# Markets for Ideas, Asymmetric Information and the Allocation of Managerial Skills*

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**Abstract**

The success of a product is often jointly determined by the usefulness of an idea and managerial skill, which are in a complementary relationship. We postulate that consumers and incoming managers cannot observe these sources of success, but rely on profit data to form their beliefs. Incoming managers take over firms, replace outgoing managers, and acquire ideas. The sales price of an idea increases with the prior to purchase unobservable usefulness of an idea. Though managerial skills are endogenously positively correlated over time, useful ideas end up under the control of better skilled managers. The market for ideas leads to positively assortative matches even though asymmetric information constitutes a friction in the matching of ideas and managers.

**Keywords:** Allocation of ideas, innovation, entrepreneurship, asymmetric information, ownership change, reputation transfer.

**JEL-classification:** L14, L15, G34.

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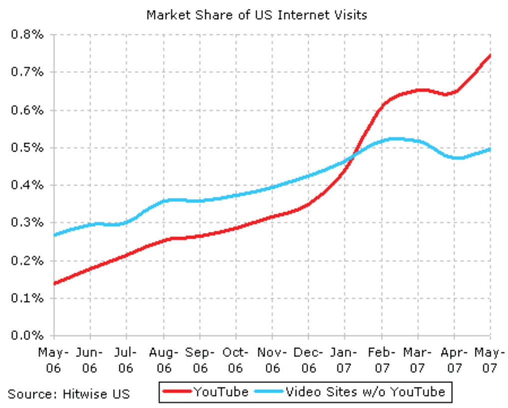
# 1 Introduction

On November 13, 2006, Google acquired YouTube for $1.65 billion. YouTube had only been founded in February 2005. Soon after the acquisition, both the monthly growth and the market share of YouTube video views overtook that of competitors (see figure 1). YouTube's revenues in 2007 were not material. But soon after the acquisition, YouTube started to show commercials before regular videos. For 2012, CitiBank estimates revenues at $1 billion.

Figure 1: Growth of YouTube Market Share




Has Google been lucky? In this paper, we develop a theory of idea trading. It suggests that the result may only partly be attributable to luck. The other part stems from a better match of ideas and managerial skill since a successful combination of idea and managerial skill is needed for firms to prosper. When planning the deal, Google could rely on publicly available information. In particular, Youtube's customer base was observable and Google may have partly inferred the quality of the underlying quality the Youtube's technology. Google could bring in managerial expertise that Youtube possibly was missing, and it is due to the institution of a *market for ideas* that firm performance improved. The rents that are generated through the rematching are partly transferred to the owner of the idea through the price Google paid to Youtube's outgoing owners.

In this paper we develop a theory of idea trading under asymmetric information. To fix ideas, initially there is an innovator (in the example, YouTube) who also acts as a manager. After some period, ideas are offered for sale. We do not require that innovators lack managerial skills, but postulate that a good innovator is not necessarily a good manager. In addition, we postulate that there is a complementary relationship between the usefulness of an idea and managerial skill: It is better for society that a good idea is skillfully managed and a bad idea less so than the reverse.

Idea and managerial skill are complements, jointly determining the consumer valuation of a product. A reallocation of managerial skill and ideas is therefore efficiency increasing and ideas are sold to managers (in the example, to Google).

Given the complementary relationship, highly skilled managers would like to purchase good ideas; less skilled managers are left with worse ideas. However, immediate perfect sorting requires that incoming managers perfectly observe the usefulness of an idea. Under perfect observability, the managerial skill of the innovator would be immaterial to the allocation after the market for ideas has cleared. More realistically, as we consider in this paper, the usefulness of an idea is unobservable to outsiders, i. e. consumers of the product and incoming managers who are the potential buyer of the idea. It appears to be an open question how the right combination of ideas and skills are formed over time under asymmetric information.

Outsiders only observe outcomes from the previous period (e.g., consumers report the quality of the outcome they observe and this becomes public information) and can try to make inferences from this observation. In market equilibrium, highly skilled managers acquire firms with successful products in the previous period. Since high profits in the previous period are more likely to have been resulted from an innovator with high managerial skills, skills of predecessors and successors are positively correlated. Furthermore, a better idea sells at a higher price. In a sense, the reputation of the innovator is transferred to the succeeding manager.

Let us look at the argument in more detail: We have to understand both the product market and the market for ideas. After a new product is released on the market, customers can judge its usefulness. However, a failure of the product does not necessarily imply that the underlying technology is not appropriate. The failure may rather be due to insufficient managerial skills of the innovator. This means that consumers face a signal extraction problem since they only observe the overall performance of the product. The start-up is then taken over by an investor who has the same information as customers. This investor obtains access to the innovator's idea and contributes with his own managerial skills, or appoints his own managers. While we may have some faith in the active screening role of the incoming investor cum manager (i. e., his expertise to judge the merits of a new technology) we take a more sceptical approach and ask whether the market system as such provides a (possibly imperfect) self-selection mechanism that improves upon the initial combination of idea and managerial skill. We may attribute higher average managerial skills to incoming investors/managers vis-à-vis outgoing innovators; however, we believe it is important to ask whether a better allocation due to a more productive input combination is the outcome of sorting in market equilibrium and not simply due to a better distribution of incoming managers.

After the transfer of ownership to the incoming manager, consumers are offered a new product which results from the combination of the previously developed idea and current managerial skills. Consumers repeatedly purchase (or there is some word-of-mouth between consumers) allowing them to update their beliefs after purchase.

While low-skilled managers taking over successful ideas are initially not detected, repeat purchases rely on previous experience which is linked to the underlying type of the manager. This gives stronger incentives for skilled incoming managers to take over those start-ups that showed a good performance. Since initial good performance relies not only on the technology but also on the managerial skills of the innovator, this skill is valuable to the innovator as it affects the price paid by the acquiring investor cum manager. Hence, (because of the signal extraction problem) in market equilibrium there is a positive correlation between the managerial skill of the innovator and the manager, although this managerial skill is initially unrelated to the "quality" of the idea. Also there is a positive correlation between technology and the managers' skill. This shows that the market mechanism improves on the initial allocation, but does not implement the first best. While the initial allocation is characterized by random matching between technology and managerial skills, the situation after ownership change is characterized by positive assortative matching.

We consider a model in which a firm is sold after the first period, and the usefulness of the idea is not revealed, but can only partially inferred from product performance. Our findings are as follows:

1. After the sale of the firm there is a positive correlation between managerial skill from period 1 onward and the quality of the technology.
2. The managerial skill in period 0 positively affects the sale price of the firm although it does not affect future performance.
3. There is a positive correlation between managerial skills in periods 0 and 1.
4. The sale of firms improves on the initial allocation and increases social welfare.

The first finding is not specific to our model. This positive correlation would even hold if incoming managers learnt the quality of the technology directly before deciding whether to buy the firm.[1] However, in a world in which the quality of the technology becomes publicly known, there is no role for managerial skills in period 0 to affect future allocations. It thus would not affect the sale price of the firm. Thus, our second finding is due to our assumption of asymmetric information. The underlying argument is that more managerial skills make good performance more likely. Due to the signal extraction problem this makes consumers trust the used technology and, therefore, the innovator's technology becomes more valuable. The lack of full information also lies at the heart of our third finding. An initial owner with higher skills will produce better products in expectations. Consumers observe only the product's performance, not the innovator's managerial skills. Therefore, they attribute some of that performance to the usefulness of the idea. But incoming managers, when deciding which firm to buy, prefer more useful ideas. The higher their own skills, the higher their willingness to pay for good ideas. As a consequence,

[1]We analyze such an alternative model in Section 4.2.

better managers end up buying better performing firms, which tend to be firms that are initially run by high-skill innovators. With respect to the social desirability of the trade in firms, we note that initially ideas and managerial skills are matched randomly. Since all good ideas tend to be bought by highly skilled managers, ideas and managerial skills end up being partly assortatively matched, which increases social welfare because of the input complementarity.

The remainder of the paper is organized as follows. After a short literature review, the following section 2 introduces the base model. Section 3 characterizes and discusses the equilibrium with positively assortative matching on the market for firms. Section 4 contains some model extensions. Section 5 concludes.

**Literature.** Our paper introduces a mechanism how "reputation" can be transferred from a manager to his successor when a firm is sold. Hence, our paper is related to other work on reputation transfer. In the seminal paper by Tadelis (1999), consumers buy products from firms. The owners of firms change unobservedly, hence reputation does not evaporate immediately with the trade of a firm. This leads to (partial) sorting, which in effect leads to reputation transfer. By contrast, trading the firm is observable in our model, but one of the factors of production is a long-run factor. This also leads to partial sorting, and to reputation transfer.

Morrison and Wilhelm (2004) look at the incentives to pass on human capital within a firm. As a consequence, also the reputation will be passed on. This phenomenon is absent in our model. In Levin and Tadelis (2005), profit sharing between team members plus reputation pooling alleviates moral hazard problems, especially at the end of a manager's career. Also Bar-Isaac (2007) models teams, where the ability of each team member cannot be observed separately. Each member has thus an incentive to work hard, because this improves also the reputation of his team mates. At retirement, they can then sell out at a higher price. Also in our setting, the types of innovators and technologies can only jointly be determined. However, our focus is on the sorting properties of the equilibrium. In Hakenes and Peitz (2007), firms acquire a customer base over some time. But losing good customers is easy, hence on the market for firms, better managers have an incentive to bid more for better firms. Again, there is partial sorting and, as a consequence, reputation transfer. A key feature of that paper is that reputation is only local. Tadelis (2002) shows that a mechanism of reputation transfer increases incentives to work, even at the end of ones career. This augments ones own reputation, which can then be sold to the next generation. We obtain a related result in section 4.3.[2] In contrast to previous work our paper considers the dynamics over a product's life cycle.

A different strand of related literature deals with the match between skilled managers and projects. For example, Gabaix and Landier (2008) hold the match between tasks

[2]Tadelis (2003) looks at prices for reputations, again in the context of reputation transfer. Andersson (2002) considers the transfer of reputation between products in the context of umbrella branding. The same applies for Hakenes and Peitz (2008), in a moral hazard problem.

and talent of CEOs responsible for the steep increase in CEO pay (before the crisis). Acharya, Pagano, and Volpin (2010) provide an example where skilled financial managers are matched to risky projects. If there is a mismatch, risk endogenously increases, possibly leading to crises.[3]

## 2 The Model

Consider an economy with three types of agents: *innovators*, *entrepreneurs cum managers*, and *consumers*. Innovators and managers use an *idea*; we may think of this as the technology of production. There are three dates ($t \in \{0, 1, 2\}$). In period $t = 0$, innovators produce and sell one unit to each consumer. At the beginning of period $t = 1$, innovators retire and sell their firm/idea to an entrepreneur cum manager.[4] In $t = 1$ and $t = 2$ each, managers produce one unit of the redesigned product and sell it to consumers. All agents are risk neutral and have perfect recall, there is no discounting.

*Innovators* have different types $S_0$, standing for the managerial quality they achieve. $\log S_0$ is normally distributed with mean 0 and variance $\sigma_0^2$, thus type $S_0$ is lognormally distributed with $\log S_0 \sim N(0, \sigma_0^2)$. The mean of $S_0$ is hence $e^{\sigma_0^2/2}$. Innovators can produce in period $t = 0$, but not in $t = 1$ and $t = 2$.

Also *managers* have different types $S_1$, standing for quality, which is lognormally distributed with $\log S_1 \sim N(0, \sigma_1^2)$. Managers can produce in period $t = 1$ and $t = 2$, but not in $t = 0$.

Finally, both innovators and managers must use an idea as a long-run factor, which we call technology. *Technologies* also come in different quality types $T$, lognormally distributed with $\log T \sim N(0, \tau^2)$. All random variables $S_0, S_1$ and $T$ are stochastically independent.

In the production process, the types of innovators (or managers) and technologies are complements. If an innovator of type $S_0$ uses a technology of type $T$, the product quality is $Q = S_0\,T$. If a manager of type $S_1$ uses technology $T$, the product quality will be $Q = S_1\,T$.

*Consumers* have unit demand for the product in each period. There is a unit mass of consumers. Goods are experience goods; consumers cannot assess the quality before consumption. They demand one unit of the product if the price is equal or below its expected valuation, otherwise they demand zero.

[3]The list and discussion of literature expanded further. For example, Andersson, Freedman, Haltiwanger, and Shaw (2009) Abowd, Haltiwanger, Lane, McKinney, and Sandusky (2007) Baker, Gibbs, and Holmström (1993, 1994a,b); Baker and Holmström (1995) should be mentioned.

[4]Note that this analysis also applies to take-over by other firms or investors who put a new manager in charge. In that case, we implicitly assume that the incentives of this firm or investor are perfectly aligned with the incentives of the new manager.

Table 1: Time Line

0 Each innovator uses its technology to produce one product and sell it to consumers.
Consumers experience the product and update their beliefs. Consumer experience becomes public information.

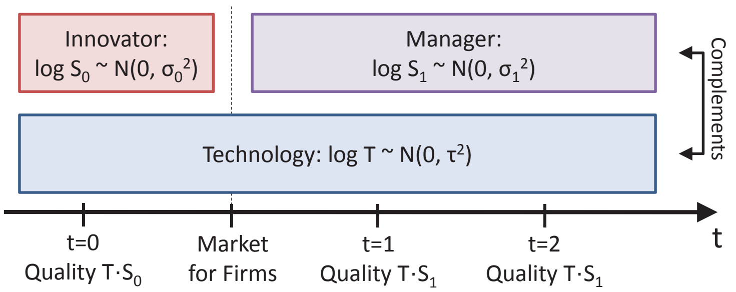
Market for firms: Innovators announce prices $P_0$ for their firms/technologies, managers decide whether to buy.

1 Managers use the technology to produce one product and sell it to consumers.
Consumers experience the product and update their beliefs.

2 Managers use the technology to produce one product and sell it to consumers.
Consumers experience the product and update their beliefs.

We solve for perfect Bayesian equilibria. The market for firms, however, is competitive. Furthermore, firms are assumed to set retail prices such that they extract the full expected surplus of the consumers. The time line is given by table 1. An illustration of the main modeling ingredients and the timing is given in figure 2.

Figure 2: Graphic Illustration of the Model




# 3 Equilibrium Characterization

**The Product Market in Period $t = 0$.** At the beginning of that date, innovators and technologies are matched randomly. Because $\log S_0$ and $\log T$ both are normally distributed and stochastically independent, also $\log S_0 + \log T$ is normally distributed and variances are added up. Namely, $\log S_0 \sim N(0, \sigma_0^2)$ and $\log T \sim N(0, \tau^2)$, hence $\log(S_0 T) = (\log S_0 + \log T) \sim N(0, \sigma_0^2 + \tau^2)$. At date 0, the product quality $Q_0 = S_0 T$ is hence lognormally distributed, with an expected value of

$$E[Q_0] = e^{\frac{\sigma_0^2 + \tau^2}{2}}. \tag{1}$$

Because consumers have unit demand, they will demand zero if the price exceeds the expected quality, otherwise they demand exactly one unit. The innovator sets the price at the expected valuation of consumers. Because there are no production costs and consumers are of unit mass, profits at date 0 are

$$\Pi_0 = E[Q_0] = e^{\frac{\sigma_0^2+\tau^2}{2}}. \tag{2}$$

These profits depend neither on the managerial skill of the innovator $S_0$ nor on the quality of the technology $T$ because both are not known to consumers at this date. This will change after date 0, when the product has been bought and consumed, and the quality $Q_0 = S_0 T$ has become publicly known.[5] At the end of date 0, the innovator sells the firm. Hence, the innovator's type will no longer be relevant for any firm, but the technology as the long-run factor remains important for the success of the product. Therefore, managers and consumers are concerned about the technology's type, which they may partially infer from the aggregate quality $Q_0$, making use of Bayes' rule. Note that managers only care about the technology's type because consumers learn: The manager's willingness to pay for the firm depends on the expected profit the new firm is going to derive, which only depends on the consumers' valuations. Managers and consumers face the same signal extraction problem. This problem is tractable because the logs of $S_0$, $T$ and $Q_0$ are normally distributed. The expected value of $T$, given the information $Q_0$ is updated to

$$E[\log T|\log Q_0] = \frac{1/\sigma_0^2}{1/\sigma_0^2 + 1/\tau^2}\log Q_0 = \frac{\tau^2}{\sigma_0^2+\tau^2}\log Q_0, \tag{3}$$

and the variance of $T$ is updated to

$$V[\log T|\log Q_0] = \frac{1}{1/\sigma_0^2 + 1/\tau^2} = \frac{\sigma_0^2\tau^2}{\sigma_0^2+\tau^2}. \tag{4}$$

How can we interpret these expression? If $S_0$ were known with certainty, i. e. $\sigma_0 = 0$, this would imply that $\log S_0 \sim N(0,0)$ and hence $\log S_0 = 0$, thus $S_0 = 1$. The expected value of $\log T$ would then be identical to $\log Q_0$, the product quality would perfectly reveal the type of the technology. Formally, $E[\log T|\log Q_0] = \log Q_0$. If, on the other hand, $T$ were known with certainty, i. e. $\tau = 0$, this would imply that $T = 1$. Because $T$ would already be known, nothing could additionally be learnt from the product quality $Q_0$. Formally, $E[\log T|\log Q_0] = \log 1 = 0$. These are two limiting cases. Since the expected value of a lognormal distribution with parameters $\mu$ and $\sigma$ is $e^{\mu+\sigma^2/2}$, the expected value of $T$ conditioned on $Q_0$ is

$$E[T|Q_0] = \exp\Big(\frac{\tau^2}{\sigma_0^2+\tau^2}(\log Q_0 + \frac{\sigma_0^2}{2})\Big). \tag{5}$$

The larger $\tau$ in relation to $\sigma_0$, the more consumers and managers can learn on the quality type $T$ from observing $Q_0$.

[5]While some consumers may learn from their experience, the assumption here is that this information is made publicly available, e. g. by the publication of consumer reports.

**The Firm Market before Period $t = 1$.** At the end of period 0, the market for firms opens. In the assortative equilibrium, better new managers buy the firms with better signals $Q_0$. Here, we only look at the consequences for the consumers' information. We will calculate the equilibrium price $P(Q_0)$ for a firm, depending on the observed product quality it has produced, further below.

The new managers' types are lognormally distributed, $\log S_1 \sim N(0, \sigma_1^2)$. Slightly abusing notation, let $S_1(Q_0)$ denote the type of manager who buys a firm that has produced quality $Q_0$. In the assortative equilibrium, $S_1(Q_0)$ is a strictly increasing function. Furthermore, because both $\log S_1$ and $\log Q_0$ are normally distributed, $\log S_1(\log Q_0)$ is a linear transformation of $\log Q_0$. This linear function must be such that it maps the mean of $\log Q_0$ onto the mean of $\log S_1$, and it transforms the standard deviation of $\log Q_0$ into the standard deviation of $\log S_1$. Since signals $Q_0$ have standard deviation $\sqrt{\sigma_0^2 + \tau^2}$ and types of managers have standard deviation $\sigma_1$, a firm with signal $Q_0$ will be mapped onto a manager with type

$$\log S_1(\log Q_0) = \frac{\sigma_1}{\sqrt{\sigma_0^2 + \tau^2}} \log Q_0, \quad \text{and thus}$$

$$S_1 = Q_0^{\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}}. \tag{6}$$

The mapping is derived under the assumption of assortative matching. This will have to be supported by a pricing function for firms, which we will be able to compute after knowing the product prices in period 1 and 2.

**The Product Market in Period $t = 1$.** Each consumer is concerned about the probability distribution over quality $Q_1$. What can he learn from the product performance in the previous period about $T$ and $S_1$? Each consumer has consumed at date 0 and thus knows $Q_0$. Therefore, he updates his beliefs about mean and variance of $\log T$ as in (4) and (3). Furthermore, because he knows that under assortative matching firms which performed better in period 0 are matched with more skilled managers at the end of date 0, he can infer $S_1$ from $Q_0$ making use of (6). Period 1 quality $Q_1|Q_0 = S_1|Q_0 \cdot T|Q_0$ will be distributed with

$$V[\log Q_1 | \log Q_0] = V[\log T | \log Q_0] = \frac{1}{1/\sigma_0^2 + 1/\tau^2} = \frac{\sigma_0^2 \tau^2}{\sigma_0^2 + \tau^2} \quad \text{and} \tag{7}$$

$$E[\log Q_1 | \log Q_0] = \Big(\frac{\tau^2}{\sigma_0^2 + \tau^2} + \frac{\sigma_1}{\sqrt{\sigma_0^2 + \tau^2}}\Big) \log Q_0 = \frac{\tau^2 + \sqrt{\sigma_0^2 + \tau^2}\,\sigma_1}{\sigma_0^2 + \tau^2} \log Q_0. \tag{8}$$

The new manager anticipates that consumers rationally hold these expectations and sets the price at which consumers are indifferent between buying and not buying. Because consumers' expectations are conditioned on $Q_0$, the product price at date 1 will also be conditioned on $Q_0$. Considering (8) and (7), profits at date 1 are

$$\Pi_1(Q_0) = E[Q_1|Q_0] = \exp\Big(\frac{\tau^2 + \sqrt{\sigma_0^2 + \tau^2}\,\sigma_1}{\sigma_0^2 + \tau^2} \log Q_0 + \frac{1}{2} \frac{\sigma_0^2 \tau^2}{\sigma_0^2 + \tau^2}\Big). \tag{9}$$

Here, the first fraction is due to the consumers' beliefs ($\tau^2$) and the assortative matching ($\sqrt{\sigma_0^2+\tau^2}\,\sigma_1$), the second fraction is due to the variance.

**The Product Market in period $t = 2$.** After the product market at date 1, $Q_1$ is public information. In the base version of the model management changes only once throughout the time horizon and thus the manager remains in the firm after period 1. Then the expected quality at date 2 will simply be the quality at date 1, $Q_2 = Q_1 = S_1\,T$. The manager will thus set the product price at $Q_2$, and the profit equals this price, $\Pi_2 = S_1\,T$.

**Firm valuations in period $t = 1$.** A manager who buys a firm that has produced quality $Q_0$ will earn $\Pi_1$ and $\Pi_2$ during the next periods. We can thus calculate his willingness to pay for such a firm, and thus calculate prices on the market for firms in equilibrium. Let $P_0(Q_0)$ denote the price investors pay in equilibrium for a firm to innovators, which depends on the public signal $Q_0 = S_0\,T$ at date 0. Then the expected profit of a manager of type $S_1$ is

$$\begin{aligned}\Pi_1 + E[\Pi_2] - P_0(Q_0) = \exp\Big(&\frac{\tau^2+\sqrt{\sigma_0^2+\tau^2}\,\sigma_1}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big)\\ &+ S_1\cdot\exp\Big(\frac{\tau^2}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big) - P_0(Q_0). \qquad (10)\end{aligned}$$

It must be optimal for the manager to pay the price $P_0$ for the firm. Which manager $S_1$ will buy a firm that has produced quality $Q_0$? In (6), we have argued that $S_1$ is a monotonic function of $Q_0$. We recall that quality $Q_0$ is "designated" for type $S_1$ with

$$S_1 = Q_0^{\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}}. \qquad (11)$$

In an assortative matching equilibrium, the pricing function $P_0(Q_0)$ should be such that type $S_1$ optimally chooses quality $Q_0$. Thus, considering the first-order condition, the marginal expected profit from a firm that has produced quality $Q_0$ must vanish,

$$\begin{aligned}0 = &\frac{\tau^2+\sqrt{\sigma_0^2+\tau^2}\,\sigma_1}{\sigma_0^2+\tau^2}\,\exp\Big(\frac{\sqrt{\sigma_0^2+\tau^2}\,\sigma_1-\sigma_0^2}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big)\\ &+ \frac{\tau^2}{\sigma_0^2+\tau^2}\,S_1\cdot\exp\Big(-\frac{\sigma_0^2}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big) - P_0'(Q_0). \qquad (12)\end{aligned}$$

Consequently, we must substitute (11) into (12), which yields

$$\begin{aligned}P_0'(Q_0) = &\frac{\tau^2+\sqrt{\sigma_0^2+\tau^2}\,\sigma_1}{\sigma_0^2+\tau^2}\,\exp\Big(\frac{\sqrt{\sigma_0^2+\tau^2}\,\sigma_1-\sigma_0^2}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big)\\ &+ \frac{\tau^2}{\sigma_0^2+\tau^2}\,Q_0^{\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}}\cdot\exp\Big(-\frac{\sigma_0^2}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big). \qquad (13)\end{aligned}$$

This differential equation must apply for the function $P_0(Q_0)$. If we can additionally identify one point $Q_0$ where we know the price $P_0(Q_0)$, we have a classical initial value problem and can solve for the complete function $P_0(Q_0)$.

For the worst quality $\log Q_0 = -\infty$, hence $Q_0 = 0$, the price must be $P_0(0) = 0$. This is seen as follows: According to (6), the associated manager type is $\log S_1 = -\infty$, hence $S_1 = 0$. This worst possible type does not have an incentive to pay anything for any technology because $S_1 T$ is always zero. He will never make any money at date 2, and he will buy the worst $Q_0$, hence he cannot sell at a positive product price at date 1.

We can now integrate both sides of (13) and explicitly solve for the pricing function

$$\begin{aligned} P_0(Q_0) = & \int_0^{Q_0} \Big[ \frac{\tau^2 + \sqrt{\sigma_0^2+\tau^2}\,\sigma_1}{\sigma_0^2+\tau^2} \exp\Big( \frac{\sqrt{\sigma_0^2+\tau^2}\,\sigma_1 - \sigma_0^2}{\sigma_0^2+\tau^2} \log Q + \frac{1}{2}\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big) \\ & + \frac{\tau^2}{\sigma_0^2+\tau^2}\, Q^{\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}} \cdot \exp\Big( -\frac{\sigma_0^2}{\sigma_0^2+\tau^2} \log Q + \frac{1}{2}\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big)\Big]\, dQ \\ & = \exp\Big(\frac{1}{2}\frac{\sigma_0^2\tau^2}{\sigma_0^2+\tau^2}\Big) \cdot \Big[\frac{\tau^4}{(\sigma_0^2+\tau^2)\,(\tau^2+\sigma_1\sqrt{\sigma_0^2+\tau^2})} + 1\Big] \cdot Q_0^{\frac{\tau^2+\sigma_1\sqrt{\sigma_0^2+\tau^2}}{\sigma_0^2+\tau^2}}. \end{aligned} \tag{14}$$

With this pricing function, one can immediately show that a manager of type $S_1$ does not benefit from buying a firm different from $S_1(Q_0)$ as given in (6).

**Summary – Equilibrium Characterization.** Summing up, we have shown that the price function $P_0(Q_0)$ constitutes a market equilibrium in which there is imperfect sorting. This is summarized by the following proposition.

**Proposition 1** *There is an equilibrium with imperfect sorting in which the equilibrium value of firms traded at the end of period 0 is given by (14).*

**Proof of proposition 1:** In our equilibrium, there is imperfect sorting because better managers buy firms with better quality, which tend to be founded by better innovators. Hence, it suffices to show that the first-order approach also fulfills the second-order condition, to see that the behavior of buyers is indeed optimal. Indeed, inserting (14) into (10), taking the second derivative with respect to $Q_0$, and respecting (11) yields

$$-\frac{\sigma_1\,\tau^2}{\sqrt{\sigma_0^2+\tau^2}^5} \exp\Big(\frac{\sigma_0^2\,\tau^2}{2\,(\sigma_0^2+\tau^2)}\Big)\, Q_0^{\frac{\tau^2}{\sigma_0^2+\tau^2} + \frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}} - 2} \tag{15}$$

as the second derivative. This is clearly negative. ■

The firm price depends on the produced quality $Q_0$: A higher observed quality $Q_0$ always leads to a higher price $P_0$. Since more skills in period 0 have a positive effect on $Q_0$, this established finding 2 of the introduction.

We are interested in the question whether "reputation" can be transferred from one generation of managers to the next. Put differently, are firms from better innovators likely to be bought by better managers? This question can be answered from two different perspectives. First, one can consider the correlation of $S_0$ and $S_1$, of the types of innovator and manager. However, the reputation of the innovator is the innovator's type $S_0$ expected by consumers, conditioned on the observed quality $Q_0$, and the manager's type $S_1$ expected by consumers, conditioned on the fact that the manager buys a firm with signal $Q_0$. The second perspective is thus to consider the conditional correlation. Let us start with the second perspective. The prior distribution of $S_0$ is $\log S_0 \sim N(0, \sigma_0^2)$. In analogy to (3) and (4), after a consumer has observed the quality $Q_0$, he updates the distribution to

$$\log S_0 | \log Q_0 \sim N\Big(\frac{\sigma_0^2}{\sigma_0^2+\tau^2}\,Q_0, \frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big). \tag{16}$$

The firm will then be bought by a manager with type $S_1$ as defined in (6). Therefore, the sorting between an innovator's logged reputation and the succeeding manager's expected logged ability is *perfect*. Now let us come back to the first perspective. What is the correlation between $S_0$ and $S_1$, or equivalently, between $\log S_0$ and $\log S_1$? The correlation between $\log S_1$ and $\log Q_0$ is perfect, consequently

$$\rho(\log S_0, \log S_1) = \rho(\log S_0, \log Q_0) = \frac{\sigma_0^2}{\sigma_0\,\sqrt{\sigma_0^2+\tau^2}} = \frac{1}{\sqrt{1+\tau^2/\sigma_0^2}}. \tag{17}$$

In the limiting case of $\tau \to 0$, the correlation converges to one. If the information on $T$ is rather precise, then $Q_0$ is nearly perfectly correlated with $S_0$. But still, managers with a higher $S_1$ buy firms with a higher initial quality $Q_0$. This means that good ideas are likely to be acquired by good managers. In the limiting case of $\sigma_0 \to 0$ or, equivalently, $\tau \to \infty$, the correlation converges to zero. If technology qualities are widely spread, then the signal $Q_0$ is heavily influenced by $T$, and relatively less influenced by $S_0$. Because of the noise, the correlation between innovator and manager types goes becomes low. Still, better managers buy firms with better qualities $Q_0$ and hence in a first-order stochastic dominance sense those with better skilled innovators (higher $S_0$). This establishes the following proposition:

**Proposition 2** *Managerial skills in periods 0 and 1 are positively (but not perfectly) correlated.*

Proposition 2 reflects finding 3 of the introduction. Since $S_1$ is an increasing function in $Q_0$ and there is a positive correlation between $\log T$ and $\log Q_0$, there is also a positive correlation between $\log T$ and $\log S_1$. This establishes finding 1 of the introduction. As we noted in the introduction, finding 1 is also preserved in a market in which asymmetric information is absent.

**Comparative statics.** The influence of the parameters $\sigma_0$, $\sigma_1$ and $\tau$ is more complex, however. The following findings are based on numerical calculations, they illustrate the influence of these parameters. For comparability, the blue curve has the parameters $\sigma_0 = 1$, $\sigma_1 = 1$ and $\tau = 1$ in all three figures reported below. Then in each figure, a different parameter is changed.

Figure 3 shows $P_0(Q_0)$ for different variances of $\log S_0$, with $\sigma_0 \in \{1, 2, 4\}$. The dashed curves give the limiting cases $\sigma_0 \to 0$ and $\sigma_0 \to \infty$. If the dispersion of $S_0$ is extremely wide ($\sigma_0 \to \infty$), then the quality $Q_0$ contains no information on the technology $T$. The product prices will not depend on $Q_0$. Therefore, the price for all firms is identical, $P_0 = e^{\tau^2/2}$. In the other extreme, if $\sigma_0 \to 0$, the type of the innovator is known, and the quality $Q_0$ perfectly reveals the technology type $T$, and the firm price reacts sensitively to the observed quality $Q_0$. For reaction of the firm price on $\sigma_0$ is ambiguous. If the observed quality is poor and $Q_0$ is small, then a large $\sigma_0$ obfuscates the information on $T$ and thus has a positive impact on the price $P_0$. The opposite is true for a larger observed quality $Q_0$.

Figure 3: Effect of $\sigma_0$ on the Firm Price

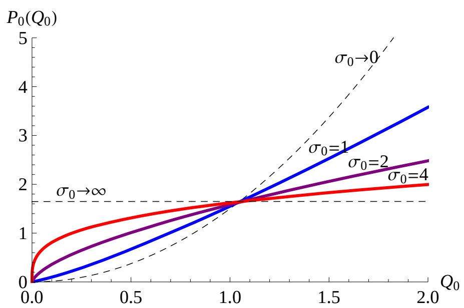
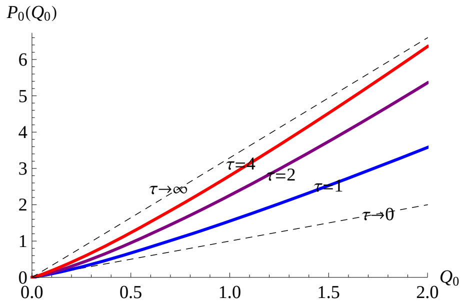



Figure 4 shows $P_0(Q_0)$ for different variances of $\log T$, with $\tau \in \{1, 2, 4\}$. The dashed curves give the limiting cases $\tau \to 0$ and $\tau \to \infty$. For $\tau \to 0$, the pricing function becomes $P_0(Q_0) = Q_0^{\sigma_1/\sigma_0}$. For the limit $\tau \to \infty$, it becomes $P_0(Q_0) = 2\,e^{\sigma_0^2/2}\,Q_0$. A comparison shows that the curve for $\tau \to \infty$ does not lie unambiguously above or below the curve for $\tau \to 0$. In general, the price therefore depends ambiguously on $\tau$. Thus, $\tau$ influences $P_0$ in several ways. First, if $\tau$ is large, then the variance of $T$ increases. It thus pays for managers to bid aggressively for firms in order not to end up with a bad quality. But second, as $\tau$ increases, the expected $T$ also increases, hence the expected $Q_0$ also increases. The same $Q_0$ is in a lower quantile if the distribution for larger $\tau$. A $Q_0$ that is a medium quality for low $\tau$ may be an extremely bad quality for larger $\tau$. Consequently, worse managers will aquire such a firm and the price that is paid for the firm will be lower. Figure 3 shows that the first channel may dominate, but this does not hold for all parameter constellations.

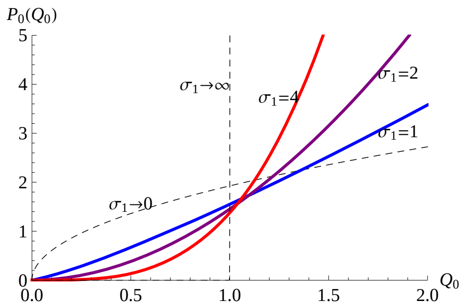
Finally, figure 5 shows $P_0(Q_0)$ for different variances of $\log S_1$, with $\sigma_1 \in \{1, 2, 4\}$. The dashed curves give the limiting cases $\sigma_1 \to 0$ and $\sigma_1 \to \infty$. For $\sigma_1 \to 0$, the function is proportional to a root of $Q_0$; for $\sigma_1 \to \infty$, it is zero for $Q_0 < 1$ and infinite

Figure 4: Effect of $\tau$ on the Firm Price




for $Q_0 > 1$. In between, $P_0$ increases in $\sigma_1$ for small qualities $Q_0$, and id decreases in $\sigma_1$ for larger qualities, with the following intuition. For a larger $\sigma_1$, there are more good managers with high skills, but also more worse managers with low skills. Due to the complementarity with their own skills, low-skilled managers will bid less for a firm with fixed technology. On the other hand, high-skilled managers bid more. This explains why, for higher $\sigma_1$, the curves moves down on the left side, and moves up on the right side.

Figure 5: Effect of $\sigma_1$ on the Firm Price




The price $P_0$ is a convex function of the initial quality $Q_0$ if

$$\sigma_1 > \frac{\sigma_0^2}{\sqrt{\sigma_0^2 + \tau^2}},$$

in words, if the dispersion between managers is large, that between technologies is large, and/or that between between innovators is small. Then, we have a superstar phenomenon. For mediocre firms that produce medium quality, the price is close to zero, but for some firms that manage to produce a good quality, the price can be very high. In the special case that the dispersion of managers and innovators is the same, $\sigma_0 = \sigma_1$, the price function is always a convex function of the initial quality.

**Welfare.** Especially when we want to compare the equilibrium with other scenarios, the welfare perspective will be interesting. In the welfare function, the transfer price $P(Q_0)$ cancels out because it is a mere transfer. Consumers pay exactly for the expected quality of products, hence their rent is zero. Thus, welfare consists only of the firm's profits. But firms have zero costs, thus welfare consists only of the firms' revenues. Finally, because the volume of products is fixed to one, welfare is identical to the sum of product prices from an ex-ante perspective,

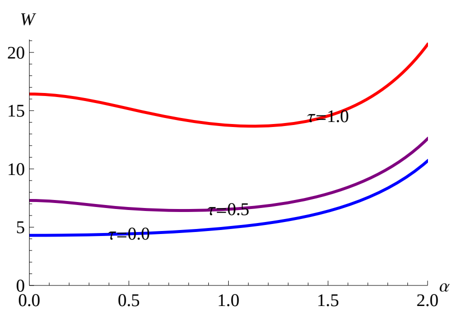
$$\begin{aligned} W &= \Pi_0 + E_0[\Pi_1] + E_0[\Pi_2] = \Pi_0 + 2\,E_0[\Pi_1] \\ &= \exp\left(\frac{\sigma_0^2+\tau^2}{2}\right) + 2\,E_0\left[\exp\left(\frac{\tau^2+\sqrt{\sigma_0^2+\tau^2}\,\sigma_1}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\right)\right]. \end{aligned} \tag{18}$$

Now because $\log Q_0$ is normally distributed with mean 0 and variance $\sigma_0^2+\tau^2$,

$$W = \exp\left(\frac{1}{2}\,(\sigma_0^2+\tau^2)\right) + 2\,\exp\left(\frac{1}{2}\,(\sigma_1^2+\tau^2) + \frac{\sigma_1\,\tau^2}{\sqrt{\sigma_0^2+\tau^2}}\right). \tag{19}$$

Welfare is increasing in $\sigma_1$ and $\tau$, but not necessarily in $\sigma_0$. The reason is, as mentioned above, that a larger $\sigma_0$ raises prices at date 0, but also creates noise in the sorting process. For large $\sigma_0$, the first effect always dominates, but for small $\sigma_0$, the second effect can be dominant, depending on parameters $\sigma_1$ and $\tau$. In figure 6, the welfare function is plotted for parameters $\sigma_1 = 1$ and $\tau \in \{0, 0.5, 1\}$, illustrating the possible non-monotonicity of $W$ in $\sigma_0$.

Figure 6: Welfare




For a welfare comparison, it is useful to consider as a benchmark a variation of the model where the firm is not sold at all. As we have shown, the market for firms is used to improve the sorting between technologies and managers. Hence, without the market and without sorting, the benefits from the complementarities will be smaller and welfare will be lower, provided that skills in the two periods are chosen from the same distribution. The equilibrium is straightforward. On the product market at date 0, the price will be $\Pi_0 = \exp((\sigma_0^2+\tau^2)/2)$. After $t = 0$, the quality $Q_0$ is revealed. Because the firm is not sold, the quality will stay at $Q_0$ for the next periods, hence the price at $t = 1$ and $t = 2$ will be $Q_0$. Taking expectations from date 0, expected prices are $E_0[\Pi_1] = E_0[\Pi_2] = \Pi_0 = \exp((\sigma_0^2+\tau^2)/2)$. Welfare thus amounts to

$$W = \Pi_0 + E_0[\Pi_1] + E_0[\Pi_2] = 3\,\exp\left(\frac{\sigma_0^2+\tau^2}{2}\right). \tag{20}$$

A comparison with (19) shows that the interim market improves welfare iff

$$\sigma_1 > \frac{\sqrt{\sigma_0^4 + \sigma_0^2\tau^2 + \tau^4} - \tau^2}{\sqrt{\sigma_0^2 + \tau^2}}, \tag{21}$$

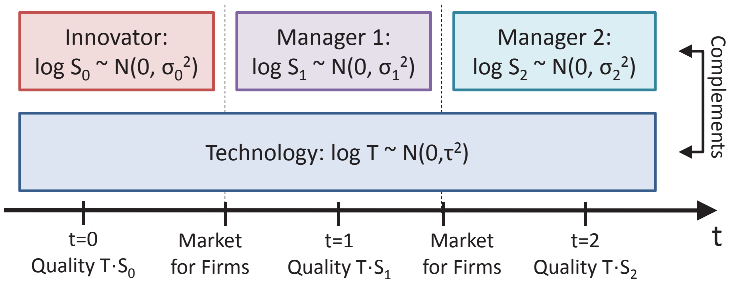
which again is smaller than $\sigma_0$. Because $E[S_0] = \exp(\sigma_0^2/2)$ and $E[S_1] = \exp(\sigma_1^2/2)$, the above condition has the following interpretation. If $\sigma_1 = \sigma_0$, i.e. if the skill distribution of new managers is the same as the one of innovators, creating a market for IPOs or takeovers always improves welfare because it leads to better matches and therefore achieves the benefits of the complementarity between managers and technologies. However, if $\sigma_0 \geq \sigma_1$ such that (21) held, innovators could do the job better than managers on average. Then introducing the sales of firms after period 0 would reduce welfare.

# 4 Extensions

## 4.1 Additional Trading of Firms

In the previous section, we assumed that the firm is traded only before date 1. Firms with better signals $S_0 T$ had a better expected performance. Thus, due to imperfect observability and complementarity, they were bought by better managers. Because the signal about technology was noisy, the matching between managers and technologies was not perfect. As we show in this subsection, by introducing an additional trading possibility, the matching is improved and welfare increases. In our setting, the matching is efficient already after the second market for firms.

Figure 7: Graphic Illustration of the Extension




For a formal analysis, let us assume that there is a third class of managers, running firms at date 2. Their types $S_2$ are lognormally distributed along $\log S_2 \sim N(0, \sigma_2)$.[6]

[6]Potentially, these managers could be the same as in period 1 but could be reallocated, such that $\sigma_2 = \sigma_1$.

In addition, there is a market for firms at the end of date 1. Figure 7 is an illustration of the timing and modeling ingredients.

We solve the model using backward induction. On the market at the end of date 1, the price $P_1$ for a firm will depends on expected future output, which again depends on observed qualities, thus $P_1(Q_0, Q_1)$. On the market at the end of date 0, the price $P_0$ depends on expected output in $t = 1$, the expected price at which the firm can be sold at date 1, both depending on the observed quality $Q_0$, hence $P_0(Q_0)$. Let us start with the discussion of $P_1(Q_0, Q_1)$.

First, we argue that $Q_1 = S_1 T$ perfectly reveals the type $T$ of the technology. At the end of date 0, the quality $S_0 T$ is known, and because there is perfect matching between $S_0 T$ and $S_1$, also type $S_1$ is known. Now at date 1, a quality $Q_1 = S_1 T$ is produced and after purchase observed by consumers. Thus, the technology type $T$ can then be perfectly inferred, $T = Q_1/S_1$. Formally, because $S_1 = Q_0^{\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}}$, we obtain

$$T = Q_0^{-\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}} Q_1. \tag{22}$$

from the viewpoint of an outsider (consumer or incoming manager). In other words, a consumer who has observed quality $Q_0$ at date 0 believes that the firm has been bought by a manager of type $S_1$, hence from now observing the quality $Q_1 = S_1 T$, he can perfectly infer the technology type $T$. The expected profit in period 2 of a manager of type $S_2$ buying a firm with technology $T$ is

$$\Pi_2 - P_1(T) = S_2 T - P_1(T). \tag{23}$$

In equilibrium, the marginal expected profit must vanish,

$$S_2 - P_1'(T) = 0. \tag{24}$$

Technology $T$ is "designated" for type $S_2$ with

$$\log S_2 = \frac{\sigma_2}{\tau} \log T, \quad \text{or} \quad S_2 = T^{\sigma_2/\tau}, \tag{25}$$

or equivalently $T = S_2^{\tau/\sigma_2}$. Substituting this into (24), considering that $P(0) = 0$ and solving the ensuing differential equation yields

$$P_1(T) = \frac{\tau}{\sigma_2 + \tau} T^{\frac{\sigma_2+\tau}{\tau}}. \tag{26}$$

Thus, a manager of type $S_2$ makes an aggregate profit of $S_2 T - P_1(T) = \frac{\sigma_2}{\sigma_2+\tau} S_2^{\frac{\sigma_2+\tau}{\sigma_2}}$.

Now consider the market at the end of date 0. The quality $Q_0$ has already been observed. As before, better managers will buy firms with higher expected quality, hence, there will be sorting between qualities $Q_0$ and types $S_1$. The allocation is, thus, as before, given by equation (6). The parameter $T$ is not yet known, but the distribution parameters are updated as before, mean and variance are given by (4) and (3). This implies that a manager of type $S_1$, buying the firm, will get a price $\Pi_1$ as in (9) at date 1. Then the quality $Q_1 = S_1 T$ will be revealed, and he will be able to sell the firm for a price

$$P_1(Q_0, Q_1) = \frac{\tau}{\sigma_2+\tau}\left(Q_0^{-\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}} Q_1\right)^{\frac{\sigma_2+\tau}{\tau}} = \frac{\tau}{\sigma_2+\tau}\left(Q_0^{-\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}} S_1\,T\right)^{\frac{\sigma_2+\tau}{\tau}}. \tag{27}$$

Note that $Q_1$ enters positively, but $Q_0$ enters negatively into the price. Why? Because if the quality $Q_0$ has been low, but still $Q_1$ is high, there can be just one reason. Because firms with low $Q_0$ are bought by managers with low type $S_1$, a high $Q_1$ can only mean that $T$ is in fact extremely high. Thus, ex post, the low $Q_0$ must have been caused by an extremely low initial $S_0$. However, because $T$ must be high, a hight price will be paid for the firm at date 0.

The cumulated profit of a manager of type $S_1$ buying a firm that has produced quality $Q_0$ is thus

$$\begin{aligned}
&\Pi_1 + E[P_1(Q_0,Q_1)|S_1] - P_0(Q_0)\\
&\quad= \exp\Big(\frac{\tau^2+\sqrt{\sigma_0^2+\tau^2}\,\sigma_1}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big)\\
&\quad+ E\Big[\frac{\tau}{\sigma_2+\tau}\left(Q_0^{-\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}} S_1\,T\right)^{\frac{\sigma_2+\tau}{\tau}}\Big] - P_0(Q_0)\\
&\quad= \exp\Big(\frac{\tau^2+\sqrt{\sigma_0^2+\tau^2}\,\sigma_1}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big)\\
&\quad+ \frac{\tau}{\sigma_2+\tau}\,S_1^{\frac{\sigma_2+\tau}{\tau}}\,Q_0^{-\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}\cdot\frac{\sigma_2+\tau}{\tau}}\,E\Big[T^{\frac{\sigma_2+\tau}{\tau}}\Big] - P_0(Q_0),
\end{aligned} \tag{28}$$

where again $T$ is lognormally distributed with parameters as in (4) and (3). Thus, $\ln T$ is normally distributed with mean (3) and standard deviation (4), and hence $\ln T^{\frac{\sigma_2+\tau}{\tau}}$ is normally distributed with $\frac{\sigma_2+\tau}{\tau}$ times the mean and $\frac{\sigma_2+\tau}{\tau}$ times the standard deviation. Hence, the expected value amounts to

$$E\Big[T^{\frac{\sigma_2+\tau}{\tau}}\Big] = \exp\Big(\frac{\sigma_2+\tau}{\tau}\,\frac{\tau^2}{\sigma_0^2+\tau^2}\,\log Q_0 + \frac{1}{2}\,\frac{\sigma_2+\tau}{\tau}\,\frac{\sigma_0^2\,\tau^2}{\sigma_0^2+\tau^2}\Big). \tag{29}$$

Substituting into (28) and taking the derivative with respect to $Q_0$, then substituting $S_1 \to Q_0^{\frac{\sigma_1}{\sqrt{\sigma_0^2+\tau^2}}}$ and solving the ensuing differential equation yields

$$P_0(Q_0) = \frac{\tau^2\left(\sqrt{\sigma_0^2+\tau^2}-\sigma_1\right)-\sigma_0^2\,\sigma_1}{\tau\,(\sigma_2+\tau)\,\sqrt{\sigma_0^2+\tau^2}}\,\exp\left(\frac{\sigma_0^2\,\tau\,(\sigma_2+\tau)}{2\,(\sigma_0^2+\tau^2)}\right) Q_0^{\frac{\tau\,(\sigma_2+\tau)}{\sigma_0^2+\tau^2}}, \tag{30}$$

for $\sigma_1 < \tau^2/\sqrt{\sigma_0^2+\tau^2}$. If this condition is not satisfied, then (formally) the price of firms becomes negative. The economic meaning is that there cannot be perfect sorting between $Q_0$ and $S_1$ in the market for firms. Why not? There is sorting in the market for firms as long as high-type managers (high $S_1$) have an incentive to buy high-quality firms (high $Q_0$). But there are now two opposing effects. First, as before, a firm with high $Q_0$ is attractive because it likely contains a technology with high $T$. On the other hand, a firm with low $Q_0$ is attractive, because if then a high quality is produced in the next period, new managers will believe that this is due to an extremely high $T$, and they will bid a very high price (even if the technology *really* has low expected quality). Hence, the equilibrium with perfect sorting of managers to observed qualities $Q_0$ cannot exist.

How can we interpret the condition $\sigma_1 < \tau^2/\sqrt{\sigma_0^2 + \tau^2}$? If $\sigma_1$ is low, then the type $S_1$ is known ex ante with fairly high precision. The quality $Q_1 = T\, S_1$ will then reveal the technology $T$ with high precision. "Faking" by (out of equilibrium) buying firms with low $Q_0$ does not pay. Hence, for low $\sigma_1$, the market for firms exhibits sorting between $Q_0$ and $S_1$. For high $\sigma_1$, the opposite holds true.

## 4.2 Observability of the Idea

In this subsection we explore the role the non-observability of quality of the technology $T$ plays. Consider a model where the type of technology or idea $T$ is perfectly revealed before the market for firms operates.

At date 0, the price will depend only on $S_0$ and $T$, and $T$ is not yet revealed. Thus, as before, the price on the product market will equal the expected quality and the period profit is $\Pi_0 = e^{(\sigma_0^2+\tau^2)/2}$. When the product is consumed, both $T$ and $S_0$ are revealed. In an assortative equilibrium, better managers with higher $S_1$ will buy firms with better qualities, higher $T$. Because $\log S_1$ has standard deviation $\sigma_1$ and $\log T$ has standard deviation $\tau$, the mapping is straightforward,

$$\log S_1(\log T) = \frac{\sigma_1}{\tau} \log T, \quad \text{thus} \quad S_1 = T^{\sigma_1/\tau}. \tag{31}$$

At date 1, consumers pay the expected quality for the good. For a firm of type $T$, they expect it has been bought by a manager of type $S_1 = T^{\sigma_1/\tau}$, hence the expected quality is $T \cdot T^{\sigma_1/\tau} = T^{1+\sigma_1/\tau}$, and thus $\Pi_1 = T^{1+\sigma_1/\tau}$. At date 2, both $T$ and $S_1$ are known, and $\Pi_2 = S_1\, T$. We can thus calculate expected profits for a manager, net of the firm's price,

$$\Pi_1 + \Pi_2 - P_0(T) = T^{1+\sigma_1/\tau} + S_1\, T - P_0(T). \tag{32}$$

The first-order condition yields

$$P_0'(T) = \frac{\tau + \sigma_1}{\tau}\, T^{\sigma_1/\tau} + S_1. \tag{33}$$

Substituting (31), bearing in mind that $P_0(0) = 0$ and integrating yields[7]

$$
\begin{aligned}
P_0'(T) &= \frac{\tau + \sigma_1}{\tau}\, T^{\sigma_1/\tau} + T^{\sigma_1/\tau} = \frac{2\,\tau + \sigma_1}{\tau}\, T^{\sigma_1/\tau},\\
P_0(T) &= \int_0^T \frac{2\,\tau + \sigma_1}{\tau}\, \tilde{T}^{\sigma_1/\tau}\, d\tilde{T} = \frac{2\,\tau + \sigma_1}{\tau + \sigma_1}\, T^{\frac{\tau+\sigma_1}{\tau}}. \qquad (34)
\end{aligned}
$$

To calculate welfare under perfect observability we need to aggregate the expected prices of products,

$$
\begin{aligned}
W &= \Pi_0 + E_0[\Pi_1] + E_0[\Pi_2] = \Pi_0 + 2\, E_0[\Pi_1]\\
&= \exp\Big(\frac{\sigma_0^2 + \tau^2}{2}\Big) + 2\, E_0\Big[\exp\Big(\frac{\tau + \sigma_1}{\tau}\, \log T\Big)\Big]\\
&= \exp\Big(\frac{\sigma_0^2 + \tau^2}{2}\Big) + 2\, \exp\Big(\frac{(\sigma_1 + \tau)^2}{2}\Big). \qquad (35)
\end{aligned}
$$

Comparing this welfare function with that of section 3, we obtain that perfect observability of the type of technology or idea $T$ increases welfare. This is hardly surprising since this will lead to perfect matches between $T$ and $S_1$ from period 1 onward, which is not the case if the idea is not observable after period 0. Formally this is confirmed by comparing (19) with (35),

$$
\exp\Big(\frac{1}{2}\,(\sigma_1^2 + \tau^2) + \frac{\sigma_1\, \tau^2}{\sqrt{\sigma_0^2 + \tau^2}}\Big) < \exp\Big(\frac{(\sigma_1 + \tau)^2}{2}\Big) \iff \tau < \sqrt{\sigma_0^2 + \tau^2}, \qquad (36)
$$

which is true. The fact that $S_0$ and $T$ are stochastically independent, and $S_1$ is a function of $T$ implies that $S_0$ and $S_1$ are uncorrelated. Thus, under perfect observability finding 3 of the introduction cannot be obtained. Put differently, the possibility to transfer reputation from one generation of managers to the next crucially depends on the fact that $T$ is not observable. The reputation of the firm can only affect expectations about the technology type if $T$ is not perfectly known. Clearly, if it is revealed, reputation transfer is not an issue.

Note that the innovator's skill does not have a positive effect on the sales price of the firm. While this does not have negative welfare consequences in a setting in which $S_0$ is not a choice variable, it is clear that in a slightly extended setting in which innovators can initially invest in their managerial skills, there is less investment in this skill under perfect observability than under the setting considered in the previous section. We return to this issue in the following section.

To derive a testable implication, let us see how the observability of $T$ affects the average price $P_0$. We find that information on the quality of the technology on average leads to a price increase, with the following reason. The type of technology $T$ and manager $S_1$ are complements. Therefore, a better matching improves the

[7] The same pricing function is obtained by taking the limit $\sigma_0 \to 0$ in (14). Bear in mind that if $\sigma_0 \to 0$, then $S_0$ converges to 1 almost surely, thus $Q_0 = T$ almost surely. The limits of the other factors are straightforward.

expected quality. Hence, especially good managers are willing to bid a higher price for the firms. While the average price increases by the information, at the lower part of the distribution, the price function is depressed by the additional information.

**Proposition 3** *The expected price $E[P_0]$ on the market for firms increases if information on the type of technology becomes public.*

**Proof of proposition 3:** The pricing function is given by (34). But $T$ is distributed with $\log T \sim N(0, \tau^2)$, Hence

$$\begin{aligned}
\log P_0(T) &= \log\Big(\frac{2\,\tau + \sigma_1}{\tau + \sigma_1}\, T^{\frac{\tau+\sigma_1}{\tau}}\Big) \\
&= \log\Big(\frac{2\,\tau + \sigma_1}{\tau + \sigma_1}\Big) + \frac{\tau + \sigma_1}{\tau} \cdot \log T \\
&\sim N\Big(\log\big(\frac{2\,\tau + \sigma_1}{\tau + \sigma_1}\big),\ (\tau + \sigma_1)^2\Big), \\
E[P_0] &= \exp\Big(\frac{(\tau + \sigma_1)^2}{2} + \log\big(\frac{2\,\tau + \sigma_1}{\tau + \sigma_1}\big)\Big) = \frac{2\,\tau + \sigma_1}{\tau + \sigma_1} \cdot e^{(\tau+\sigma_1)^2/2}. \qquad (37)
\end{aligned}$$

The type of the innovator plays no role, $\sigma_0$ thus does not occur in the equation. Now we need to compare this average with the average price in the original model, with unobservable $T$. For equation (1), we have argued that $Q_0 = S_0\,T$ is distributed with $\log Q_0 = \log S_0 + \log T \sim N(0, \sigma_0^2 + \tau^2)$. Using (14), we get

$$\begin{aligned}
\log P_0(T) &= \frac{1}{2}\,\frac{\sigma_0^2\tau^2}{\sigma_0^2{+}\tau^2} + \log\Big(\frac{\tau^4}{(\sigma_0^2{+}\tau^2)\,(\tau^2{+}\sigma_1\sqrt{\sigma_0^2{+}\tau^2})} + 1\Big) + \frac{\tau^2 + \sigma_1\sqrt{\sigma_0^2{+}\tau^2}}{\sigma_0^2 + \tau^2}\log Q_0 \\
&\sim N\Big(\frac{1}{2}\,\frac{\sigma_0^2\tau^2}{\sigma_0^2{+}\tau^2} + \log\big(\frac{\tau^4}{(\sigma_0^2{+}\tau^2)\,(\tau^2{+}\sigma_1\sqrt{\sigma_0^2{+}\tau^2})} + 1\big),\ \frac{(\tau^2 + \sigma_1\sqrt{\sigma_0^2{+}\tau^2})^2}{\sigma_0^2{+}\tau^2}\Big), \\
E[P_0] &= \exp\Big(\frac{(\tau^2 + \sigma_1\sqrt{\sigma_0^2{+}\tau^2})^2}{2\,(\sigma_0^2{+}\tau^2)} + \frac{\sigma_0^2\tau^2}{2\,(\sigma_0^2{+}\tau^2)} + \log\big(\frac{\tau^4}{(\sigma_0^2{+}\tau^2)\,(\tau^2{+}\sigma_1\sqrt{\sigma_0^2{+}\tau^2})} + 1\big)\Big) \\
&= \Big(\frac{\tau^4}{(\sigma_0^2{+}\tau^2)\,(\tau^2{+}\sigma_1\sqrt{\sigma_0^2{+}\tau^2})} + 1\Big) \cdot e^{\frac{\sigma_0^2\tau^2 + (\tau^2+\sigma_1\sqrt{\sigma_0^2+\tau^2})^2}{2\,(\sigma_0^2+\tau^2)}} \\
&= \Big(\frac{\tau^4}{(\sigma_0^2{+}\tau^2)\,(\tau^2{+}\sigma_1\sqrt{\sigma_0^2{+}\tau^2})} + 1\Big) \cdot e^{\frac{\sigma_1^2+\tau^2}{2} + \frac{\sigma_1\,\tau^2}{\sqrt{\sigma_0^2+\tau^2}}}. \qquad (38)
\end{aligned}$$

Now the additional information on $T$ depresses the average price if (37) falls short of (38). The proof remainder of the proof contains some clutter. We give some intermediate steps. First, comparing the terms, we see that the factor $\exp(\frac{\sigma_1^2+\tau^2}{2})$ can be canceled out on both sides. Rearranging terms, using the estimation $e^x \geq x + 1$ for all $x$, and substituting $\sqrt{\sigma_0^2 + \tau^2} \to x$, and multiplying out the fractions on both sides, we obtain the sufficient condition

$$(\sigma_1 + 2\tau)X^3(\sigma_1 X + \tau^2) - (\sigma_1 + \tau)(X - \sigma_1\tau X + \sigma_1\tau^2)(\sigma_1 X^3 + \tau^2 X^2 + \tau^4) \geq 0. \qquad (39)$$

This term is positive for all $X > \tau$, as can be shown by considering the Taylor series at the point $X = \tau$. The term (39) itself vanishes for $X = \tau$. The first, second, third, and fourth derivatives with respect to $X$ contain only positive components. The fifth and higher derivatives vanish. This shows that the term (39) is positive for all parameters $\sigma_0$, $\sigma_1$ and $\tau$. This again implies that (37) exceeds (38). ■

### 4.3 Moral Hazard

Up to now, we have assumed that the types of technologies and managers were fixed. Let us now analyze an effort choice. For exposition, let us assume that the innovator knows the technology type $T$ and can alter his type from $S_0$ to $e\,S_0$ by exerting an effort $e$ at cost $c\,e^2/2$. The effort choice is unobservable. We want to argue that the innovator has an incentive to exert effort in order to raise the initial quality $Q_0$, which will then raise the selling price $P_0$. Potentially, the effort choice may even exceed the first best, which is given by maximizing $e\,S_0\,T - c\,e^2/2$, thus $e^* = S_0\,T/c$.

In equilibrium, the price at date 0 will be independent from the effort choice; the effort choice will only be anticipated. Only the selling price of the firm will be relevant. Let us assume for now that the selling price has the form $P_0(Q_0) = k\,Q_0^\kappa$. We will later show that this assumption is correct. Then, the innovator will maximize

$$P_0(Q_0) - c\,e^2/2 = k\,e^\kappa\,S_0^\kappa\,T^\kappa - c\,e^2/2, \tag{40}$$

which is maximized by

$$k\,\kappa\,e^{\kappa-1}\,S_0^\kappa\,T^\kappa = c\,e \implies e^{2-\kappa} = k\,\kappa\,S_0^\kappa\,T^\kappa/c \implies e^* = (k\,\kappa\,S_0^\kappa\,T^\kappa/c)^{\frac{1}{2-\kappa}}. \tag{41}$$

This effort choice alters the distribution of qualities $Q_0$ at date 0. We have

$$\begin{aligned}
\log Q_0 = \log(e\,S_0\,T) &= \log\Big(S_0^{\frac{2}{2-\kappa}}\,T^{\frac{2}{2-\kappa}}\,\big(\frac{\kappa\,k}{c}\big)^{\frac{1}{2-\kappa}}\Big) \\
&= \frac{2}{2-\kappa}\,\log S_0 + \frac{2}{2-\kappa}\,\log T + \frac{1}{2-\kappa}\,\log\frac{\kappa\,k}{c} \\
&\sim N\Big(\frac{1}{2-\kappa}\,\log\frac{\kappa\,k}{c};\ \frac{2}{2-\kappa}\,(\sigma_0^2+\tau^2)\Big).
\end{aligned} \tag{42}$$

Remember that $\log T \sim N(0,\tau^2)$ and $\log S_0 \sim N(0,\sigma_0^2)$. Thus both $\log T$ and $\log S_0$ add to the variance of $\log Q_0$, but not to the mean. The constant $\frac{1}{2-\kappa}\,\log(\kappa\,k/c)$ only adds to the mean, not to the variance.

When $Q_0$ is observed, the beliefs about the distribution of $T$ are updated. The new parameters of the lognormal distribution are

$$\begin{aligned}
E[\log T|\log Q_0] &= \frac{\tau^2}{\frac{2}{2-\kappa}\,(\sigma_0^2+\tau^2)+\tau^2}\Big(\log Q_0 - \frac{1}{2-\kappa}\,\log\frac{\kappa\,k}{c}\Big) \\
&= \frac{(2-\kappa)\,\tau^2}{2\,\sigma_0^2+(4-\kappa)\,\tau^2}\Big(\log Q_0 - \frac{1}{2-\kappa}\,\log\frac{\kappa\,k}{c}\Big),
\end{aligned} \tag{43}$$

$$V[\log T|\log Q_0] = \frac{2\,\tau^2\,(\sigma_0^2+\tau^2)}{2\,\sigma_0^2+(4-\kappa)\,\tau^2}. \tag{44}$$

After this updating exercise, incoming managers buy technologies on the market for firms. There is a one-to-one mapping between qualities $Q_0$ and buyers $S_1$,

$$\log S_1(\log Q_0) = \sigma_1 \sqrt{\frac{2-\kappa}{2\,(\sigma_0^2+\tau^2)}}\,\Big(\log Q_0 - \frac{1}{2-\kappa}\,\log\frac{\kappa\,k}{c}\Big) \tag{45}$$

At date 1, the price on the market is independent of the actual value of $T$ and $S_1$. It will, however, depend on $Q_0$ and the consumers' expectations about the buying manager, which is given by (45). Thus,

$$E[\log Q_1|\log Q_0] = \Big(\frac{(2-\kappa)\,\tau^2}{2\,\sigma_0^2+(4-\kappa)\,\tau^2} + \sigma_1\sqrt{\frac{2-\kappa}{2\,(\sigma_0^2+\tau^2)}}\Big)\Big(\log Q_0 - \frac{1}{2-\kappa}\,\log\frac{\kappa\,k}{c}\Big), \tag{46}$$

$$V[\log Q_1|\log Q_0] = \frac{2\,\tau^2\,(\sigma_0^2+\tau^2)}{2\,\sigma_0^2+(4-\kappa)\,\tau^2}. \tag{47}$$

The expected quality, and thus the price and the firm profit, are given by

$$\begin{aligned}\Pi_1(Q_0) = \exp\Big[&\Big(\frac{(2-\kappa)\,\tau^2}{2\,\sigma_0^2+(4-\kappa)\,\tau^2} + \sigma_1\sqrt{\frac{2-\kappa}{2\,(\sigma_0^2+\tau^2)}}\Big)\Big(\log Q_0 - \frac{1}{2-\kappa}\,\log\frac{\kappa\,k}{c}\Big)\\ &+\frac{1}{2}\cdot\frac{2\,\tau^2\,(\sigma_0^2+\tau^2)}{2\,\sigma_0^2+(4-\kappa)\,\tau^2}\Big]\end{aligned} \tag{48}$$

After this market, the quality $Q_1$ is revealed, such that at date 2, the price will equal $\Pi_2 = Q_1 = S_1\,T$. Hence, the new manager will want to know what type of firm he is buying. The distribution of $\log T$ is given in (43) and (44). Consequently, the expected $T$ is

$$E[T|Q_0] = \exp\Big(\frac{(2-\kappa)\,\tau^2}{2\,\sigma_0^2+(4-\kappa)\,\tau^2}\,\Big(\log Q_0 - \frac{1}{2-\kappa}\,\log\frac{\kappa\,k}{c}\Big) + \frac{1}{2}\cdot\frac{2\,\tau^2\,(\sigma_0^2+\tau^2)}{2\,\sigma_0^2+(4-\kappa)\,\tau^2}\Big). \tag{49}$$

The expected profit of a manager buying a firm with observed quality $Q_0$ consists of the profit at date 1, the profit at date 2, net of the price $P_0(Q_0)$. The derivative of this profit with respect to $Q_0$ must vanish, otherwise the manager would buy a different firm,

$$\begin{aligned}0 = &\Big(\frac{(2-\kappa)\,\tau^2}{2\,\sigma_0^2+(4-\kappa)\,\tau^2} + \sigma_1\sqrt{\frac{2-\kappa}{2\,(\sigma_0^2+\tau^2)}}\Big)\cdot\Pi_1(Q_0)\\ &+ S_1\cdot\frac{(2-\kappa)\,\tau^2}{2\,\sigma_0^2+(4-\kappa)\,\tau^2}\cdot E[T|Q_0] - P_0'(Q_0)\end{aligned} \tag{50}$$

Remember that $S_1$ is a function of $Q_0$, as given by (45). After substitution, we are left with an ordinary differential equation in $Q_0$. The fact that $P_0(0) = 0$ provides us with a boundary, hence we can use straight integration to solve the problem, yielding

$$P_0(Q_0) = \frac{\sigma_1\, X/Y + 2\,(4-2\,\kappa)}{\sigma_1\, X/Y + (4-2\,\kappa)} \cdot \frac{e^{(\sigma_0^2+\tau^2)\,Y}\; Q_0^{\sigma_1\, X/2+(2-\kappa)\,Y}}{(\kappa\, k/c)^{\sigma_1/\sqrt{(4-2\kappa)(\sigma_0^2+\tau^2)}+Y}}$$

$$\text{with} \quad X = \sqrt{\frac{4-2\kappa}{\sigma_0^2+\tau^2}} \quad \text{and} \quad Y = \frac{\tau^2}{2\,\sigma_0^2 + (4-\kappa)\,\tau^2}. \tag{51}$$

Remember that we have assumed initially that $P_0(Q_0) = k\, Q_0^\kappa$, where $k$ and $\kappa$ were yet to be determined. We have now found that $P_0(Q_0)$ has indeed required form, where

$$\kappa = \frac{1}{2}\,\sigma_1 \sqrt{\frac{4-2\,\kappa}{\sigma_0^2+\kappa^2}} + \frac{(2-\kappa)\,\tau^2}{2\,\sigma_0^2+(4-\kappa)\,\tau^2} \tag{52}$$

is the exponent over $Q_0$, and $k$ is the remaining factor. Equation (52) does not contain $k$, so we can solve for $\kappa$ first, and then set the remaining factor in (51) equal to $k$ and solve for $k$. Even a closed-form algebraic solution is possible, but it is too messy to be written down. Instead, consider a numerical example. As for the figures, let $\tau = 1$, $\sigma_0 = 1$ and $\sigma_1 = 1$, and in addition set $c = 1$. Then the solution for $\kappa$ and $k$ is $\kappa = 0.7843$ and $k = 1.6265$. Due to (40), we know that $e^* = (k\,\kappa\, S_0^\kappa\, T^\kappa/c)^{\frac{1}{2-\kappa}} = 1.2218\,(S_0\, T)^{0.6452}$. Remember that the first-best choice of effort is $e^* = S_0\, T/c = S_0\, T$. Therefore, if $S_0\, T$ is small, the innovator will choose too high effort; if it is large, he will exert too little effort. The critical value in this numerical example is $S_0\, T = 1.7591$, or $\log(S_0\, T) = 0.5647$. Because the median log is zero, the innovator is more likely to work too hard in this example.

Let us discuss (52) is some more detail. For $\kappa = 0$, the left side is zero but the right side is positive. For $\kappa = 2$, it is the other way round. Both sides are continuous in $\kappa$, hence (52) must have a solution $\kappa \in (0; 2)$. As a consequence, independent of the parameter choice, there is always a critical value for $S_0\, T$ such that the innovator exerts too much effort if $S_0\, T$ is below this critical value, and vice versa.

# 5 Conclusion

In the life cycle of a firm, an ownership change can be an incisive event, especially if new management is brought in. We have constructed a model of ownership change, taking into account both the improved sorting that such a market provides and reputation considerations. Some background technology that is a complement in the production process works like a storage device for reputation. The noisier the type of the technology, the higher the correlation of types between initial innovators and managers will be, thus the better reputation transfer will work.

On the other hand, the variance between types of technologies also influences the prices paid in the market for firms. If the variance is extreme, this implies that good technologies differ a lot from bad ones. Hence, managers can benefit from paying a high price for firms. This may serve as an incentive for innovators to work hard, to drive up the price of their firm. Hence, creating a market for trading the firms' long term asset (i. e., their technology) benefits the efficient allocation of resources. The "reputation" of innovators partly determines the firm value due to imperfect observability.

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