

# Simultaneous Signaling in Markets with Private Demand Information

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## Abstract

As is well-known from the literature on oligopoly games with incomplete information, firms have an incentive to share private demand information. However, if verifiability of reported demand data is not guaranteed, the models imply that firms send misleading information. We derive a costly information sharing device via simultaneous signaling of private information that allows for an advantageous demand revelation, even in absence of verifiability. It can be shown that, in case of gamma distributed demand variables, the expected gross gains from information revelation exceed the expected cost of signaling if the skewness of the distribution is sufficiently large and the market under consideration is sufficiently heterogeneous.

Keywords: Private information; simultaneous signaling; heterogeneous markets

JEL Classification: C73, D82, L13

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## 1 Introduction

The strategic interaction of firms is typically characterized by problems of asymmetric information. Firms are usually better or at least earlier informed about their own cost and demand parameters than about those of their rivals. Therefore, it is an important issue in the industrial organization literature to study the incentives of firms to exchange private information (see, e.g. Vives 1999, ch. 8). The literature on information sharing in oligopoly is large. However, most models deal with Cournot competition in homogeneous markets. Only few papers deal with price competition in heterogeneous markets, even if this mode of competition seems to be relevant in many industries. Gal-Or (1986) and Sakai (1991) study the expected gains of exchanging cost information, whereas Vives (1984) and Sakai (1986) analyze the exchange of demand information. The authors derive the well known result that firms *ex ante* have no incentive to share private cost information but indeed have an incentive to share private demand information. These results, summarized in the rather general duopoly model of Raith (1996), show that the decisive factors for information sharing include not only the mode of competition (prices or quantities) but also the kind of private information (common value or private value) and the relation of the products (substitutes or complements). In some cases firms expect to benefit from information sharing, in other cases the reverse is true.

The cited models have in common that they ignore the firms' possibility of strategically misinforming competitors by assuming that firms can agree on revealing information as soon as it becomes privately known. As an alternative, they (implicitly) assume an outside institution, such as a trade association, which is able to transfer true information. The role of this association is to ensure that no firm can deviate from its precommitment to disclose the true information. Such an institution may exist in a regulated environment but it will hardly emerge in a competitive market. Without precommitment, however, the decision to disclose depends on the realization of the private information. For a large subset of realizations, firms will prefer not to disclose. Even if firms are willing to report about their private information, the question arises whether they have an incentive to disclose the true value or whether they find it in their best interest to mislead the rivals. So far, this crucial question has only been addressed by Ziv (1993) in the context of private cost information in a homogeneous Cournot market. By introducing a costly two-sided

signaling mechanism, he solves the precommitment problem by showing that, if it is not too costly for firms, their optimal strategy comprises to truthfully disclose their private information by sending a message. Of course, competitors could send no message at all, but if this strategy is interpreted as the worst possible case, such behavior will certainly be dominated by sending a signal.

The intention of the present paper is twofold: First, it complements Ziv's results by analyzing the information revelation strategies of firms in case of private demand information instead of private cost information. Thereby, we generalize the model by accounting for heterogeneous markets and by allowing not only for quantity but also for price competition. These extensions prove to be essential since the information revelation behavior in the sequentially rational equilibrium turns out to depend decisively on the degree of market heterogeneity. Secondly, we provide a condition necessary for firms to agree on implementing such a revelation mechanism. We show that, depending on mean and variance of the distribution of the demand variable, the expected gross gains from information revelation exceed the expected cost of signaling only if the market is sufficiently heterogeneous.

The remainder of the paper is structured as follows: Section 2 introduces the model and determines the expected gains from information sharing. Section 3 proves that, without a costly signaling device implementing an information exchange, firms have no incentive to truthfully reveal private demand information. Section 4 derives such a signaling mechanism implementing a truthful exchange of private information if the expected gross gains from information revelation exceed the expected cost of signaling. Section 5 derives the condition under which risk-neutral firms should agree to implement the proposed signaling device and presents a parametric specification of the model appropriate to study the role of market heterogeneity. Section 6 concludes.

## 2 The Basic Model of Competition

We consider a simultaneous-move game between two firms producing differentiated products. Each strategy  $s_i, i = 1, 2$ , belongs to the positive real line and the profit functions  $\pi^i(s_i, s_j)$  are twice continuously differentiable. If the decision variables are prices, the game is one of price competition, if they are quantities, it is one of quantity competition. To keep the model analytically tractable, we rely on quadratic

profit functions

$$\pi^i(s_i, s_j) = s_i(a_i - bs_i + ds_j), \quad i, j = 1, 2, i \neq j,$$

where  $|d| < b$  and the random demand variables  $a_i$  are independent and identically distributed on the support  $[\underline{a}, \bar{a}]$ . To avoid negative quantities or prices for  $d < 0$ , we have to assume that  $2b\underline{a} + d\bar{a} > 0$ . The parameter  $d$  measures the degree of substitutability of products.<sup>1</sup> If  $d = 0$ , the price of each variety depends only on the quantity of the variety produced and vice versa. In the case of quantity competition, the products of the two firms are substitutes (complements) according to whether  $d < 0$  ( $d > 0$ ). In the case of price competition, they are substitutes (complements) depending on  $d > 0$  ( $d < 0$ ).

The game consists of three stages. In the first stage, each firm  $i$  receives private information about the realization of its demand parameter  $a_i$ . In the second stage, it can send a message  $\hat{a}_i \in [\underline{a}, \bar{a}]$  about this parameter. After receiving the rival's message, firms simultaneously compete in quantities or prices in the third stage of the game. The payoffs are the resulting profits net of the message cost.

As a benchmark we will first derive the ex ante expected firm profits in the two cases of truthful information sharing and information concealing.<sup>2</sup>

## 2.1 Equilibrium in Case of Private Demand Information

If firms conceal their private information, the game is one of quantity or price competition with incomplete demand information and the expected profit of firm  $i$  is

$$E^i \pi^i = s_i(a_i - bs_i + dE^i(s_j)), \quad i, j = 1, 2, i \neq j.$$

Since the firms' demand variables are assumed to be identically distributed, it is easy to solve for the firms' strategies in the Bayesian equilibrium, given their private

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<sup>1</sup> We treat  $d$  as a constant parameter. As an alternative,  $d$  could be treated as a random variable as well (see, e.g. Chokler et al. 2006).

<sup>2</sup> As Raith (1996) has shown, it is never optimal for a firm to only partially share information by sending a signal with intermediate noise to the rivals.

demand information. The first-order conditions lead to the equilibrium strategies

$$s_i = \frac{(2b-d)a_i + dE(a)}{2b(2b-d)}, \quad i = 1, 2.$$

The resulting expected profits are

$$E_p^i \pi^i = b \left( \frac{(2b-d)a_i + dE(a)}{2b(2b-d)} \right)^2, \quad i = 1, 2,$$

where the subscript  $p$  denotes the case of private information. The ex ante expected profits, before firms have learned their own demand information, are

$$EE_p \pi = \frac{b}{(2b-d)^2} E(a)^2 + \frac{1}{4b} Var(a), \quad (1)$$

where  $Var(a) = E(a^2) - E(a)^2$  denotes the variance of the demand distribution.

## 2.2 Equilibrium in Case of Truthful Information Sharing

In case of a truthful exchange of demand information the firms' Nash-equilibrium strategies in the third stage are

$$s_i = \frac{2ba_i + da_j}{4b^2 - d^2}, \quad i, j = 1, 2, i \neq j,$$

and lead to the profits

$$\pi_s^i = b \left( \frac{2ba_i + da_j}{4b^2 - d^2} \right)^2, \quad i, j = 1, 2, i \neq j,$$

where the subscript  $s$  denotes the case of information sharing. If the disclosure of information is anticipated, the ex ante expected profits are

$$E\pi_s = \frac{b}{(2b-d)^2} E(a)^2 + \frac{b(4b^2 + d^2)}{(4b^2 - d^2)^2} Var(a). \quad (2)$$

The difference  $\Delta \equiv E\pi_s - EE_p \pi$  indicates whether truthful information sharing increases the firms' expected profits at a point in time where own demand parameters

are still unknown. If firms are risk-neutral they prefer truthful information sharing to information concealing if  $\Delta > 0$ .

*Proposition 1:* Before firms have learned the realization of their demand parameters, they strictly prefer truthful information sharing to information concealing, independently of whether the products are substitutes or complements.

*Proof:* From (1) and (2), the difference between the ex ante expected profits is

$$\Delta = \frac{(12b^2 - d^2)d^2}{4b(4b^2 - d^2)^2} \text{Var}(a) > 0 \quad \forall |d| \in (0, 1]. \quad (3)$$

Of course, if the decision variables  $s_i$  are strategically independent, i.e.  $d = 0$ , there is no gain from information sharing. In cases of strategic substitutes or complements, however, firms strictly prefer a truthful information transfer to a concealment of information. The expected gains are increasing in the variance  $\text{Var}(a)$  and in the substitution parameter  $|d|$ . The maximum expected gains can be realized for the values  $d = b$  (including the case of price competition for market shares) and  $d = -b$  (including the case of Cournot competition).

### 3 The Truth-Telling Problem

The preceding analysis of the expected gains from information sharing refers to a situation where firms do not know the realization of own demand variables. Once they have learned their private demand information they have an incentive to strategically lie. To demonstrate this effect, we assume that each firm, after having learned its private demand information, can send a message  $\hat{a}_i$  about  $a_i$ , whereby it is not confirmed to the truth.

Maximizing the expected profits

$$E^i \pi^i = s_i(a_i - bs_i + dE^i(s_j)), \quad i, j = 1, 2, i \neq j,$$

leads the reaction functions

$$s_i = (a_i + dE^i(s_j))/(2b), \quad i, j = 1, 2, i \neq j.$$

At the same time firms take expectations on each rival's strategy according to

$$E^i(s_j) = (E^i(a_j|\hat{a}_j) + dE^j(s_i|\hat{a}_i))/(2b), \quad i, j = 1, 2, i \neq j.$$

Inserting this expectation into the reaction functions gives

$$s_i = (2ba_i + dE^i(a_j|\hat{a}_j) + d^2E^j(s_i|\hat{a}_i))/(4b^2), \quad i, j = 1, 2, i \neq j. \quad (4)$$

Since the profit function is quadratic, it proves convenient to adopt Radner's (1962) approach and assume linear solution equations of the form

$$s_i = \xi_1 a_i + \xi_2 E^j(a_i|\hat{a}_i) + \xi_3 E^i(a_j|\hat{a}_j), \quad i, j = 1, 2, i \neq j,$$

implying that

$$E^j(s_i|\hat{a}_i) = (\xi_1 + \xi_2)E^j(a_i|\hat{a}_i) + \xi_3 E^i(a_j|\hat{a}_j), \quad i, j = 1, 2, i \neq j.$$

Substituting these two expressions into (4) enables us to equate the yet unknown coefficients to find  $\xi_1 = 1/(2b)$ ,  $\xi_2 = d^2/[2b(4b^2 - d^2)]$ , and  $\xi_3 = d/(4b^2 - d^2)$ , implying the strategies

$$s_i = \frac{(4b^2 - d^2)a_i + d^2 E^j(a_i|\hat{a}_i) + 2bd E^i(a_j|\hat{a}_j)}{2b(4b^2 - d^2)}, \quad i, j = 1, 2, i \neq j.$$

The expected profits are

$$E^i \pi^i = \frac{[(4b^2 - d^2)a_i + d^2 E^j(a_i|\hat{a}_i) + 2bd E^i(a_j|\hat{a}_j)]^2}{4b(4b^2 - d^2)^2}, \quad i, j = 1, 2, i \neq j. \quad (5)$$

*Proposition 2:* Without a costly signaling mechanism, firms have no incentive to send true messages about their private demand information. Instead, each firm has an incentive to signal the highest possible value of its demand parameter, i.e.  $\hat{a}_i = \bar{a}$ .

*Proof:* It is clear that  $E^j(a_i|\hat{a}_i) \in [\underline{a}, \bar{a}]$ ,  $i, j = 1, 2, i \neq j$ . Hence, the derivative

$$\frac{dE^i \pi^i}{dE^j(a_i|\hat{a}_i)} = \frac{(4b^2 - d^2)d^2 a_i + 2bd^3 E^i(a_j|\hat{a}_j) + d^4 E^j(a_i|\hat{a}_i)}{2b(4b^2 - d^2)^2}$$

is positive for all  $a_1, a_2, E^1(a_2|\hat{a}_2), E^2(a_1|\hat{a}_1)$ , given our assumption that  $2b\underline{a} + d\bar{a} > 0$ . This implies that each firm has an incentive to signal the highest possible value of its demand parameter. Since these signals are not credible, rivals will not react on them. What is needed for a truth-telling perfect Bayesian equilibrium is a mechanism device that implements revelation of the private information.

## 4 Simultaneous Signaling of Demand Information

In this section we derive a signaling mechanism that indeed implements truth-telling behavior of both firms. The mechanism involves consistent beliefs of each firm about the rival's behavior. In the resulting sequentially rational equilibrium, truth-telling is not an ad hoc assumption as in the predecessor models of information sharing but a consequence of an incentive-compatible device, whereby it is in the best interest of the firms to send true messages.

In the first stage of the game, each firm receives private demand information. When sending a signal in the second stage, each firm has to take into account the implications on competition in the third stage. It is clear from Proposition 2 that each firm has an incentive to convince the rival that the realization of its demand variable  $a_i$  equals the highest possible value  $\bar{a}$ . This implies that in order to induce the firm to send the truthful lower message, it must incur some cost when announcing high demand. Adopting the procedure suggested by Ziv (1993), we therefore introduce a costly signaling device for firms to send a message about the realization of their demand parameter.

We start by defining the cost function  $f(\hat{a}_i)$  indicating the amount of money that firm  $i$  has to pay when sending the message  $\hat{a}_i$ . Such spending can be observed, for example, in dissipative advertising activities of firms - or all other activities appropriate to observably burn money.<sup>3</sup> The introduction of a costly signaling device defines a signaling game where the cost  $f(\hat{a}_i)$  of sending the message is the signaling cost. Of course, a firm does not have to send an explicit message at all. But if this strategy is interpreted as the worst possible case, such behavior will be dominated by sending a signal.

Taking into account the signal cost  $f(\hat{a}_i)$  when sending the message  $\hat{a}_i$ , the expected net profits (5) extend to

$$E_f^i \pi^i = \frac{[(4b^2 - d^2)a_i + d^2 E^j(a_i | \hat{a}_i) + 2bd E^i(a_j | \hat{a}_j)]^2}{4b(4b^2 - d^2)^2} - f(\hat{a}_i), \quad i, j = 1, 2, i \neq j, \quad (6)$$

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<sup>3</sup> As an alternative, rather than burning money, rivals may exchange transfer payments, thereby reducing net signaling cost.



where the subscript  $f$  denotes the expected profits net of signaling cost. We characterize a separating perfect Bayesian equilibrium in which the costly message serves as a signal of the demand parameter. Let  $E^j(a_i|\hat{a}_i)$  be the belief of firm  $j$  that relates firm  $i$ 's message  $\hat{a}_i$  to its demand parameter  $a_i$ . Thus, when firm  $i$  sends the message  $\hat{a}_i$  (or the signal  $f(\hat{a}_i)$ , respectively), it is inferred to be characterized by the demand parameter  $E^j(a_i|\hat{a}_i) \in [\underline{a}, \bar{a}]$ . A perfect Bayesian equilibrium requires that for each firm  $i$  the expected profit (6) is maximized with respect to the message  $\hat{a}_i$ . In addition to these incentive compatibility constraints, firms' beliefs must be consistent with the equilibrium play, that is,  $E^j(a_i|\hat{a}_i) = a_i$ . It is straightforward to derive the incentive-compatible signal-cost function in the perfect Bayesian equilibrium.

*Proposition 3:* In the separating perfect Bayesian equilibrium the signal-cost function is determined by

$$f(a_i) = \frac{bd^2(a_i^2 - \underline{a}^2) + d^3(a_i - \underline{a})E(a)}{(4b^2 - d^2)^2} > 0, \quad i = 1, 2, \quad \forall |d| \in (0, 1].$$

*Proof:* The expected profits (6) are maximized when the first-order conditions hold, i.e.

$$\frac{dE^i \pi_f^i}{d\hat{a}_i} = \frac{(4b^2 - d^2)d^2 a_i + 2bd^3 E^i(a_j|\hat{a}_j) + d^4 E^j(a_i|\hat{a}_i)}{2b(4b^2 - d^2)^2} \cdot \frac{dE^j(a_i|\hat{a}_i)}{d\hat{a}_i} - \frac{df(\hat{a}_i)}{d\hat{a}_i} = 0.$$

In order to induce consistent beliefs, these conditions must be fulfilled at  $\hat{a}_i = a_i$ , implying  $E^j(a_i|\hat{a}_i) = a_i$  and, hence,  $\frac{dE^j(a_i|\hat{a}_i)}{d\hat{a}_i} = 1$ . Since the information transfer is assumed to be simultaneous, firm  $i$  does not know  $a_j$  when sending its signal, so that  $E^i(a_j|\hat{a}_j) = E^i(a_j) = E(a)$ . Using these expressions, we can solve for the marginal signal-cost function

$$\left. \frac{df(\hat{a}_i)}{d\hat{a}_i} \right|_{\hat{a}_i = a_i} = \frac{2bd^2 a_i + d^3 E(a)}{(4b^2 - d^2)^2}, \quad i = 1, 2.$$

Integrating and inserting the initial condition  $f(\underline{a}) = 0$  leads to the signal-cost function

$$f(a_i) = \frac{bd^2(a_i^2 - \underline{a}^2) + d^3(a_i - \underline{a})E(a)}{(4b^2 - d^2)^2}, \quad i = 1, 2, \quad (7)$$

which is positive for all  $|d| \in (0, b]$ , given our assumption that  $2b\underline{a} + d\bar{a} > 0$ . Thus, each firm invests a positive amount of money if its realized demand parameter is higher than the worst one, i.e.  $a_i > \underline{a}$ .

If firms agree to implement the derived signaling mechanism, information sharing results as a feature of the equilibrium strategy. Each firm reveals its private demand information because this behavior maximizes its expected profit. Consequently, the messages are credible and obviously taken into account by firms when setting quantities or charging prices in the last stage. A crucial condition for the firms to implement the signaling device is, however, that the expected signal cost does not overcompensate the expected gross gains from a truthful information exchange.

## 5 Conditions for Implementing the Signaling Device

Whether firms will reach an agreement on implementing the proposed signaling device depends on whether the expected gross gains from information sharing exceed the expected signaling cost. Having solved for the signal-cost function, it is straightforward to derive net gains.

*Proposition 4:* The expected gross gains from information sharing exceed the expected signal cost if and only if

$$(8b^2 - d^2)d^2Var(a) - 4bd^2[(b + d)E(a)^2 - b\underline{a}^2 - d\underline{a}E(a)] > 0. \quad (8)$$

*Proof:* In the separating perfect Bayesian equilibrium the expected value of the signal cost as given in (7) is

$$Ef(a) = \frac{bd^2(E(a^2) - \underline{a}^2) + d^3(E(a) - \underline{a})E(a)}{(4b^2 - d^2)^2}.$$

Using the expression for  $\Delta$  in (3), we derive the expected net gains from signaling true demand information as

$$\Delta - Ef(a) = \frac{(12b^2 - d^2)d^2Var(a) - 4bd^2[b(E(a^2) - \underline{a}^2) + d(E(a)^2 - \underline{a}E(a))]}{4b(4b^2 - d^2)^2}.$$

Thus, whether information signaling will occur or not, depends decisively on the degree of product differentiation as well as on the first two moments of the distribution function of the random demand variables.

In order to make sharper predictions we present a parametric specification by assuming that the random demand variables are gamma distributed on the support  $[\underline{a}, \bar{a}] = [0, \infty]$ .<sup>4</sup> The rather general gamma distribution seems to be an appropriate example for the empirical description of demand uncertainty since it is nonnegative and is skewed to the right, i.e. to the realization of high values of the demand variables. Its probability density function is

$$g(a) = \frac{n^m}{\Gamma(m)} a^{m-1} e^{-na} , \quad \Gamma(m) = \int_0^\infty x^{(m-1)} e^{-x} dx ,$$

with the shape parameter  $m > 0$  and the inverse scale parameter  $n > 0$ . The distribution has mean  $E(a) = m/n$ , variance  $Var(a) = m/n^2$ , and skewness  $Skew(a) = 2/\sqrt{m}$ .

*Proposition 5:* In case of the gamma distribution, the expected gross gains from signaling exceed the expected cost of signaling only if the skewness of the distribution is sufficiently large (small values of  $m$ ) and the products are sufficiently differentiated (small values of  $d$ ).

*Proof:* Inserting mean and variance as well as the limits of the support into (8) leads to the inequality

$$m < \frac{8b^2 - d^2}{4b(b + d)} . \quad (9)$$

In order for  $d$  to be an indicator of market heterogeneity we define  $b \equiv 1 + d$  such that the profit functions read as  $\pi^i(s_i, s_j) = s_i(a_i - s_i + d(s_j - s_i))$ . In the limit case of  $d = 0$  the heterogeneous market is completely separated into two monopoly markets where firms do no longer interact. In the opposite limit case with  $d \rightarrow \infty$  the market approaches homogeneity. Using this specification, condition (9) simplifies to

$$m < \frac{8 + 16d + 7d^2}{4 + 12d + 8d^2} , \quad (10)$$

where the right-hand side is monotonically decreasing in  $d$ , i.e. increasing in market heterogeneity.

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<sup>4</sup> Ziv (1993) has demonstrated his results by using a binary distribution of the unit cost of production. A discrete probability function, however, is not an appropriate example for a mechanism relying on the revelation of information about continuous random variables.

Figure 1: Parameter range for information signaling

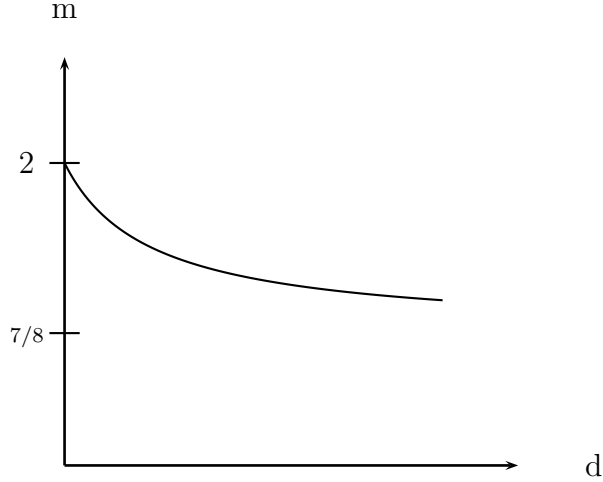


Figure 1 illustrates the region in the two-dimensional parameter space  $(d, m)$  where the condition for expected profitability of the information sharing device by costly signaling is fulfilled. Obviously, inequality (10) is generally fulfilled if  $m \leq 7/8$ , but never fulfilled if  $m > 2$ . For all intermediate values of  $m$  the parameter  $d$  has to be sufficiently small. As an example, in the case of  $m = 1$ , where the gamma distribution degenerates to the exponential distribution with p.d.f.  $g(a) = ne^{-na}$ ,  $d < 2(\sqrt{2} + 1) \approx 4.83$  must hold for (10) to be fulfilled.

Thus, in the basic model of duopolistic price competition with substitute products, firms will agree to implement the proposed signaling device and will reveal their private demand information by sending true messages, if the products are sufficiently differentiated.

## 6 Conclusion

The revelation of private demand information increases the expected profits of competitors in a market, and consequently firms are interested in sharing this information. The information exchange, however, cannot be done without a truth-telling mechanism, since firms could realize even higher gains by claiming to be larger than they really are. There are at least three channels through which firms can infer

the private demand information of rivals. The first is the existence of an outside institution, such as a trade association, which is able to transfer true information. However, even if such an institution may exist in a regulated environment it will hardly emerge in a competitive market. The second channel is an intertemporal transmission of private demand information. Of course, in a repeated game of competition with serially correlated or even constant demand parameters, firms can simultaneously signal their private information by strategic price and quantity decisions in previous periods (see, e.g., Caminal 1990). This paper has presented a third channel by introducing a two-sided signaling mechanism implementing information revelation even in a one-period (but multi-stage) game. Due to the cost of signaling, firms will agree to apply such a mechanism only if the expected gains from information sharing exceed the expected cost of signaling. We have shown that in a basic model of price competition this condition is fulfilled if the gamma distribution of the random demand variables is sufficiently skewed to the right and/or the market under consideration is sufficiently heterogeneous. Thus, in the absence of an outside institution and in a context where intertemporal learning is disabled due to serially uncorrelated demand variables, signaling may turn out as an advantageous strategy for firms to truthfully exchange private demand information.

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