_1, n_2\}$.

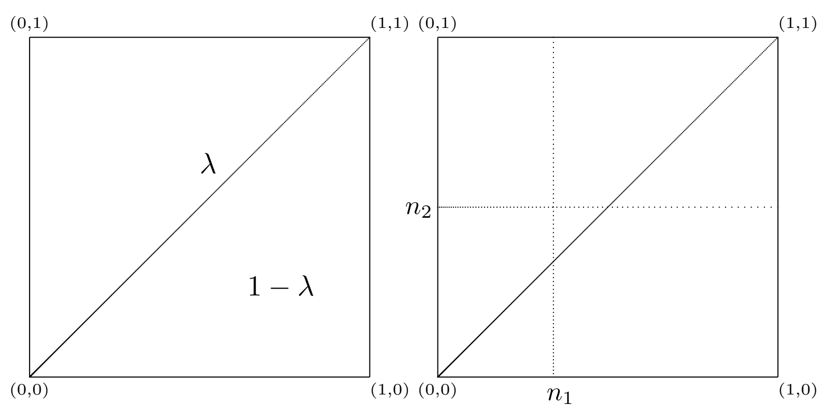

Figure 1: Positive Correlation

Likewise, we can express negative correlation by distributing a mass λ on the line from $(0,1)$ to $(1,0)$ (rather than on the line from $(0,0)$ to $(1,1)$). The larger is λ, the more negative is the correlation of preferences. Analyzing the effect of viewer preference correlation on the advertising levels we obtain the following result:

Proposition 3. *The equilibrium advertising levels are (weakly) decreasing in the correlation of viewers' preferences.*

Proof: See the Appendix

To build intuition, consider the extreme cases of perfect correlation and independence. If correlation between q_1 and q_2 is perfectly positive, in our model all viewers are distributed on the 45-degree line. But this implies that at a symmetric equilibrium, $D_1 = D_2 = 0$, i.e., all viewers watch either both platforms or none. If now one platform lowers its advertising level, the viewer composition of this channel changes, such that its new viewers are pure single-homers, that is, they all exclusively watch this platform. Since these exclusive viewers are very valuable, the incentive for a platform to lower its advertising level is relatively large.

By contrast, if q_1 and q_2 are independent, all viewers are uniformly distributed on the unit square. Thus, by lowering its advertising level, a platform receives both single- and multi-homing viewers. Since the viewer composition of new viewers is less valuable than in case of perfect positive correlation, the incentives to lower the advertising level is reduced, leading to a larger advertising level in equilibrium. If correlation is positive but not perfect, both effects are at work. However, the more positive the correlation is, the higher is the mass of exclusive viewers that a platform can get when lowering the advertising level. Thus, equilibrium advertising levels are decreasing with the correlation if it is positive.

We now turn to the other extreme, the case of perfectly negative correlation. In that case if advertising levels are not too large, i.e., $n_1 = n_2 \leq 0.5$, the majority of viewers exclusively watch either platform 1 or platform 2. However, by reducing its advertising level, the new viewers that a platform gets are already watching the other platform and are therefore not very valuable. Thus, the incentive to reduce the advertising level is small. As a consequence, the equilibrium amount of advertising is relatively large and, as the correlation becomes more negative, advertising levels increase. As we show in the proof, if correlation is highly negative, that is, many viewers are distributed on the line from $(0, 1)$ to $(1, 0)$, then $n_1^\star = n_2^\star = 0.5$ and the equilibrium advertising levels do not change if the correlation varies. However, for moderately negative correlation, advertising levels strictly rise if the correlation becomes more negative.

In sum, our framework allows for an analysis of viewer preference correlation and shows that advertising levels are lowest if this correlation is highly positive. In this case stations compete for viewers that have similar preferences for both programmes which induces the stations to lower their advertising levels. The analysis also shows that in a Hotelling world in which correlation is perfectly negative, advertising levels are particularly high.

6 Entry

We now turn to the case of market entry. Such an analysis allows us to compare advertising levels in case of a single station with the case of competition.[16] It is also at the heart of our empirical analysis in

[16]To avoid confusion, note that this exercise is different than the previous comparison between duopoly competition and a monopolist operating two platforms.

which we can observe entry of different stations in the U.S. television industry in our panel data set.

Suppose there is only one platform. The viewer demand of this platform i is given by $d_1 \equiv \text{Prob}\{q_i - \gamma n_i \geq 0\}$. Differentiating the profit function $\Pi_i = d_i \phi_i(n_i)$ with respect to n_i yields a first-order condition of

$$\frac{\partial d_i}{\partial n_i}\phi_i + d_i \frac{\partial \phi_i}{\partial n_i} = 0.$$

To compare the advertising level of a single platform with the equilibrium level of the platform in duopoly competition, we can divide d_i into two viewer sets. The first is the set that continues to watch only station i even if the rival station j is present, while the second set watches both stations after entry of station j.[17] In the notation for the demand schedules introduced in Section 3, the first set is D_i while the second set is D_{12}. We then have $d_i = D_i + D_{12}$. The first-order condition can then be rewritten as

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i}\phi_i + D_{12}\frac{\partial \phi_i}{\partial n_i} = 0, \tag{9}$$

which characterizes the platform's choice.[18] Comparing (9) with the equilibrium advertising level in duopoly, implicitly given by (4), we obtain:

Proposition 4. *Advertising levels in case of duopoly are larger than in case of monopoly if*

$$-\frac{\partial D_{12}}{\partial n_i}(\phi_1 + \phi_2 - \phi_{12}) > D_{12}\left(\frac{\partial \phi_i}{\partial n_i} - \frac{\partial \phi_{12}}{\partial n_i}\right), \tag{10}$$

where all functions are evaluated at the equilibrium advertising levels in duopoly.

Proof: See the Appendix

Since $\phi_1 + \phi_2 - \phi_{12} > 0$, condition (10) is fulfilled if $\partial \phi_i / \partial n_i - \partial \phi_{12}/\partial n_i$ is small. The intuition behind the result is the following: Since multi-homing viewers are less valuable for platforms, the foregone benefit from losing a multi-homing viewer is relatively small. Therefore, the platform has a larger incentive to increase its advertising level. By contrast, in the absence of a rival platform, the firm can also extract the full benefit from these potentially multi-homing viewers, implying that a platform without a rival has a smaller incentive to reduce its advertising level. This intuition can be seen in the left-hand side of (10), which is a multiple of $\phi_1 + \phi_2 - \phi_{12}$. This term measures the reduced value of overlapping viewers. So the lower ϕ_{12} (relative to ϕ_1 and ϕ_2), the lower the left-hand side of (10), and the higher the likelihood that advertising levels rise after entry.

To provide more precise conclusions and compare our results with previous studies, let us put more structure on the advertising technology. In particular, suppose the functional form is either a polynomial,

$$(i) \quad \phi_i(n_i) = n_i^{1/a} \quad \text{and} \quad \phi_{12}(n_1, n_2) = (n_1 + n_2)^{1/a} \quad ,$$

[17] This can be done because the aggregate viewer demand of platform i does not depend on n_j implying that $d_i(n_i) = D_i(n_i, n_j) + D_{12}(n_i, n_j)$.

[18] Under our assumptions, the profit function of monopolist is strictly concave because

$$\frac{\partial^2 \Pi^m}{\partial (n_i)^2} = \phi_i\left(\frac{\partial^2 D_i}{\partial (n_i)^2} + \frac{\partial^2 D_{12}}{\partial (n_i)^2}\right) + 2\frac{\partial \phi_i}{\partial n_i}\left(\frac{\partial D_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i}\right) + \frac{\partial^2 \phi_i}{(\partial n_i)^2}(D_i + D_{12}) < 0.$$

or negative exponential,

$$(ii) \quad \phi_i(n_i) = 1 - e^{-bn_i} \quad \text{and} \quad \phi_{12}(n_1, n_2) = 1 - e^{-b(n_1+n_2)}.$$

Since ϕ is increasing in the advertising level but is concave, the parameter restriction for a and b is that $a \in (1, \infty)$ and $b \in (0, \infty)$. For $a \to \infty$ and $b \to \infty$, the advertising technology resembles the one of Anderson, Foros and Kind (2010) in which overlapping viewers are of zero value. This is the case because then $\phi_i(n_i) = 1$, $i = 1, 2$, while $\phi_{12}(n_1, n_2) = 1$ as well.

Now consider the polynomial advertising technology given by (i), and use it in (10). We obtain $\phi_1 + \phi_2 - \phi_{12} = n_i^{1/a} + n_j^{1/a} - (n_i + n_j)^{1/a}$, while $\phi_i/\partial n_i - \partial\phi_{12}/\partial n_i = 1/a(n_i)^{(1-a)/a} - 1/a(n_i + n_j)^{(1-a)/a}$. It is easy to see that for $a \to \infty$, the first expression becomes 1 while the second expression becomes 0. But this implies that for a large enough (10) is always satisfied and the advertising levels rise with entry. By contrast, for a close to 1, both expression are very small, and whether advertising increases with entry depends on the difference between D_{12} and $-\partial D_{12}/\partial n_i$. We obtain the same result for the exponential advertising technology form (ii).[19] The next proposition summarizes this analysis:

Proposition 5. *Suppose that the advertising technology is given by either* (i) *or* (ii)*. Then for* a *or* b *large enough, the advertising level increases with entry while for* a *close to 1 or* b *close to 0, the advertising level increases with entry if and only if* $-\partial D_{12}(n_1^d, n_2^d)/\partial n_i > D_{12}(n_1^d, n_2^d)$*.*

The proposition shows that if the advertising technology is highly concave, which implies that overlapping viewers are of low value, entry leads to a rise in the advertising level. The intuition is that the negative effect of losing viewers through additional advertising becomes small, so stations increase their advertising levels. By contrast if the advertising technology is only mildly concave, the result is less clear-cut and depends on the specifics of the demand function. Therefore, our analysis generalizes Anderson, Foros and Kind (2010), who consider the case of an advertising technology with zero value for overlapping viewers.

So far we focused on differences in the advertising technology when analyzing the effects of entry. However, our framework also allows to consider how viewers' preferences influence the entry effects. This is particularly important for the empirical analysis since changes in the advertising technology are much less clear-cut than differences in the correlation of viewers' preferences between stations. Hence, the obtained result can be tested in the empirical analysis.

As in the last section, we proceed with a simple example. Nevertheless, the intuitions extent to more general cases. Consider the same demand structure and advertising technology as introduced in the last section. That is, viewers are uniformly distributed on the unit square and correlation can be expressed by the mass of viewers on the 45-degree line or on the line from $(0, 1)$ to $(1, 0)$. The advertising technology is given by $\phi_i(n_i) = 1 - e^{-n_i}$ and $\phi_{12}(n_1, n_2) = 1 - e^{n_1+n_2}$. When comparing the advertising levels in the case of a single station with the one under duopoly, we obtain the following:

[19] Here, $\phi_1 + \phi_2 - \phi_{12} = 1 - e^{-an_i} - e^{-an_j} + e^{-a(n_i+n_j)}$ and $\phi_i/\partial n_i - \partial\phi_{12}/\partial n_i = a(e^{-an_i} - e^{-a(n_i+n_j)})$. For $a \to \infty$ the first expression equals 1 while, by using the rule of L'Hospital, the second expression equals zero. For a close to zero, both expressions are also close to zero.

Proposition 6. *The equilibrium advertising level with entry is lower than without entry if the correlation of viewers' preferences is positive but it is higher with entry than without if the correlation is negative. For independent distribution of viewers' preferences the advertising volumes in both cases coincide.*

Proof: See the Appendix

The intuition behind the result is as follows: if correlation is positive, the composition of viewers consists of many multi-homing viewers. When a new channel enters, most viewers are of low value, as they are not exclusive. By reducing its advertising level, a platform can obtains some exclusive viewers, who otherwise do not join any platform. In the meantime, it does not lose much on its existing viewers, as these viewers are non-exclusive. As a consequence, equilibrium advertising levels are lower with competition. So with positive correlation we obtain the same result as derived in previous literature with single-homing viewers, i.e., competition leads to a fall in the advertising level. However, the intuition for these results is different in the two cases. In the case of single-homing viewers, viewers switch to the competitor if advertising levels on a platform rise, thereby confining these advertising levels. In our case, if correlation becomes more positive, exclusive viewers become scarce. Thus, platforms reduce their advertising levels to get some of these viewers.

By contrast, if correlation is negative, entry leads to an increase in advertising levels. The intuition is that a platform attracts many multi-homing viewers under duopoly when lowering the advertising level. Since these viewers are of lower value than the exclusive viewers that a monopolist can attract, the incentives to lower advertising levels are diminished, leading to more advertising after entry.

An important implication of this analysis is that the entry of FOX News should have led to an increase in the advertising level of other stations like e.g., CNN, for which it is likely that viewer preferences are negatively correlated. However, for platforms with positive correlation, e.g., sports programs, our model predicts the opposite. As we will demonstrate later, this prediction is validated by the empirical analysis.

7 Viewer Pricing

In this section we consider the possibility of platforms to charge viewers who watch their program. In particular, we are interested if the neutrality result carries over the the case of viewer pricing and if the result on excessive provision of advertising continues to hold. The analysis is also relevant for policy implications, because additional pricing instrument can possibly revert the results obtained earlier. As we will show, this is not the case.

Let p_i denote the viewer price at platform i. Platforms set the prices in the first stage before viewers decide which channel to watch. Otherwise, the model is the same as described in Section 3. In line with the literature, we restrict the viewer charge to be non-negative, since viewer subsidies seem to be difficult to implement.[20] The utility of a viewer of type q_i from watching platform i is then given by $q_i - \gamma n_i - p_i$.

[20]For example, as Anderson and Coate (2005) point out, even if monitoring viewer behavior is possible, it is impossible to know whether the viewer is paying attention.

The demand schedules of Section 2 are then given by

$$
\begin{aligned}
\text{Multi-homers}: &\quad D_{12} \equiv \text{Prob}\{q_1 - \gamma n_1 - p_1 \geq 0 \ ; \ q_2 - \gamma n_2 - p_2 \geq 0\}, \\
\text{Single-homers}_1: &\quad D_1 \equiv \text{Prob}\{q_1 - \gamma n_1 - p_1 \geq 0 \ ; \ q_2 - \gamma n_2 - p_2 \leq 0\}, \\
\text{Single-homers}_2: &\quad D_2 \equiv \text{Prob}\{q_1 - \gamma n_1 - p_1 \leq 0 \ ; \ q_2 - \gamma n_2 - p_2 \geq 0\}, \\
\text{Zero-Homers}: &\quad D_0 \equiv 1 - D_1 - D_2 - D_{12}.
\end{aligned}
$$

We first turn to the comparison of advertising levels in duopoly and in monopoly. The profit function of platform i in duopoly is

$$\Pi_i^d = \omega\left(D_i\phi_i + D_{12}(\phi_{12} - \phi_j)\right) + p_i(D_i + D_{12}).$$

Differentiating with respect to n_i and p_i, we obtain first-order conditions of

$$\frac{\partial \Pi_i^d}{\partial n_i} = \omega\left[\frac{\partial D_i}{\partial n_i}\phi_i + D_i\phi_i' + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_j) + D_{12}\frac{\partial \phi_{12}}{\partial n_i}\right] + p_i\left(\frac{\partial D_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i}\right) = 0 \tag{11}$$

and

$$\frac{\partial \Pi_i^d}{\partial p_i} = \omega\left[\frac{\partial D_i}{\partial p_i}\phi_i + \frac{\partial D_{12}}{\partial p_i}(\phi_{12} - \phi_j)\right] + D_i + D_{12} + p_i\left(\frac{\partial D_i}{\partial p_i} + \frac{\partial D_{12}}{\partial p_i}\right) = 0. \tag{12}$$

Since by our assumptions on viewer demand and advertising technology, the second-order conditions are satisfied, equations (11) and (12) determine the equilibrium advertising level and viewer charge in duopoly.

The profit function of a monopolist is

$$\Pi^m = \omega\left(D_1\phi_1 + D_2\phi_2 + D_{12}\phi_{12}\right) + p_1 D_1 + p_2 D_2 + (p_1 + p_2)D_{12}.$$

Differentiating this function with respect to n_i and p_i, and using that $\partial D_j/\partial n_i = -\partial D_{12}/\partial n_i$ and $\partial D_j/\partial p_i = -\partial D_{12}/\partial p_i$, it is easy to check that we obtain the same first-order conditions as in (11) and (12). Therefore, advertising levels in monopoly and duopoly are again the same. Thus, we obtain the following proposition:

Proposition 7. *The neutrality result that $n_i^d = n_i^m$ carries over to the case of viewer pricing.*

The result shows that viewer pricing does not change the similarity in the trade-off for a monopolist and a duopolist. So, the neutrality between the two scenarios does not depend on the number of pricing instruments but is inherent in the either/both structure of competition.

However, the advertising level is affected by the possibility of viewer charges. Since viewer charges provide platforms with an additional revenue source, channels substitute some advertising revenues for viewer revenues, thereby reducing the advertising level. The next result shows, however, that this can never result in an under-provision of advertising.

Proposition 8. *Suppose that the equilibrium advertising level is positive. Then, even with viewer pricing, the equilibrium advertising levels are inefficiently high.*

Proof: See the Appendix

The intuition for this result is that by viewer charges platforms can only extract a part of the viewer surplus but not the full surplus. This implies that platforms have an incentive to increase the advertising levels beyond the socially efficient one to obtain higher profits. The exception is the case in which it is optimal to use only viewer prices as a revenue source. In that case the socially optimal level coincides with the equilibrium level because viewer disutility from advertising is so large, that platforms prefer to set advertising to zero, thereby having a larger viewer base and being able to charge higher prices. However, in all other cases the equilibrium advertising level exceeds to welfare optimal one.

This result contrasts with the one of Anderson and Coate (2005), who find that, if two platforms are active, advertising levels with viewer pricing are below the socially optimal one. This is due to the fact that they consider an inelastic viewer demand, i.e., the market is covered, and allow for heterogeneous advertisers. This combined with direct competition for viewers, as is the case in the either/or framework, results in under-provision. In our case of either/both competition, only indirect competition takes place. As we show in the proof of Proposition 8, if in this case only the first part of Anderson and Coate (2005) is present, i.e., viewer demand is inelastic but advertisers are homogeneous, then viewer pricing leads to the socially efficient advertising level. So in line with our remarks in Subsection 4.2, if advertisers are heterogeneous, it is no longer necessarily the case, that there is over-provision of advertising. However, as Proposition 8 demonstrates, the tendency that the market provides too much advertising is not reverted, even if viewer pricing is allowed for.

When comparing welfare in case of viewer pricing with the case of no viewer pricing, our results are similar to those of Anderson and Coate (2005). As they, we find that viewer pricing can lead to a rise or fall of social welfare. Welfare can rise because advertising levels get lower and if viewers strongly dislike advertising, this leads to a welfare improvement. However, the full price for viewers is larger in case of viewer charges, implying that more viewers switch off, leading to a reduction in welfare. As shown in the appendix, the distributional consequences of viewer prices are that viewer utility falls and also advertising revenues fall. Hence, the possibility to charge viewers redistributes revenue from viewers and advertisers to stations. Again, these results are in line with the ones of previous models.

From a policy perspective, our analysis casts doubt on arguments that viewer pricing corrects inefficiencies in the TV market. If viewers can watch multiple channels, competition between channels does not lead to a change in advertising level—the neutrality result—and so channels use the pricing instrument mainly to extract more viewer rent, thereby reducing demand. In fact, this can be observed in several countries in which pay-tv channels have a small number of subscribers although they provide high-quality content.

8 Empirical Evidence

Our data is provided by Kagan-SNL a highly regarded proprietary source for information on broadcasting markets. The data consists of a time series of 68 basic cable channel cross-sections, covering the period from 1989 to 2002. That is, channels received by a cable subscriber on the basic lineup. It covers almost all of the cable industry advertising revenues (75% of all industry revenue is generated by the biggest 20 networks in our dataset). The cross section contains data on subscribers, advertising revenues, programming expenses, cash flow and prime-time rating. Most importantly for each channel/year we have information on the average number of 30-second advertising slots per hour of programming (in jargon "avails"). Finally we have a record of all new network launches that occurred in our sample period, a total of 43 launches.

We hand-picked the most significant entry events that occurred in our sample period to eyeball the impact of entry of well known networks. Ideally we could test our model by checking whether the observed outcomes are consistent with viewer behavior. Unfortunately we have no measure of overlapping viewership. Instead we use the analysis of section 6 that maps preferences in user behavior. Needless to say, we don't observe preferences either. However we can make reasonable assumptions on preferences by slicing-up our data set in different categories. In what follows we consider three categories consistent, arguably, with positive correlation, negative correlation and no correlation. We postulate preferences for all-news stations to be negatively correlated. For example we postulate Fox News viewers to have a low valuation for CNBC and viceversa. Similarly we postulate preferences for info-tainment channels (the three biggest being Discovery Channel, Lifetime Television and the Weather Channel) to be independent. Finally we look at sports assuming that those who watch ESPN are more likely to watch ESPN2.

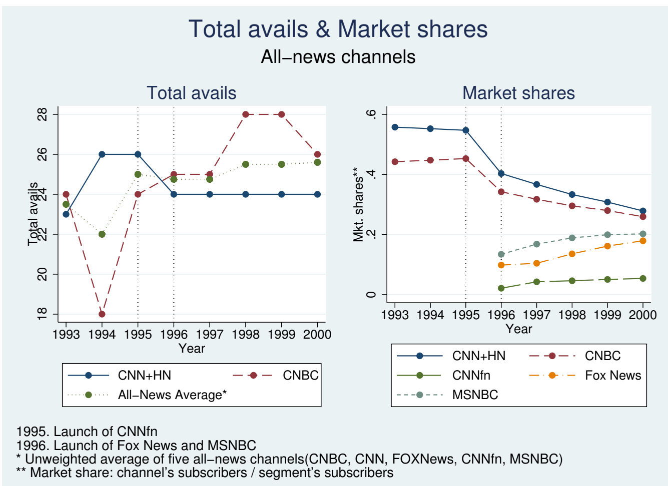

Fig. 2: All-news segment.

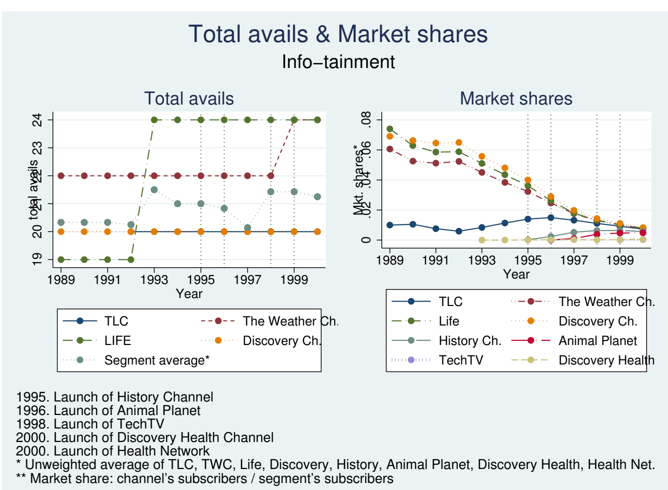

Fig. 3: Info-tainment segment.

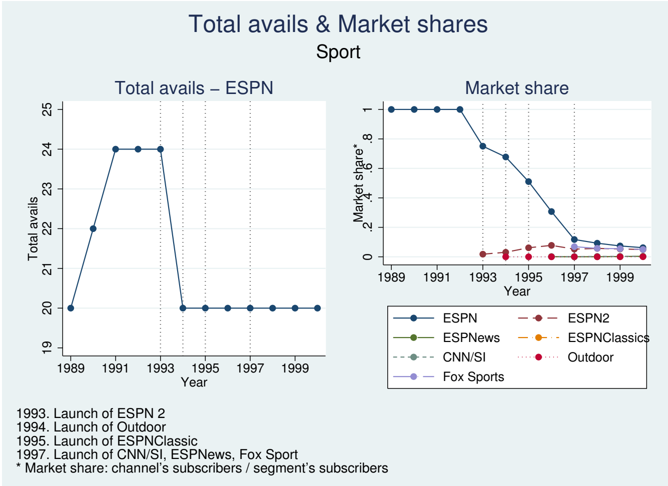

Fig. 4: Sports segment.

Consider first "all-news" channels. The left panel in Figure 2 plots average avails over time. The right panel shows the relative sizes of the different players considered.

The three substantial launches in news in our sample period are CNN financial, Fox News and MSNBC.[21] [22] We register an increase of the number of avails contextual with these events (some refer to this fact as the "Fox News effect" order "Fox News puzzle"). Info-tainment is a category whose broadcast programming do not fall in music, sports, news, kids or pure entertainment (comedy, drama, movies, shows) category.

Figure 3 shows that despite a good deal of entry between 1995 and 2000 the strategic choices of the four biggest channels didn't change, save for an *increase* from 22 to 24 slots per hour registered in 1998 operated by the Weather channel. The sport category is by far the most profitable (in terms of ad revenues) but also the more concentrated. Up until 1993 ESPN is the only all sports channel in our dataset. We speculate that this is a byproduct of exclusivity in broadcasting rights of major events. ESPN substantially decreased its advertising levels following the launch of ESPN2. We also obtained similar patterns for the kids segment and movie segment (the relative figures are relegated at the end of this document).

8.1 Regression analysis

In what follows we attempt to estimate the impact of entry on the incumbents' choices of ad levels. There are 816 potential observations in our data set (68 channels times 12 years - from 1989 to 2000). Over the course of the years we observe entry by a total of 43 channels. The panel is thus unbalanced reducing the number of observations to less than half of that. Using information contained in the channel description

[21] In what follows we ranked entry events by looking at the market share in terms of subscribers five years after entry and focused on the impact of channels whose market share after entry was higher than 1%. We include a list of all entry events in all categories in the appendix.

[22] Given the yearly frequency of our dataset, and since we are looking at strategic choices if entry occurs after the sixth month of a calendar year, we plot a dotted line on the following year.

Table 1: Definitions, means and standard deviations (SD) of variables

Variable	Definition (mean, SD)
Avails	Channel's yearly average 30 seconds slots per hour of programming (Mean = 21.91, SD = 3.58).
Incumbent	Number of channels in the basic cable lineup. From 26 in 1989 and to 69 in 2000.
Incumbent_j	Number of channels operating in the same segment j. The categories being entertainment, news, sport.
HHI_j	Herfindal concentration index. Sum of the squares of the market shares of the channels operating in the same segment. Market shares defined as the ratio of each channel's market subscribers to the number of subscribers in each segment, at time t.
Programming expenses	Includes both purchased program rights and expenses for production of original programming for a basic cable network. Units in millions of USD. (mean = 80.64, SD = 112.90).
Gross revenues	Income earned by Cable TV companies from all business activities. 1 Unit = $1 million. (Mean = 131.90, SD = 186.23)
Subscribers	Number of potential viewers. In millions (mean = 38.90, SD = 25.42)
Real GDP index	Gross Domestic Product in 2000 USD (at Purchasing Power Parity)

we partitioned channels into three categories: sports, news, entertainment.[23] Table 1 contains definitions, means and standard deviations of the primary variables in the data set.

The empirical strategy is to regress strategic choices (here the logarithm of average number of hourly avails) on a measure of entry and a number of controls. More precisely we estimate a static linear model with unobserved heterogeneity of the form:

$$ln(y_{ijt}) = \alpha + \beta * \text{Incumbents}_{jt} + \gamma X_{it} + \tau_t + \eta_i + \nu_{it}, \tag{13}$$

where β is the parameter of interest and η_i is treated as a fixed effect. The dependent variable is a direct measure of supply choices of a channel i in segment j in year t. The main explanatory variable "incumbent_j" measures the number of firms that are present in segment j at time t. In addition, since it could take some time for new entrants to become active on the advertising market, we repeat the analysis using a lagged measure of entry as the main explanatory variable.

Needless to say this strategy has several pitfalls. In particular the issue of entry endogeneity on incumbent performance. In general it is hard to instrument for entry. A paved road is the exploitation of policy changes or technological shocks that lowered entry barriers. Unfortunately this did not happen in our sample period (at least as far as we know). Our main source of concern is the increase in advertising prices per viewer in broadcasting markets registered at the end of the nineties. Real prices per viewer per avail more than double. This (we believe) is due to sustained GDP growth. The conjecture is that

[23] See the appendix for a list of channels and channel assignments to categories (or market segments).

Table 2: Dependent Variable: Hourly Avails

	(1)	(2)	(3)	(4)	(5)
Number incumbent (same segment)	0.00940***	0.00956***	0.00956***	0.00956***	
	(0.002)	(0.002)	(0.002)	(0.002)	
Number incumbent (same segment) (t-1)					0.00981***
					(0.002)
Programming expenses		-0.00005	-0.00005	-0.00005	-0.00054*
		(0.000)	(0.000)	(0.000)	(0.000)
Gross Revenue		-0.00020	-0.00020	-0.00020	-0.00004
		(0.000)	(0.000)	(0.000)	(0.000)
Subscribers		-0.00029	-0.00029	-0.00029	-0.00038
		(0.001)	(0.001)	(0.001)	(0.001)
Real GDP index				0.00149	0.00127
				(0.001)	(0.001)
Constant	2.73335***	2.75381***	2.75381***	2.60521***	2.65521***
	(0.053)	(0.064)	(0.064)	(0.115)	(0.109)
Segment fixed effect	No	No	Yes	Yes	Yes
Channel fixed effect	Yes	Yes	Yes	Yes	Yes
Time fixed effect	Yes	Yes	Yes	Yes	Yes
Observations	414	413	413	413	393
R-squared	0.816	0.820	0.820	0.820	0.831

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

firms advertise more during booms because the opportunity cost of not informing is higher. (ω increases). On the other hand a booming economy doesn't imply that viewers spend more time watching TV. That attention is still scarce. So higher demand inflates also the opportunity cost of *not* increasing ad-levels. We don't observe the advertisers' demand for ad-slots (we only observe the platforms' choices). To disentangle increases due changes in market structure from increases due to demand side factors, we use two different proxies for advertisers demand. First current and lagged GDP measures. Second the U.S. Consumer Confidence Index (CCI).[24] We don't use prices because these are endogenously determined. Supply side changes would wash out and confound the effect of changes on the demand side when measured by prices. We also include controls for the year, segment, number of subscribers, programming expenses and gross advertising revenues.

Results are presented in Table II (next page). In summary, in all our specifications we find a significative and large impact of market structure on the number of avails. This effect is there regardless of whether we consider lagged or current dependent variables and is robust to a number of controls.

[24]This measure is an indicator of the optimism of consumers on the state of the economy and hence is a predictor of consumer spending.

9 Conclusion

This paper presented a media market model with either/both competition on the viewer side. The model allows for general viewer demand and advertising technologies. In this framework, a neutrality result between competition and joint ownership emerges, that is, the advertising level is the same in the case of duopoly and in the case in which both stations are under the control of a single owner. Moreover, for both market structures, there is a tendency of excessive provision of advertising as compared to the socially optimal level. Market entry (if it leads to an increase in the number of channels) leads to an increase in the advertising level if preference correlation across channels is negative but lowers advertising levels for positive correlation. This result is validated by a simple empirical analysis. Finally, the possibility to charge viewers does not alter the neutrality result and does only partly correct the excessive provision of advertising.

A fundamental question for which our theory might serve as a useful building block is how these considerations would change the incentives towards programming. Supposing one could affect the competition mode and the degree of overlap in viewership, through an appropriate choice of programming, our model would allow to draw implications for the emerging TV landscape.

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10 Appendix

Proposition 0

Consider the following three assumptions:

$A1$ Platforms are not too asymmetric.

$A2$ The expression $\alpha\Big\{d_i(\alpha n^d)\phi(n^d) - d_i((1-\alpha)n^\star)\phi(n^\star) + t_i^\star\Big\}$, where $n_d = \arg\max_{n_i} d_i(\alpha n_i)\phi(n_i)$ is maximized at $\alpha = 1$.

$A3$ For all $n \geq n_i^\star$, as n_i changes, the change in aggregate demand of a platform is large relative to the change in ϕ_i, that $|\partial d_i(n_i)/\partial n_i| >> \partial \phi_i(n_i)/\partial n_i$.

Suppose that $A1$, $A2$, and $A3$ hold. Then, there is a equilibrium in the game with posted contracts, that is outcome-equivalent to the equilibrium of the game in the main text.

Proof:

Suppose that in the game with posted contracts each platform offers a contract with $n_i = n_i^{star}$, where $n_i^\star$ is implicitly determined by (4) and a transfer

$$t_i = D_i(n_i^\star, n_j^\star)\phi_i(n_i^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_j(n_j^\star)\right). \tag{14}$$

These contracts will be accepted by all advertisers. Since advertising levels are the same as in the equilibrium of the model in the main text, viewerships are also the same. Therefore, this candidate equilibrium is outcome-equivalent to the equilibrium of the model in teh main text.

Let us now consider if there exists a profitable deviation from this candidate equilibrium. We first show that there can be no profitable deviation contract of platform i that still induces full advertiser participation on platform j but a smaller participation on platform i. Let x_i denote the fraction of advertisers who accept the offer of platform i.

Given (t_i, n_i) and advertisers' choices x_i, the equilibrium payoff of platform i is equal to $x_i t_i$. Note that $u(n_i, \infty) - u(0, \infty) > 0$ is a lower bound of the incremental value of accepting i's offer. It follows all contracts characterized by $n_i > 0$ and $0 < t_i < u(n_i, \infty) - u(0, \infty)$ are accepted by all advertisers and guarantee a strictly positive payoff. Therefore if (t_i, n_i) is a best reply then $t_i, n_i > 0$ and $x_i > 0$.

Next consider a candidate contract (n_i, t_i). Suppose that $x_i < 1$ so that platform i's equilibrium payoff is $t_i x_i$. Now consider the following alternative contract: $(x_i n_i, x_i t_i)$. Note that total advertising on channel i is equal to $x_i n_i$. So platform i is at least as attractive even if $x_i = 1$. Note moreover that by independence the advertisers' payoff, rejecting i's offer does not change with i's offer. Finally, note that because ϕ_i and ϕ_{12} are strictly concave in n_i, the incremental value of accepting offer $(x_i n_i, x_i t_i)$ must exceed $x_i t_i$ for all levels of advertiser participation. So all advertisers would accept $(x_i n_i, x_i t_i)$ regardless. It follows that platform i can marginally increase $x_i t_i$ while still getting full participation and therefore profits are strictly higher than $x_i t_i$. It follows that no offer inducing a level of participation $x_i < 1$ can be part of a best reply.

Now suppose platform i deviates from the candidate equilibrium in such a way that it induces a fraction α of the advertisers to single-home on its platform while the remaining fraction $1 - \alpha$ single-homes on platform j.

Then the largest possible transfer that platform i can ask is bounded above by

$$t_i^d = \left(D_i(\alpha n_i^d, (1-\alpha)n_j^\star) + D_{12}(\alpha n_i^d, (1-\alpha)n_j^\star)\right)\phi_i(n_i^d) - u_{shj},$$

where n_i^d denotes the optimal deviation advertising level and u_{shj} denotes the payoff of an advertiser who chooses to reject the contract of platform i and instead single-homes on platform j. To determine u_{shj} we determine the payoff, that an advertiser obtains, when accepting only the contract of platform j, which offers the equilibrium contract, after platform i has deviated to induce all advertisers to single-home. We obtain that

$$u_{shj} = \left(D_j((1-\alpha)n_j^\star, \alpha n_i^d) + D_{12}((1-\alpha)n_j^\star, \alpha n_i^d)\right)\phi_j(n_j^\star) - t_j^\star =$$

$$\left(D_j((1-\alpha)n_j^\star, \alpha n_i^d) + D_{12}((1-\alpha)n_j^\star, \alpha n_i^d)\right)\phi_j(n_j^\star) - D_j(n_j^\star, n_i^\star)\phi_j(n_j^\star - D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_i(n_i^\star)\right).$$

The associated profit of platform i is then αt_i^d. Hence, deviating is not profitable if

$$\alpha\Big\{\left(D_i(\alpha n_i^d, (1-\alpha)n_j^\star) + D_{12}(\alpha n_i^d, (1-\alpha)n_j^\star)\right)\phi_i(n_i^d) - \left(D_j((1-\alpha)n_j^\star, \alpha n_i^d) + D_{12}((1-\alpha)n_j^\star, \alpha n_i^d)\right)\phi_j(n_j^\star)$$

$$+ D_j(n_j^\star, n_i^\star)\phi_j(n_j^\star + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_i(n_i^\star)\right)\Big\}$$

$$< D_i(n_i^\star, n_j^\star)\phi_i(n_i^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_j(n_j^\star)\right).$$

Defining $d_i(n_i) \equiv D_i(n_i, n_j) + D_{12}(n_i, n_j)$, we can rewrite the first two terms of the left-hand side of this inequality to get

$$\alpha\Big\{d_i(\alpha n_i^d)\phi_i(n_i^d) - d_j((1-\alpha)n_j^\star)\phi_j(n_j^\star) + D_j(n_j^\star, n_i^\star)\phi_j(n_j^\star + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_i(n_i^\star)\right)\Big\}$$

$$< D_i(n_i^\star, n_j^\star)\phi_i(n_i^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_j(n_j^\star)\right).$$

Now suppose that the two platforms are symmetric. Then the condition boils down to

$$\alpha\Big\{d_i(\alpha n^d)\phi(n^d) - d_i((1-\alpha)n^\star)\phi(n^\star)\Big\} - (1-\alpha)\left(D_i(n^\star, n^\star)\phi(n^\star) + D_{12}(n^\star, n^\star)\left(\phi_{12}(n^\star, n^\star) - \phi(n^\star)\right)\right) < 0, \tag{15}$$

with the abbreviation that $n_i^\star = n_j^\star = n^\star$, $n_i^d = n^d$, $\phi_i(\cdot) = \phi_j(\cdot) = \phi(\cdot)$.

By $A2$, the left-hand side is maximized if the number single-homing advertisers on platform i is the largest possible, which is the case when $\alpha = 1$.

Rewriting the condition gives

$$d_i(n^d)\phi(n^d) - d_i(0)\phi(n^\star) < 0. \tag{16}$$

If n^d were below $n^\star$, then $d_i(n^d) < d_i(0)$ and $\phi(n^d) < \phi(n^\star)$, implying that the inequality is for sure fulfilled. If n^d is above $n^\star$, then $A3$ ensures that it is fulfilled as well.

As a consequence, if platforms are symmetric, a deviation is not profitable. Hence, this is also the case if platforms are not too asymmetric.

Proof of Proposition 2:

We first look at the last three terms in W, i.e., $\omega D_2\phi_2 + \omega D_{12}\phi_{12}$. Taking the derivative of these terms gives[25]

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\phi_i' + \frac{\partial D_j}{\partial n_i}\phi_j + \frac{\partial D_{12}}{\partial n_i}\phi_{12} + D_{12}\frac{\partial \phi_{12}}{\partial n_i}. \tag{17}$$

It is easy to check that the first principal minors of the Hessian, i.e., $\partial^2\Pi^m/\partial(n_i)^2$ are both negative if the assumptions on the demand schedule and the probabilities ϕ_k, $k = 1, 2, 12$, are fulfilled. Checking that the determinant of Hessian is positive, i.e., $\left(\partial^2\Pi^m/\partial(n_1)^2\right)\left(\partial^2\Pi^m/\partial(n_2)^2\right) - \left(\partial^2\Pi^m/(\partial n_1\partial n_2)\right)^2 > 0$, we obtain that this is indeed the case if $|\partial D_i/\partial n_i| \geq |\partial D_i/\partial n_{-i}|$, $\left|\partial^2 D_i/\partial(n_i)^2\right| \geq \left|\partial^2 D_i/\partial n_i\partial n_{-i}\right|$ and $\left|\partial^2\phi_i/\partial(n_i)^2\right| \geq \left|\partial^2\phi_i/\partial n_i\partial n_{-i}\right|$. Therefore, the last three terms are concave in n_i.

We can now use $\partial D_{12}/\partial n_i = -\partial D_j/\partial n_i$ in (17) to obtain after rearranging

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\phi_i' + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_j) + D_{12}\frac{\partial \phi_{12}}{\partial n_i}.$$

From (4) we know that at $n_i = n^d$ the last expression equals zero.

However, the first terms in W are the utilities of the viewers which are strictly decreasing in n_i. As a consequence, the first-order condition with respect to n_i of W evaluated at $n_i = n_i^d$ is strictly negative, which implies that there is too much advertising. ■

Proof of Proposition 3:

We start with the case of positive correlation. As is evident from Figure 1, at $n_1 = n_2$ the demand function of the λ-types exhibits a kink. This is the case because $D_1 = D_2 = 0$ for the λ-types at $n_1 = n_2$ but D_i becomes positive if channel i reduces n_i slightly. Since there is a positive mass of λ-types, demand is kinked at this point.

To avoid this problem and be able to use differentiation techniques, we perturb the model by assuming that the λ-types are not just distributed on the 45-degree line but on the area that includes the space in ϵ-distance around the 45-degree line and we will later let ϵ go to zero. This preference configuration with the ϵ-area is displayed in Figure 2 on the left-hand side. The advantage of this formulation is that, as shown in the right-hand side of Figure 2, both D_1 and D_2 for the λ-types are strictly positive at $n_1 = n_2$. Therefore, when slightly changing n_i around a symmetric equilibrium, the profit function Π_i changes continuously, allowing us to apply differentiation techniques. After letting $\epsilon \to 0$, we obtain the equilibrium that arises when approaching the framework with viewers distributed just on the 45-degree line.

We can now derive the demand functions for the viewers located on different points on the unit square. In the following we denote the demands for viewers in the ϵ-area by D_1^e, D_2^e and D_{12}^e and the demands by the viewers outside this area by D_1^s, D_2^s and D_{12}^s. This is illustrated in Figure 3.[26]

[25]For simplicity we omit the arguments of the functions in the following.

[26]D_{12}^s shows up twice just to express that both areas belong to D_{12}^s.

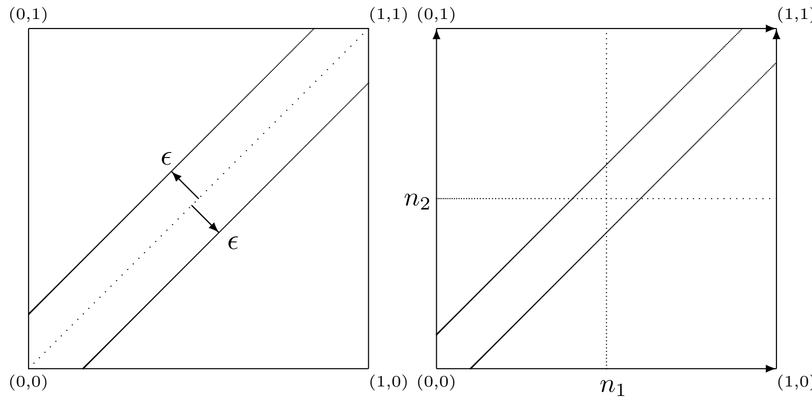

Figure 5: An Area with Positive Correlation

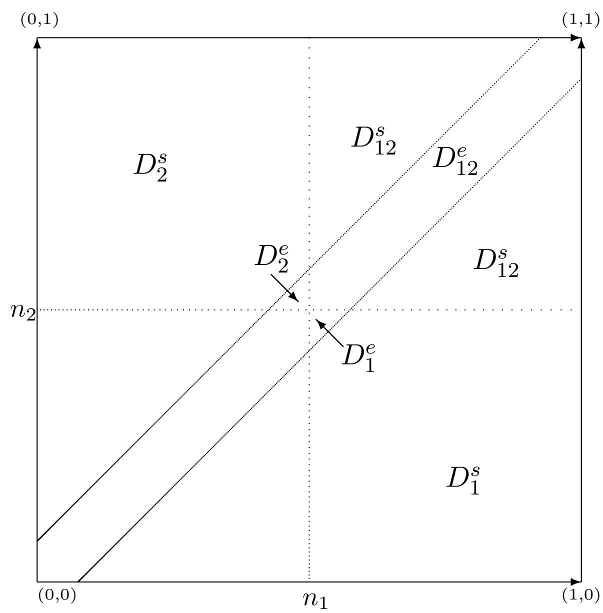

Figure 6: Demands

We first determine the ϵ-area. Doing so yields that its volume is $2\epsilon(\sqrt{2}-\epsilon)$. Then calculating the demands D_1^e and D_2^e, we obtain from Figure 5 that they are given by the triangulars starting at the intersection point between the lines representing n_1 and n_2 and the lines confining the ϵ-area. Calculating these demands gives

$$D_1^e = \frac{(\sqrt{2}\epsilon + n_2 - n_1)^2}{2} \quad \text{and} \quad D_2^e = \frac{(\sqrt{2}\epsilon - n_2 + n_1)^2}{2}.$$

From that we can easily deduce D_1^s and D_2^s to get

$$D_1^s = (1-n_1)n_2 - D_1^e \quad \text{and} \quad D_2^s = (1-n_2)n_1 - D_1^e.$$

Similarly, determining the demands for multi-homing viewers, we obtain

$$D_{12}^s = \frac{1}{2}\left(1 - n_2 - \sqrt{2}\epsilon\right)^2 + \frac{1}{2}\left(1 - n_1 - \sqrt{2}\epsilon\right)^2,$$

implying that

$$D_{12}^e = (1-n_1)(1-n_2) - D_{12}^e.$$

The profit function of channel i in duopoly is given by

$$\Pi_i^d = \omega \left[(\lambda D_i^e + (1-\lambda) D_i^s)(1-e^{-n_i}) + (\lambda D_{12}^e + (1-\lambda D_{12}^s)(e^{-n_{-i}} - e^{-(n_1+n_2)}) \right] \tag{18}$$

leading to a first-order condition of

$$\frac{\partial \Pi_i^d}{\partial n_1} = \left(\lambda \frac{\partial D_i}{\partial n_i} + (1-\lambda) \frac{\partial D_i^s}{\partial n_i} \right) (1-e^{-n_i}) + (\lambda D_i + (1-\lambda) D_i^s) e^{-n_i}$$

$$+ \left(\lambda \frac{\partial D_{12}}{\partial n_i} + (1-\lambda) \frac{\partial D_{12}^s}{\partial n_i} \right) (e^{-n_{-i}} - e^{(n_1+n_2)}) + (\lambda D_{12} + (1-\lambda) D_{12}^s) e^{-(n_1+n_2)} = 0, \tag{19}$$

where the partial derivatives of the different demand regions with respect to n_i can be easily calculated from the demands given above.

Using that at a symmetric equilibrium $n_1 = n_2 = n^\star$ and letting $\epsilon \to 0$, we obtain that $n^\star$ is implicitly given by

$$\lambda n^\star - n^\star - \frac{\lambda}{2} + e^{-n^\star} \left[\lambda + 3n^\star + \lambda (n^\star)^2 - 1 - (n^\star)^2 - 3\lambda n^\star \right] \tag{20}$$

$$+ e^{-2n^\star} \left[2 + (n^\star)^2 + 2\lambda n^\star - \frac{\lambda}{2} - 3n^\star - \lambda (n^\star)^2 \right] = 0.$$

At $\lambda = 0$, we obtain

$$e^{-n^\star} \left[\left(3n^\star - (n^\star)^2 - 1 \right) + e^{-n^\star} \left(2 + (n^\star)^2 - 3n^\star \right) \right] = n^\star.$$

Solving this for $n^\star$ we obtain that there is a unique solution given by $n^\star = 0.443$. Similarly, at $\lambda = 1$, (20) writes as

$$e^{-2n^\star} \left(\frac{3}{2} - n^\star \right) = \frac{1}{2}.$$

Solving this yields $n^\star = 0.396$.

To determine how $n^\star$ changes with λ we can apply the Implicit Function Theorem to the first-order condition (19) and then evaluate it a symmetric equilibrium $n_1^\star = n_2^\star$. After letting $\epsilon \to 0$ we obtain

$$\text{sign} \left\{ \frac{dn^\star}{d\lambda} \right\} = \text{sign} \left\{ -\frac{1}{2} + n^\star - e^{-n^\star} \left(3n^\star - 1 - (n^\star)^2 \right) - e^{-2n^\star} \left(\frac{1}{2} + (n^\star)^2 - n^\star \right) \right\}.$$

It is easy to verify that for all values of $n^\star \in [0.396, 0.443]$ the sign of $dn^\star/d\lambda$ is strictly negative. But this implies that for all $\lambda \in [0,1]$, $n^\star$ is strictly decreasing with λ.

We now turn to the case of negative correlation. Here the analysis is simpler. However, we need to distinguish between two cases, namely, the one in which D_{12}^e is positive and the one in which it is zero. The first case is displayed on the left-hand side of Figure 4 and the second case on the right-hand side.

As is easy to check in the first case demand of the λ-types are given by

$$D_1^e = n_2, \quad D_2^e = n_1, \quad \text{and} \quad D_{12}^e = (1 - n_1 - n_2),$$

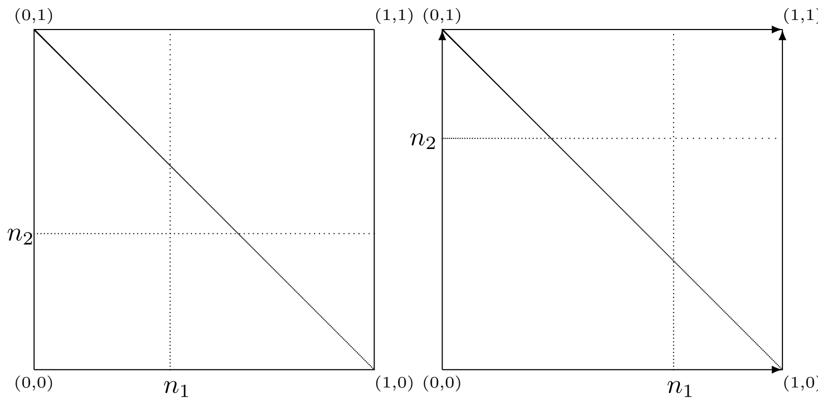

Figure 7: Negative Correlation

while the second case demands are

$$D_1^e = 1 - n_1, \quad D_2^e = 1 - n_2, \quad \text{and} \quad D_{12} = 0.$$

For the $1-\lambda$-types we have

$$D_1^s = (1-n_1)n_2 \quad D_2^s = (1-n_2)n_1 \quad D_{12}^s = (1-n_1)(1-n_2)$$

independent of the case under consideration.

We start with the first case. Here, we need to take into account that the demand configuration in this case can only be an equilibrium if $n_1 + n_2 \leq 1$ since otherwise we would have $D_{12}^e = 0$. The profit functions and the first-order conditions can be written as in (18) and (19), just with the adapted demand function. We can then again solve the first-order conditions for the symmetric equilibrium. Here we obtain that $n^\star$ is defined by

$$(\lambda-1)n^\star + e^{-n^\star}\left[3n^\star + (\lambda-1)(n^\star)^2 - 2\lambda n^\star - 1\right] \tag{21}$$

$$+e^{-2n^\star}\left[2 + \lambda n^\star - 3n^\star - (\lambda-1)(n^\star)^2\right] = 0.$$

Applying the Implicit Function Theorem we get

$$\text{sign}\left\{\frac{dn^\star}{d\lambda}\right\} = \text{sign}\left\{n^\star - e^{-n^\star}n^\star(2-n^\star) - e^{-2n^\star}n^\star(1-n^\star)\right\},$$

which is positive for all $n^\star \in [0.443, 0.5]$. Inserting $n^\star = 0.5$ into (21) and solving for λ, we obtain that $\lambda = 0.529$. Therefore, a symmetric equilibrium exists with the demand configuration given by case 1 as long as $\lambda \leq 0.529$.

We can do the same analysis for the second case in which D_{12}^e is equal to zero. However, building the first-order conditions for this case and solving for the symmetric equilibrium we obtain that for all $\lambda \in [0,1]$, $n^\star < 0.5$ implying that this demand configuration can never be an equilibrium.

Therefore, for $\lambda > 0.529$ the only symmetric equilibrium is that both channels set $n_i^\star$ exactly equal to 0.5, leaving D_{12}^s just equal to zero. Lowering the advertising level is not profitable since this does not lead to increase in D_i^e because then the case $D_i^e = n_{-i}$ becomes relevant. However, also increasing the advertising level is not profitable since then D_i^e falls by too much due to the fact that the case $D_i^e = 1-n_i$

is relevant. As a consequence, we obtain that for negative correlation $n^\star$ is weakly increasing over the range $\lambda \in [0,1]$; $n^\star = 0.443$ at $\lambda = 0$, $n_i^\star$ strictly increases up to $n^\star = 0.5$ at $\lambda = 0.529$ and stays at this level for $\lambda \in [0.529, 1]$. ■

Proof of Proposition 4:

Suppose that n_i^d were equal to the optimal n_i of a single-station monopolist. Then, the right-hand sides of (4) and (9) must be the same. Subtracting the right-hand side of (4) from the one of (9), we obtain

$$\frac{\partial D_{12}}{\partial n_i}(\phi_1 + \phi_2 - \phi_{12}) + D_{12}\left(\frac{\partial \phi_i}{\partial n_i} - \frac{\partial \phi_{12}}{\partial n_i}\right), \tag{22}$$

where all functions are evaluated at the equilibrium levels in duopoly. After rearranging we obtain that (22) is negative if (10) holds. But if (22) is negative, this implies that at $n_i = n_i^d$ the first-order condition of a monopolist is negative. But the fact that the first-order condition of a monopoly is negative in n_i^d implies that n_i^d is larger than the advertising level chosen by a monopolist. ■

Proof of Proposition 6:

Keeping the demand notation as it was derived using Figure 3, the profit function of a monopolist owning a single channel can be written as

$$\Pi_i^m = \omega\left[(\lambda D_i^e + (1-\lambda)D_i^s + \lambda D_{12}^e + (1-\lambda D_{12}^s))(1-e^{-n_i})\right],$$

which leads to first-order condition of

$$\frac{\partial \Pi_i^m}{\partial n_i} = \left(\lambda \frac{\partial D_i^e}{\partial n_i} + (1-\lambda)\frac{\partial D_i^s}{\partial n_i} + \lambda \frac{\partial D_{12}^e}{\partial n_i} + (1-\lambda)\frac{\partial D_{12}^s}{\partial n_i}\right)(1-e^{-n_i})$$

$$+(\lambda D_i^e + (1-\lambda)D_i^s + \lambda D_{12}^e + (1-\lambda)D_{12}^s)e^{-n_i} = 0.$$

Inserting the respective values into this first-order condition and rearranging it can be written as

$$e^{-n_i^\star}(2 - n_i^\star) = 1.$$

Therefore, $n_i^\star$ is independent of λ. Solving for $n_i^\star$ yields $n_i^\star = 0.443$. This corresponds to the equilibrium under duopoly for independent viewerships. Since we know that $n_i^\star < 0.443$ for positive correlation and $n_i^\star > 0.443$ for negative correlation, the result follows. ■

Proof of Proposition 8:

Rewriting the conditions (11) and (12), which determine the equilibrium advertising levels and the viewer prices, yields

$$\frac{\partial D_i}{\partial n_i}(\omega\phi_i(n_i) + p_i)\,\frac{\partial D_{12}}{\partial n_i}\left(\omega(\phi_{12}(n_i,n_j) - \phi_j(n_j)) + p_i\right) = -\omega\left(D_i\phi_i' + D_{12}\frac{\partial \phi_{12}}{\partial n_i}\right) \tag{23}$$

and

$$\frac{\partial D_i}{\partial p_i}\left(\omega\phi_i(n_i)+p_i\right)\frac{\partial D_{12}}{\partial p_i}\left(\omega(\phi_{12}(n_i,n_j)-\phi_j(n_j))+p_i\right)=-\left(D_i+D_{12}\right). \quad (24)$$

To determine the relationship between $\partial D_i/\partial n_i$ and $\partial D_i/\partial p_i$, we write $D_i = \int_{\gamma n_i+p_i}^{\infty}\int_0^{\gamma n_j+p_j} h(q_i,q_j)dq_jdq_i$ and $D_{12}=\int_{\gamma n_i+p_i}^{\infty}\int_{\gamma n_j+p_j}^{\infty} h(q_i,q_j)dq_jdq_i$. This implies that

$$\frac{\partial D_i}{\partial n_i}=-\gamma\int_0^{\gamma n_j+p_j} h(\gamma n_i+p_i,q_j)dq_jdq_i, \quad \frac{\partial D_i}{\partial p_i}=-\int_0^{\gamma n_j+p_j} h(\gamma n_i+p_i,q_j)dq_jdq_i,$$

$$\frac{\partial D_{12}}{\partial n_i}=-\gamma\int_{\gamma n_j+p_j}^{\infty} h(\gamma n_i+p_i,q_j)dq_jdq_i \quad \text{and} \quad \frac{\partial D_{12}}{\partial p_i}=-\int_{\gamma n_j+p_j}^{\infty} h(\gamma n_i+p_i,q_j)dq_jdq_i.$$

Therefore, $\partial D_i/\partial n_i=\gamma\partial D_i/\partial p_i$ and $\partial D_{12}/\partial n_i=\gamma\partial D_{12}/\partial p_i$. As a consequence, if the monopolist varies n_i by Δn_i, demand changes in the same way as when the monopolist varies by p_i by $\Delta p_i=\gamma\Delta n_i$.

We can now determine the optimal level of advertising from (23) and (24). Inserting $\partial D_i/\partial n_i = \gamma\partial D_i/\partial p_i$ in (23) and then dividing (23) by (24), we obtain, after rearranging,

$$\gamma=\frac{\omega\left(D_i\phi_i'+D_{12}\frac{\partial\phi_{12}}{\partial n_i}\right)}{D_i+D_{12}}. \quad (25)$$

Now we turn to the socially optimal advertising level. From Subsection 4.2, social welfare is given by

$$W=\int_{\gamma n_1}^{\infty}\int_0^{\gamma n_2} q_1-\gamma n_1 h(q_1,q_2)dq_2dq_1+\int_0^{\gamma n_1}\int_{\gamma n_2}^{\infty} q_2-\gamma n_2 h(q_1,q_2)dq_2dq_1$$

$$+\int_{\gamma n_1}^{\infty}\int_{\gamma n_2}^{\infty} q_1-\gamma n_1+q_2-\gamma n_2 h(q_1,q_2)dq_2dq_1+\omega D_1\phi_1+\omega D_2\phi_2+\omega D_{12}\phi_{12}. \quad (26)$$

We know that viewer demand D_i and D_{12} fall with n_i. Suppose to the contrary that D_i and D_{12} would not change with n_i. Differentiating (26) with respect to n_i then gives

$$-\gamma D_i-\gamma D_{12}+\omega D_i\phi_i'+\omega D_{12}\frac{\partial\phi_{12}}{\partial n_i}=0.$$

Rearranging this yields (25).

Therefore, the advertising level n_i implicitly determined by (25) provides an upper bound on the socially optimal level of advertising, which obtains when viewer demand is inelastic. But since D_i and D_{12} fall with n_i, the socially optimal level must be lower than the one prescribed by (25). This shows that the socially optimal level is lower than the one with viewer pricing, provided that the latter is positive. ■

Proof of the Reduction of Viewer Surplus and Advertiser Revenue with Pricing:

We start with a comparison of the equilibrium advertising levels in case of viewer pricing and in case without. In case of viewer pricing, the equilibrium advertising level is given by the derivative of $\omega\left(D_1\phi_1+D_2\phi_2+D_{12}\phi_{12}\right)+p_1D_1+p_2D_2+(p_1+p_2)D_{12}$ with respect to n_i. By contrast, in case without viewer pricing the equilibrium advertising level is given by the derivative of $\omega\left(D_1\phi_1+D_2\phi_2+D_{12}\phi_{12}\right)$ with respect to n_i. Since $p_1,p_2\geq 0$ and $\partial D_i/\partial n_i<0$, $\partial D_{12}/\partial n_i<0$ and $\partial D_j/\partial n_i=-\partial D_{12}/\partial n_i$, the derivative of $p_1D_1+p_2D_2+(p_1+p_2)D_{12}$ with respect to n_i is negative. This implies that the first-order condition with respect to n_i in case of viewer pricing is negative at the equilibrium value of n_i for the case without viewer pricing. As a consequence, the equilibrium advertising level with viewer pricing is below the one without viewer pricing. If viewer demand is also lower with viewer pricing than without, this implies that advertising revenue is lower.

The monopoly profit function in case of viewer pricing can be written as

$$D_1 (\omega\phi_1 + p_1) + D_2 (\omega\phi_2 + p_2) + D_{12} (\omega\phi_{12} + p_1 + p_2).$$

Therefore, for any demand segment, the monopolist has two revenue sources. It can either use advertising or viewer pricing or both. This depends on the shape of the per-viewer revenues of advertising ($\omega\phi_i$ and $\omega\phi_{12}$), the shape of the per-viewer revenue of pricing (p_i) and how the viewer demand reacts to changes in the advertising level and the viewer price.

Suppose that the monopolist uses both revenue sources, advertising and pricing. Since $\phi_i(n_i)$ and $\phi_{12}(n_i, n_j)$ are concave in n_i, the per-viewer revenue from advertising is also concave in n_i. By contrast, the per-viewer revenues from pricing p_i is linear. Since $\partial D_i/\partial n_i = \gamma \partial D_i/\partial p_i$ and $\partial D_{12}/\partial n_i = \gamma \partial D_{12}/\partial p_i$, it must be that the first marginal unit of revenue comes from advertising. This is because due to the shapes of the demand functions and the revenue functions, the marginal revenue from advertising is decreasing more strongly than the one from pricing. If advertising were not used for the first unit of revenue, it will be never be used.

Now if the monopolist increases its advertising further, at some point the marginal revenue from viewer pricing equals the marginal revenue from advertising, since otherwise, the monopolist will not use both revenue sources. At this point, the monopolist will start to use pricing as well.

Let us now consider the monopolist's optimal advertising level when pricing is not possible, denoted by $n_i^\star$. If the marginal per-viewer revenue of viewer pricing is lower than the one of advertising even at this point, pricing will not be used. Therefore, the optimal solution with and without pricing is the same. Hence, viewer surplus and advertising revenue are unchanged. By contrast, if viewer pricing will be used, we have that at $n_i^\star$ the marginal per-viewer revenue with must be (weakly) larger than without pricing. In addition, we know that the monopolist can induce the same aggregate demand via increasing n_i by 1 unit and via increasing p_i by $\Delta p_i = \gamma \Delta n_i$. This implies that at the point $n_i = n_i^\star$ and $p_i = 0$, the monopolist obtains a larger marginal revenue when viewer pricing can be used. Therefore, the monopolist optimally raises either p_i at this point, inducing a smaller demand than without viewer pricing. As a consequence, not only viewer surplus but also advertising revenue falls. ■

10.1 Heterogeneous Advertisers

The goal of this section is to show that the basic trade-off driving the neutrality result is robust to allowing for heterogeneous advertisers. In case of heterogeneous advertisers, the optimal individual advertising amount and therefore also the payment is different for different advertiser types, which makes the analysis more complicated. We therefore do not provide a direct extension of the main model but analyze the situation with posted contracts, in which each platforms now offers a menu of contract, i.e., a price schedule for different intensities of advertising, and advertisers and viewers make their decisions conditional on these menus. This allows to look at different contracts in a direct way.

The above duopoly model is extended as follows. At stage 1 each channel simultaneously posts a price schedule, that is a mapping from quantity of ads to prices $t_i : [0, \overline{n}] \to \mathbb{R}$, where $\overline{n}$ is arbitrarily large. At stage 2 each advertiser observes the posted schedules and chooses its preferred intensity (possibly zero) on each platform. We restrict $t_i(0) = t_j(0) = 0$. Note that all advertisers would rather not contract with i than pay a positive price for $n_i = 0$. So this restriction is without loss of generality. The value of informing a viewer, ω, is private information and distributed according to a smooth c.d.f. F with support $[\underline{\omega}, \overline{\omega}]$ that satisfies the monotone hazard rate property. We assume $\underline{\omega} \geq 0$. Given $(t_1(n_1), t_2(n_2))$, type ω's payoff from choosing quantity (n_1, n_2) depends on all other advertisers' choices, as these determine the total quantity of ads on each channel and in turn viewers' demand. In what follows we define this aggregate advertising level by $N_i = \int_{\underline{\omega}}^{\overline{\omega}} n_i(\omega') dF(\omega')$, $i = 1, 2$. We also define $N = (N_1, N_2)$ as the total quantities of ads on each channel. In the advertiser game, the issue of multiplicity of equilibria might arise. To focus on the platforms' choices, we assume away coordination issues, and suppose that realized advertising levels are continuous, with respect to the uniform norm, in the price schedules chosen by the platforms. We fix the continuation equilibrium for the rest of the analysis. The continuity assumption is a very reasonable one in our game in which advertising exerts a disutility on viewers, implying that the game between advertisers exhibits negative externalities. Therefore, the standard problem of equilibrium multiplicity and discontinuity in games with positive network externalties does not arise in our setting.

We now characterize channel i's best reply, that is, the price schedule t_i that maximizes its payoff given t_j. With an abuse of notation we keep denoting $\omega u(n_1, n_2, N)$ the surplus of advertiser ω from advertising intensities (n_1, n_2). Note however that this function is only well defined given the price schedules which here are omitted as arguments. So if $n_i(\omega, (t_1, t_2))$ denotes the optimal quantity chosen by type ω, then i's problem, given the rival's price schedule t_j is well defined and equal to (arguments omitted):

$$\max_{t_i(\cdot)} \int_{\underline{\omega}}^{\overline{\omega}} t_i(n_i(\omega)) dF(\omega).$$

The above can be expressed as a standard screening problem:

$$\max_{t_i(\cdot), n_i(\cdot), \omega_0} \int_{\omega_0}^{\overline{\omega}} t_i(n_i(\omega)) dF(\omega) \qquad \text{subject to} \qquad n_i(\omega) = \arg\max_{n_i} v_i^d(n_i, \omega, N) - t_i(n_i)$$
$$v_i^d(n_i(\omega), \omega, N) - t_i(n_i(\omega)) \geq 0 \ \text{ for all } \omega \geq \omega_0.$$

Here $v_i^d(n, \omega, N) := \max_y \omega u(n, y, N) - t_j(y) - (\max_{y'} \omega u(0, y', N) - t_j(y'))$, with $u(n, y, N) \equiv\!= D_i(N_1, N_2)\phi_i(n) + D_j(N_1, N_2)\phi_j(n_j) + D_{12}(N_1, N_2)\phi_{12}(n, n_j)$, denotes the net value of advertising intensity n on channel i to type ω. This is the value of contracting with i given $t_j(n_j)$. It equals the maximum value of the allocation n minus the outside option of dealing with j exclusively. Note that in any pure strategy equilibrium channel i behaves as a monopolist facing a mass one of advertisers with v_i^d as their indirect utility function. Provided that this function satisfies standard regularity conditions in the screening literature, it is possible to apply the canonical methodology developed by Mussa and Rosen (1978) or Maskin and Riley (1984) to characterize i's best reply. As in Martimort and Stole (2009), v_i^d is said to be regular if it is continuous, monotone in ω and displays strict increasing differences in (n, ω). Our assumptions on the viewer demands $D_i(n_1, n_2)$ and $D_{12}(n_1, n_2)$ and on the advertising technology $\phi_i(n_i)$ and $\phi_{12}(n_1, n_2)$ ensure that v_i^d is continuous and monotonically increasing in ω. It also has strict increasing differences in (n, ω) for values of n that are not very large and therefore will never constitute an optimal solution. An equilibrium (t_1^d, t_2^d) is said to be regular if the induced indirect utility functions are regular.[27]

We contrast platform i's best reply with the optimal price schedule that a hypothetical multi-channel monopolist would choose for platform i given an arbitrary price schedule t_j. As our benchmark, the monopolist is restricted to post two independent price schedules t_i and t_j. For a reason that will be clear later on, we allow the monopolist to charge a participation fee t_0 to all advertisers choosing ad intensities other than $(0,0)$. The monopolist profits are (arguments omitted)

$$\max_{t_i(\cdot),t_j(\cdot),t_0} \int_{\underline{\omega}}^{\overline{\omega}} t(n_1(\omega), n_2(\omega))dF(\omega),$$

with

$$t(n_1(\omega), n_2(\omega)) = \begin{cases} t_0 + t_1(n_1(\omega)) + t_2(n_2(\omega)) & \text{if } (n_1(\omega), n_2(\omega)) \neq (0,0) \\ 0 & \text{otherwise.} \end{cases}$$

Once more, it is possible to derive the induced indirect utility function $v_i^m(n,\omega,N) = \max_y \omega u(n,y,N) - t_j(y) - t_0 - \sup\left\{\max_{y'} \omega u(0,y',N) - t_j(y') - t_0, 0\right\}$ and express the above as a standard screening problem as follows:

$$\begin{aligned} &\max_{\{t_i(\cdot)\}_{i=1}^2, \{n_i(\cdot)\}_{i=1}^2, \omega_0, t_0} \int_{\omega_0}^{\overline{\omega}} t(n_1(\omega), n_2(\omega))dF(\omega) \\ &\text{subject to } \; n_i(\omega) = \arg\max_{n_i} v_i^m(n_i,\omega,N) - t_i(n_i) \\ &v_i^m(n_i(\omega),\omega,N) - t_i(n_i(\omega)) \geq 0 \;\; \text{for all } \omega \geq \omega_0. \end{aligned}$$

A solution to the monopoly problem $(t_1^m), t_2^m))$ is said to be regular if the induced indirect utility functions are regular. Let $n_i(\omega)$ denote the optimal allocation given ω_0 and $\Lambda^m(n_i(\omega),\omega,N) = v_i^d(n(\omega),\omega,N) - (1-F(\omega))/f(\omega)(\partial v_i^d(n(\omega),\omega,N))/(\partial\omega)$ the associated virtual surplus function. Finally we assume that the profit function $\int_{\omega_0}^{\overline{\omega}} \Lambda^m(n_i(\omega),\omega,N)dF(\omega)$ is quasi-concave with respect to ω_0.

Proposition 9. *Suppose that (t_1^m, t_2^m) is a regular solution of the multi-channel monopoly problem. Let $n_1(\omega)$ and $n_2(\omega)$ be the induced allocation of ads. Then there is a regular equilibrium (t_1^d, t_2^d) of the corresponding duopoly game that induces the same allocation of ads.*

Proof:

Given (t_i, t_j), type ω's payoff from choosing quantity (n_1, n_2) depends on all other advertisers' choices, as these affect viewers' behavior. Given the optimal choice of all other types ω', denoted $n(\omega')$, the problem of type ω is given by

$$\begin{aligned} (n_1(\omega), n_2(\omega)) := \arg\max_{(n_1,n_2)} &\, \omega D_1(N_1,N_2)\phi_1(n_1) + \omega D_2(N_1,N_2)\phi_2(n_2) \\ &+ \omega D_{12}(N_1,N_2)\phi_{12}(n_1,n_2) - t_1(n_1) - t_2(n_2). \end{aligned}$$

The above operator maps the space of $n_1(\cdot), n_2(\cdot)$ schedules into itself. As mentioned previously,

[27] As we shall see, the corresponding virtual surplus is given by $v_i^d(n,\omega,N) - (1-F(\omega))/f(\omega)\partial v_i^d(n,\omega,N)/\partial\omega$. Again, our assumptions on viewer demand and on the advertising technology ensure strict quasi-concavity in n and the monotone hazard rate property ensures increasing differences in (n,ω) for values of n that are not too large.

we assume that for each pair of price schedules the realized aggregate advertising levels $N_i(t_i, t_j)$ and $N_j(t_j, t_i)$ are continuous in the price schedules. Define: $\nu := (N_i(t_i, t_j), N_j(t_j, t_i))$ as the total quantities of ads on each platform in equilibrium as a function of the schedules posted. We can then write

$$u(n_i, n_j, \nu) = D_i(\nu)\,\phi_i(n_i) + D_j(\nu)\,\phi_j(n_j) + D_{ij}(\nu)\,\phi_{ij}(n_i, n_j).$$

Now consider the problem of a duopolist i who chooses a price schedule to maximize profits $\int_{\underline{\omega}}^{\overline{\omega}} t_i(n_i(\omega))dF(\omega)$, given its rival's choice $t_j(n_j)$. This problem can be rewritten as a standard screening problem where the maximization is over the set of all monotone allocations $n_i(\omega)$, provided the associated transfer is such that the allocation is incentive compatible and individually rational:

$$\max_{\omega_0, n_i(\omega)} \int_{\omega_0}^{\overline{\omega}} t_i(n_i(\omega))dF(\omega)$$

$$\text{subject to} \quad n_i(\omega) = \arg\max_{n_i} v_i^d(n_i, \omega, N) - t_i(n_i)$$
$$v_i^d(n_i(\omega), \omega, N) - t_i(n_i(\omega)) \geq 0 \quad \text{for all } \omega \geq \omega_0.$$

Denote by $n_j^\star(n, \omega)$ the quantity that type ω optimally buys from platform j when buying quantity n from platform i. Then, the net contracting surplus for type ω is

$$v_i^d(n, \omega, \nu) = \max_y \omega u(n, y, \nu) - t_j(y) - (\max_{y'} \omega u(0, y', \nu) - t_j(y')) \tag{27}$$
$$= \omega u(n, n_j^\star(n, \omega), \nu) - t_j(n_j^\star(n, \omega)) - \big(\omega u(0, n_j^\star(0, \omega, \nu)) - t_j(n_j^\star(0, \omega))\big) \tag{28}$$

Incentive compatibility requires $n_i(\omega) = \arg\max_n v_i^d(n, \omega, \nu) - t_i(n)$. So by definition we have:

$$v_i^d(n_i(\omega), \omega, \nu) - t_i(n_i(\omega)) = \max_{y, y', n} \big\{\omega u(n, y, \nu) - t_j(y) - (\omega u(0, y', \nu) - t_j(y')) - t_i(n)\big\}$$

By the envelope theorem the derivative of the above with respect to ω is

$$u(n, n_j^\star(n_i(\omega), \omega), \nu) - u(0, n_j^\star(0, \omega), \nu)$$

Since the above pins down the growth rate of the agent's payoff, we have that $\max_{\omega_0, n_i(\cdot)} \int_{\omega_0}^{\overline{\omega}} t_i(\omega)$ subject to the two constraints above equals

$$\max_{n_i(\cdot), \omega_0} \int_{\omega_0}^{\overline{\omega}} \Big\{\omega u(n_i(\omega), n_j^\star(n_i(\omega), \omega)) - \omega u(0, n_j^\star(0, \omega)) - t_j(n_j^\star(n_i(\omega), (\omega))) + t_j(n_j^\star(0, (\omega)))$$
$$- \int_{\omega_0}^{\omega} \big[\omega u(n, n_j^\star(n_i(z), z), \nu) - \omega u(0, n_j^\star(0, z), \nu)\big]\, dz\Big\} dF(\omega)$$
$$= \max_{\omega_0, n_i(\cdot)} \int_{\omega_0}^{\overline{\omega}} \Big\{v_i^d(n_i, \omega, \nu) - \underbrace{\int_{\omega_0}^{\omega} \big[\omega u(n, n_j^\star(n_i(z), z), \nu) - \omega u(0, n_j^\star(0, z), \nu)\big]\, dz}_{\text{information rent}}\Big\} dF(\omega),$$

Integrating the double integral by parts gives:

$$\max_{n_i(\cdot),\omega_0} \int_{\omega_0}^{\overline{\omega}} \omega u(n_i(\omega), n_j^\star(n_i(\omega),\omega)) - \omega u(0, n_j^\star(0,\omega)) - t_j(n_j^\star(n_i(\omega),(\omega))) + t_j(n_j^\star(0,(\omega))) +$$
$$- \frac{1-F(\omega)}{f(\omega)}(u(n, n_j^\star(n_i(\omega),\omega),\nu) - u(0, n_j^\star(0,\omega),\nu))\, dF(\omega)$$

The duopolist's best reply allocation $n_i^d(\omega)$ solves

$$\max_{n_i(\cdot),\omega_0} \int_{\omega_0}^{\overline{\omega}} \left(\omega - \frac{1-F(\omega)}{f(\omega)}\right)(u(n_i(\omega), n_j^\star(n_i(\omega),\omega)) - u(0, n_j^\star(0,\omega))) \\ - \left(t_j(n_j^\star(n_i(\omega),\omega)) - t_j(n_j^\star(0,\omega))\right) dF(\omega) \tag{29}$$

From now on we will refer to the integrand function as $\Lambda^d(n_i(\omega),\omega,\nu)$. Recall that solving a canonical screening problem usually involves maximizing the integral over all types served of the "full utility" of type ω minus its informational rent, expressed as a function of the allocation itself. The "full utility" here is the incremental value $u(n_i(\omega), n_j^\star(n_i(\omega),\omega)) - u(0, n_j^\star(0,\omega))$, minus the difference in transfers.

Now consider the monopolist's problem, which is to choose a pair of price schedules and a participation fee $t_0 \le \bar{t} < +\infty$, where $\bar{t}$ is arbitrarily large. Without loss of generality, we restrict $t_j(0), t_i(0) \le 0$. Analogous to the duopoly case, this is due to the fact that conditional on paying the participation fee, all advertisers can guarantee a zero allocation at zero price at either platform. In the following, we define $\tilde{t}_i(n_i(\omega)) \equiv t_i(n_i(\omega)) + \bar{t}_i$, where $\bar{t}_i$ is a constant to be determined by the monopolist. Given $t_j(\cdot)$ the monopolist's problem is

$$\max_{t_i(\cdot),t_0,\bar{t}_i,\bar{t}_j} \int_{\underline{\omega}}^{\overline{\omega}} (\tilde{t}_i(n_i(\omega)) + \tilde{t}_j(n_j(\omega)) + t_0)\mathbf{I}(n_i(\omega) + n_j(\omega) > 0) dF(\omega), \tag{30}$$

where $\mathbf{I}$ is an indicator function equal to 1 whenever the argument is true and zero otherwise. The net contracting surplus corresponding to type ω as a function of the allocation is

$$v_i^m(n,\omega,\nu) = \max\nolimits_y \omega u(n,y,\nu) - t_j(y) - \bar{t}_j - t_0 - \sup\left\{\max_{y'} \omega u(0,y',\nu) - t_j(y') - \bar{t}_j - t_0, 0\right\}. \tag{31}$$

As in the previous case, the problem given by (30) can be rewritten as a standard incentive problem of the form

$$\max_{t_i(\cdot),n_i(\cdot),t_0,\bar{t}_i,\bar{t}_j} \int_{\omega_0}^{\overline{\omega}} (\tilde{t}_i(n_i(\omega)) + \tilde{t}_j(n_j(\omega)) + t_0)\mathbf{I}(n_i(\omega) + n_j(\omega) > 0) dF(\omega),$$

subject to $n_i(\omega) = \arg\max_n v_i^m(n,\omega,\nu)$ (incentive compatibility) and $v_i^m(n,\omega,\nu) - t_i(n_i(\omega)) - \bar{t}_i \ge 0$ (individual rationality) for all $\omega \ge \omega_0$. By the envelope theorem the derivative of $v_i^m(n_i(\omega),\omega,\nu)$ with respect to ω is

$$u(n_i(\omega), n_j^\star(n,\omega),\nu) - \mathbf{I}(\omega,t_0)u(0, n_j^\star(0,\omega),\nu),$$

where $\mathbf{I}(\omega,t_0)$ is an indicator function equal to 1 if $\max_{y'}\omega u(0,y',\nu) - t_j(y') - t_0 > 0$ and zero otherwise. This coupled with individual rationality implies

$$t_i(n_i(\omega)) = v_i^m(n,\omega,\nu) - \int_{\omega_0}^{\overline{\omega}} \left(u(n_i(z), n_j^\star(n_i(z),z),\nu) - \sup\{u(0, n_j^\star(0,z),\nu)\}\right) dz.$$

Plugging this into the objective function we obtain

$$\max_{n_i(\cdot),\omega_0,t_0,\bar{t}_j,\bar{t}_i} \int_{\omega_0}^{\overline{\omega}} \Big\{ \max_y \omega u(n_i(\omega), y, \nu) - \sup\Big\{ \max_y^{\prime} \omega u(0, y', \nu) - t_j(y') - \bar{t}_j - t_0, 0\Big\} \tag{32}$$
$$- \int_{\omega_0}^{\omega} \left(u(n_i(z), n_j^\star(n_i(z), z), \nu) - \mathbf{I}(\omega, t_0) u(0, n_j^\star(0, z), \nu)\right) dz \Big\} dF(\omega).$$

Since $\bar{t}$ is arbitrarily large and t_0 can be as large as $\bar{t}$, there exists a t_0 such that $t_0 > |\bar{t}_j|$. This implies that for t_0 large enough, $\sup\Big\{\max_y' \omega u(0, y', \nu) - t_j(y') - \bar{t}_j - t_0, 0\Big\} = 0$ and $\mathbf{I}(\omega, t_0) = 0$. In addition, (32) is increasing in t_0. Hence, $t_0 = \bar{t}$ and the monopolist's problem boils down to

$$\max_{n_i(\cdot),\omega_0} \int_{\omega_0}^{\overline{\omega}} \left\{ \max_y \omega u(n_i(\omega), y, \nu) - \int_{\omega_0}^{\omega} u(n_i(z), n_j^\star(n_i(z), z), \nu) dz \right\} dF(\omega).$$

Using the same technique as in the duopoly case, this gives

$$\max_{n_i(\cdot),\omega_0} \int_{\omega_0}^{\overline{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)}\right) u(n_i(\omega), n_j^\star(n_i(\omega), \omega)) dF(\omega) \tag{33}$$

The above integrand, labeled $\Lambda^m(n_i(\omega), \omega, \nu)$, reflects the "full surplus" internalization feature of our monopolist, similar to the homogeneous case. Here transfers do not show up because advertisers do not have the option to buy only one contract.

By our regularity assumptions, a solution exists to both problems: $(n_i^m(\omega), \omega_0^m), (n_i^d(\omega), \omega_0^d)$. To show that the allocation in both problems is the same, we need to establish that the optimal $n_i(\omega)$ equals the $\arg\max_q$ of $\Lambda^d(q, \omega, \nu)$ and of $\Lambda^m(q, \omega, \nu)$ and that the indifferent advertiser ω_0 is also the same.

Let us first consider the schedule keeping the marginal advertiser, ω_0^m and ω_0^d, respectively, fixed in both problems, and assume that the marginal advertiser is the same, i.e., $\omega_0^m = \omega_0^d$. The only difference between monopoly and duopoly is that in duopoly there is an additional term $t_j^*(n_j^*(n_i(\omega), \omega)$, that depends on n_i. However, applying the envelope theorem, it is evident from the definition of $v_i^d(n, \omega, \nu)$ given in (27) and (28) that when differentiating the integrand of the duopolist's problem given by (29) with respect to n_i, we can ignore the (indirect) effect of n_i on $n_j^\star$. The same argument applies to the monopolist's problem given by (33), as can be seen from $v_i^m(n, \omega, \nu)$ in (31). Therefore, the optimal solution for a duopolist and a monopolist coincide.

Under the assumption that $\omega_0^m = \omega_0^d$, we thus have established the following result:

$$n_i^m(\omega) = \begin{cases} n_i^d(\omega) & \omega \geq \omega_0^m \\ 0 & \text{otherwise} \end{cases}$$

The result basically says that neutrality carries over on the "intensive" margin. That is, conditional on ω getting some positive allocation both a monopolist and a duopolist best respond to some t_j by offering the same allocation. This is true because the maximizations problems with respect to $n_i(\cdot)$ are equivalent for a monopolist and duopolist, if $w_0^m = w_0^d$.

We now turn to the extensive margin and will establish that $\omega_0^m = \omega_0^d$. First, note that $\Lambda^d = 0$ at $n_i = 0$ for all ω. The increasing differences property $\Lambda_{n_i,\omega}^d \geq 0$ implies that the optimal allocation is weakly monotone.[28] As a consequence, the marginal type is defined as the highest type for which $n_i(\omega) = 0$. Therefore, we have $n_i^d(\omega) = 0$ for all $\omega \leq \omega_0^d$.

Further note $\Lambda^d(n_i^d(\omega), \omega, \nu) \geq 0$ because $\Lambda^d(0, \omega, \nu) = 0$ for all ω is a lower bound on $\Lambda^d(x, \omega)$, $x \geq 0$. By definition of ω_0^d, in a right neighborhood $n_i^d(\omega) > 0$; therefore, $u(n_i(\omega), n_j^*(n_i(\omega), \omega)) - u(0, n_j^*(0, \omega)) > 0$ and $t_j(n_j^*(n_i(\omega), \omega)) - t_j(n_j^*(0, \omega)) \geq 0$. Hence, $\Lambda^d(n_i(\omega), \omega, \nu) \geq 0$ only if $\omega - (1 - F(\omega))/f(\omega) \geq 0$ in a right neighborhood of ω_0^d. By continuity and the monotone hazard rate property we have $\omega - (1 - F(\omega))/f(\omega) \geq 0$ for all $\omega \geq \omega_0^d$. It follows that $\Lambda^m(n_i^m(\omega), \omega, \nu) \geq 0$ for all $\omega \geq \omega_0^d$.

Now suppose that the monopolist would exclude the marginal type ω for which $\Lambda^m(n_i^m(\omega), \omega, \nu) \geq 0$. This would obtain a first-order loss but only a second-order gain. This is because the type pays a (weakly) positive transfer (recall $n_j(\omega) \geq 0$ and therefore $t_j(n_j(\omega)) \geq 0$) but $n_i(\omega)$ is arbitrarily close to zero, so the gain for all other advertisers when excluding the marginal type becomes negligible. Therefore, it is a local maximum to serve the marginal type for whom $\Lambda^m(n_i^m(\omega), \omega, \nu) \geq 0$. But since the profit function is quasi-concave in ω_0, this is also a global maximum. Hence, $\omega_0^m \leq \omega_0^d$. This coupled with the fact that $n_i^m(\omega) = n_i^d(\omega)$ implies that the marginal price schedules must coincide: $t_i^m(n) = t_i^d(n)$. As a consequence, $\omega_0^m = \omega_0^d$. ■

The proposition establishes that if an allocation is implemented by a monopoly owner of both platforms, then the corresponding allocation is also an equilibrium of the duopoly game.

[28]Even without increasing differences, incentive compatibility would restrict us to optimize with respect to monotone $n_i(\omega)$ only.