Strategic delegation and dynamic competition

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Abstract

The separation of ownership and control in firms brings up the issue of how and what to delegate to managers. There exists a large body of literature that analyzes strategic delegation in which owners understand the incentives that managers face when they operate in imperfectly competitive product markets. In this paper we analyze strategic delegation under the assumption that firms operate in dynamic oligopolies. We derive the optimal strategic incentive in this setting and point out that the dynamic incentive can be replicated by a static one in which firms play a conjectural variations equilibrium in the output market.

Key words: Strategic delegation, dynamic competition and incentives, strategic equilibrium incentives

1 Introduction

Traditional microeconomic price theory assumes that firms maximize profits. While this assumption holds true for many firms it is true that large companies are characterized by a separation of ownership and control (management) which makes the managerial decision processes a rather complex issue. In fact, many firms can be singled out in which profit maximization is not the primary objective of management. At the end of the fifties Baumol pointed out that managers may be guided by other objectives than pure profit-maximization (Baumol, 1958), and suggested a sales-maximization model as alternative. It

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did not take long until managerial economics explored the incentives of managers not to follow profit maximization. Marris (1963) developed a model in which managers are more interested in increasing growth rates of the enterprise rather than maximizing profits or firm value. This insight led to a large stream of research which dealt with alternative firm objectives, in general. Since that time there exist many empirical studies, which support this hypothesis. [1] However, yet it is not completely clarified whether this tendency to act in a different manner than owner managed firms happens purely from the manager's intrinsic motives, or it is affected by the contract design of the incentive contracts for managers.

In the meantime it is well known that decisions which are reached apart from pure profit maximizing preferences may generate competitive advantage. In this sense hiring an agent that is not primarily interested in maximizing profits or firm value works as credible commitment. After Thomas Schelling (1960) pointed out that there exist competitive impacts of commitment strategies when designing a manager's actions, researchers started to investigate the strategic use of contracts. This view was first introduced by the path-breaking papers of Vickers (1985), Fershtman (1985), Fershtman and Judd (1987), and Sklivas (1987), VFJS for short. These contributions offer a game-theoretic explanation for managers' nonprofit-maximizing behavior. VFJS consider the separation of owners (principals) and managers (agents), and examine a two-stage game, where in the first stage (the 'contract stage') the owner writes his manager's contract, which is publicly announced, before competition evolves in the second stage. In all these models, according to the contract, managers receive a bonus or have to pay a fine which is proportional to a (linear) combination of profits and output. In the consecutive stage (the 'market stage'), firms' managers decide on output, using the utility function directed by the contract, because their compensation depends on it. FVJS show that in a quantity competition game by hiring a manager it is possible to acchieve Stackelberg leadership. [2] But if all competitors decide to separate ownership and control they end up in a prisoners dilemma. There is recent work going on where researchers focus on the optimal delegation schemes if different con-

[1] For instance see Gugler et al. (2004) or Gugler et al. (2003).

[2] An empirical affirmation is shown e.g. by Irwin (1991). He demonstrates how the Dutch East India Company attained market leadership over its rivals, the British East India Company by offering its managers an incentive scheme with a direct sales component while the managers of the British East India Company were offered a purely profit-based incentive scheme. In an experiment Fershtman and Gneezy (2001) examined the hypothesis that the appointment of managers would create competitive effects and found some support for it. In contrast to that in a different experiment where owners can choose between a pure profit based contract and a contract with an additional sales bonus, Huck et al. (2004) find that, although theory predicts to pick the sales contract, it is only rarely selected in their experimental market.

tracts are available. But the results hold with some qualitative differences in the whole class of delegation schemes. See for instance Jansen et al. (2007a), Jansen et al. (2007b) and Dierkes (2004).

However, the strategic delegation literature focuses primarily on static, mainly two stage games. A few attempts are done to make competition more dynamic. Mujumdar and Pal (2007), Kopel and Löffler (2007), for example, analyze the effects of strategic delegation in a game with two production periods before the market clears.

In this paper we go one step further and examine strategic delegation in a dynamic setting. We formulate a two stage game in which firm owners first set the optimal contract (the level of deviation from profit maximization) and managers in the second stage of the game chose their dynamic product market strategy. Managers in our paper play a dynamic Cournot game under the assumption of an infinite planning horizon. For dynamic quantity competition with linear contracts we prove that our game admits a unique symmetric closed-loop equilibrium that corresponds to equilibrium quantities that are larger than static Cournot quantities. This pro-competitive behavior causes owners to choose a negative contract, hence it is in the interest of the owners to induce managers to a less aggressive output market strategy. In our dynamic game, using managers as strategic device, is Pareto-optimal for all participants. To relate our results to the standard literature with the static strategic delegation, we reformulate our dynamic model and show that the closed-loop equilibrium in the quantity setting stage corresponds to a conjectural variations equilibrium with constant and negative conjectures. This quantity setting behavior results in negative contracts.

Our paper is organized as follows. In the next Section we present the dynamic two stage game. In the first stage owners set the incentives for managers and in the second stage managers play a dynamic Cournot game in the output market. In Section 3 we derive strategic incentives when managers play a dynamic infinite horizon quantity game. In Section 4 we provide a detailed analysis of these incentives and show that the dynamic incentives can be replicated by a static conjectural variations equilibrium in the output market. Finally, Section 5 concludes the paper.

2 The model

In the literature there exist numerous models that capture structural dynamics in competition. One possible approaches to make the static Cournot model dynamic is the one with sticky prices introduced by Fershtman and Kamien (1987) (see also Dockner, 1988). Fershtman and Kamien (1987) study dynamic duopolistic competition under the assumption that the market demand curve is described by a first order differential equation. This equation relates at each point in time the price of the commodity to the rate of change of the price and the outputs of both firms in the market. The dynamic demand equation is interpreted as a price adjustment mechanism in the sense that the market price of a homogeneous product does not adjust instantaneously to the price indicated by its static demand function at the given level of output. Thus, the evolution of the price through time is a linear function of the difference between the current price and the price given by the demand function at the current level of output. The duopolists can make use of that price lag, i.e. prices are sticky.

To fix ideas, consider a duopoly market with two firms producing a homogeneous output. Both firms are assumed to operate with decreasing returns to scale technologies described by the cost functions

$$C_i(q_i(t)) = c_i q_i(t) + \frac{1}{2} q_i^2(t), \tag{1}$$

where $q_i(t) > 0$ is the output of firm i at time t and c_i is a constant. The equilibrium price at time t is related to total output $Q(t) = q_1(t) + q_2(t)$ by means of a linear inverse demand function

$$p^\infty(t) = a - (q_1(t) + q_2(t)). \tag{2}$$

According to Fershtman and Kamien (1987) the market demand function faced by the firms is not given by the static equation (2), they face the dynamic equation

$$p(t) = a - (q_1(t) + q_2(t)) - \frac{1}{s}\dot{p}(t), \tag{3}$$

with $0 < s < \infty$, instead. Upon rewriting equation (3) it can be given the interpretation of a first order adjustment process

$$\dot{p}(t) = s\left(a - (q_1(t) + q_2(t)) - p(t)\right). \tag{4}$$

In this formulation the actual market price, $p(t)$, at time t is not equal to its equilibrium level $p^\infty(t)$, but moves towards it. The constant s determines the speed of adjustment. In the limiting case $s \to \infty$ actual prices jump instantaneously to their equilibrium levels. The use of the dynamic demand equation (3) and hence that of the first order adjustment process is economically justified because (3) is consistent with utility maximization by consumers. It can be derived if the (domestic) consumers' utility function depends on current and past levels of consumption of the traded good. As long as marginal utility of current consumption depends on past levels of consumption the market price will be sticky. However, as $s \to \infty$, (3) is a special case of the static inverse demand function (2). In this case, marginal utility of current consumption is independent of past level.

Given demand and cost conditions as in (3) and (1) and assuming that firms maximize the discounted stream of profits over an infinite planning horizon the dynamic Cournot model with sticky prices becomes

$$\max \left\{ J_i = \int_0^\infty e^{-rt} \left[p(t) q_i(t) - c_i q_i(t) - \frac{1}{2} q_i^2(t) \right] dt \right\}, \tag{5}$$

the initial condition $p(0) = p_0 \geq 0$.

Problem (5) with (4) is a two person nonzero-sum differential game with an infinite horizon. The quantities $q_i(t)$ are the control variables and the market price $p(t)$ is the state variable. If we substitute the dynamic demand equation (4) into the objective functions (5) of the firms we get

$$\int_0^\infty e^{-rt} \left\{ \left[a - q_i(t) - q_j(t) \right] q_i(t) - \frac{1}{s} \dot{p}(t) q_i(t) - c_i q_i(t) - \frac{1}{2} q_i(t) \right\} dt. \tag{6}$$

When $s \to \infty$ the price instantaneously jumps to its level given by the demand curve (2) and the payoff function become

$$\Pi_i = \int_0^\infty e^{-rt} \left\{ \left[a - q_i(t) - q_j(t) \right] q_i(t) - c_i q_i(t) - \frac{1}{2} q_i(t) \right\} dt. \tag{7}$$

This is a continuous time repeated game for which all the structural dynamics are removed.

In addition the owners of the firms can delegate the quantity decision to managers. The managers are compensated on the basis of profits and quantities sold, i.e. the compensation functions are

$$U_i = \Pi_i + \alpha_i \int_0^\infty q_i(t) dt \tag{8}$$

where the incentive parameter α_i is determined by the owners in advance such that their own profits are maximized.[3] Renegotiation of the contract is

[3] With this assumption we follow Vickers (1985). Fershtman and Judd (1985) suggest to use a combination of profits and sales revenues instead. Our results do not change for this alternative compensation scheme.

not possible. The total compensation of the manager is given by $A_i + B_i U_i$, where A_i, B_i are appropriately selected by the owners such that the total compensation equals the reservation utility of the managers. In what follows for simplicity we assume that the reservation utility is zero (see also Kräkel, 2004; Fershtman and Judd, 1987). Notice that if the value of the incentive parameter is zero, then the managers act just the same way as the owner would. On the other hand, selecting $\alpha_i > 0$ ($\alpha_i < 0$) motivates the manager to act more (less) aggressively in the product market by offering a higher (smaller) quantity.

As long as the managers are responsible for determination of the production quantity the maximization problem of equation (5) has to be modified to

$$\max \left\{ U_i = \int_0^\infty e^{-rt} \left[p(t)q_i(t) - c_i q_i(t) - \frac{1}{2} q_i^2(t) + \alpha_i q_i(t) \right] dt \right\}. \tag{9}$$

If firms employ closed-loop decision rules in the sticky price model they design their equilibrium decision rules as functions of the current market price. Strategic responses are taken care of through the price variable that reflects the latest available information for the players. Any action that is taken by a duopolist causes the market price to change and hence triggers a reaction by the rival. We discuss existence, uniqueness, and stability of a closed-loop equilibrium of the introduced sticky price model.

3 Dynamic quantity competition and strategic incentives

In this section we analyze the two stage game outlined in the preceding discussion. There are two distinct groups of decision makers involved in this game. In the first stage owners set their equilibrium incentives. These equilibrium incentives are set by an appropriate choice of the parameter α_i. These parameters measure how strongly managers should take other than profit maximizing objectives into account. Owners set these incentives under the assumption that they act sequentially rational. This implies that the choose the parameters α_i in such a way that they fully take into account which incentives they have on the dynamic production strategies taken by the managers in the production stage. The second stage of the game is a dynamic quantity game that managers play over an infinite horizon. For this dynamic quantity game we make use of the sticky price model analyzed by Fershtman and Kamien (1987). Managers choose Markovian strategies in this game by explicitly taking the chosen incentives into account.

To solve the dynamic two stage game we use backward induction and analyze the production stage (stage II) first. Hence, we assume constant and given incentives α_i and derive optimal dynamic quantity choices. In a second step we than solve the incentive problem faced by the owners. In the first stage this incentive game is played and equilibrium incentives are derived on the basis of the optimal firms' value functions. Let us derive the optimal quantity choices of the managers under the assumption that incentives are constant and fixed.

Proposition 1 *Under the assumption of constant and fixed incentives α_i there exists a globally and asymptotically stable Markov (subgame-perfect) equilibrium for the dynamic Cournot game with sticky prices. This equilibrium is unique within the class of linear Markov strategies.*

Proof A closed-loop (subgame-perfect) Nash equilibrium has to satisfy the Hamilton-Jacobi-Bellman equations

$$rU^1 = \max_{q_1} \left\{ (p - (c_1 - \alpha_1))q_1 - \frac{1}{2}q_1^2 + U_p^1 s\left[a - q_1 - q_2 - p\right] \right\}, \tag{10}$$

$$rU^2 = \max_{q_2} \left\{ (p - (c_2 - \alpha_2))q_2 - \frac{1}{2}q_2^2 + U_p^2 s\left[a - q_1 - q_2 - p\right] \right\}, \tag{11}$$

where $U_p^i \equiv \frac{\partial U^i}{\partial p}$ and U_p^i are the current-value utility functions of the two managers respectively. For simplicity but without loss of generality we will assume $c_1 = c_2 = 0$. Since our problem admits a linear quadratic structure, we guess quadratic value functions of the form

$$U^1 = \frac{1}{2}Kp_0^2 - E_1 p_0 + g_1 \tag{12}$$

$$U^2 = \frac{1}{2}Kp_0^2 - E_2 p_0 + g_{2.} \tag{13}$$

The variables K and E_i can be interpreted as the quadratic and linear cost terms, whereas g_i is the revenue perceived by the managers. This setup results in optimal quantities given by the decision rules [4]

$$q_1(t) = p(t) + \alpha_1 - sKp(t) + sE_1 \tag{14}$$

$$q_2(t) = p(t) + \alpha_2 - sKp(t) + sE_2. \tag{15}$$

Substitution of (12) to (15) into (10) and (11) implies that the constants K, E_1 and E_2 have to satisfy the following set of equations (see Fershtman and Kamien 1987, and Dockner 1988),

[4] In order that the equilibrium quantities are positive we have to restrict attention to the case where the initial price is higher than $\max\left\{\frac{c_1 - E_1}{1 - sK}, \frac{c_2 - E_2}{1 - sK}\right\}$. That is what we assume to hold throughout. Otherwise the proof needs to be modified (see Fershtman and Kamien 1987, for details).

$$-rK + 1 - 6sK + 3s^2K^2 = 0 \tag{16}$$
$$E_1(3s - 2s^2K + r) + K(as - s\alpha_1 - s^2E_2) + \alpha_1 = 0 \tag{17}$$
$$E_2(3s - 2s^2K + r) + K(as - s\alpha_2 - s^2E_1) + \alpha_2 = 0 \tag{18}$$
$$rg_1 - E_1\left(\frac{1}{2}s^2E_1 + sE_2 - as + s\alpha_1\right) = 0 \tag{19}$$
$$rg_2 - E_2\left(\frac{1}{2}s^2E_2 + sE_1 - as + s\alpha_2\right) = 0 \tag{20}$$

in order that the quadratic value functions (12) and (13) satisfy the Bellman equations (10) and (11).

A solution to (16) is given by

$$K = \frac{r + 6s \pm \sqrt{(r+6s)^2 - 12s^2}}{6s^2}. \tag{21}$$

As Fershtman and Kamien (1987) we can argue that the stable solution corresponds to the negative root of (21). Once we have found K, we can easily solve for E_1, E_2, g_1 and g_2 from (17) to (20).

$$E_i = \frac{aKs(s(Ks-3) - r) + (Ks-1)\left(K\alpha_j s^2 + (r + s(3-2Ks))\alpha_i\right)}{(r + s(3-Ks))(r - 3s(Ks-1))} \tag{22}$$
$$g_i = \frac{E_i\left(\frac{1}{2}s^2E_i + sE_j - as + s\alpha_i\right)}{r} \tag{23}$$

With these constants the equilibrium decision rules become

$$q_i(p(t)) = (1 - sK)p(t) + (sE_i + \alpha_i). \tag{24}$$

From the explicit solution of K it is obvious that $1 - sK > 0$ holds, whenever we choose the negative root of (21). This implies that firms in equilibrium increase their output with increasing market price. Stability of the equilibrium is guaranteed whenever $2sK - 3 < 0$ is satisfied. This, however, is implied when we take the negative root of (21). Hence, we can conclude that the optimal state trajectory is given by

$$p(t) = p^\infty + (p_0 - p^\infty)e^{s(2sK-3)t}, \tag{25}$$

where p^∞ is the steady state price that is given by

$$p^\infty = \frac{a - \alpha_1 - \alpha_2 - s(E_1 + E_2)}{3 - 2sK}. \tag{26}$$

Thus, the closed-loop equilibrium is globally and asymptotically stable, i.e. the equilibrium price converges to its steady state level independent of the initial condition. ■

In (26) we can see that the steady state price p^{∞} depends crucially on the incentive parameters α_i. If managers receive a bonus for selling more products competition becomes fiercer and the price decreases.

Now, as the managers strategies are determined we can solve for the optimal incentive contract that maximizes firm profits.

Proposition 2 *Investors write the incentive contract such that the incentive parameters α_i are chosen negative. As a consequence managers perceive higher linear costs than in reality. Hence, they act retentive on the product market. As a result profits are higher compared to owner driven firms.*

Proof After deriving the constants K , E_i and g_i it is now possible to calculate the optimal incentive parameter of the incentive contract. The utility function of the owners is similar to the utilities of their manager but diverge only in one part. We get

$$\Pi_i = U_i - \int_0^{\infty} \alpha_i q_i(t)dt = \frac{1}{2}Kp_0^2 - E_i p_0 + g_i - \int_0^{\infty} \alpha_i q_i(t)dt \tag{27}$$

and the maximizing condition becomes

$$\frac{\partial \Pi_i}{\partial \alpha_i} = \frac{p_0^2}{2}\frac{\partial K}{\partial \alpha_i} - \frac{\partial E_i}{\partial \alpha_i}p_0 + \frac{\partial g_i}{\partial \alpha_i} - \frac{\partial}{\partial \alpha_i}\left(\int_0^{\infty} \alpha_i q_i(t)dt\right). \tag{28}$$

The first term on the right side of (28), the quadratic cost term, is zero as K is independent of α_i. The second term $-\frac{\partial E_i}{\partial \alpha_i}$ is always positive. This term represents the change of linear costs with respect to the change of alpha. That for it has to be positive as rising α_i lowers the linear cost structure perceived by the manager. The term $\frac{\partial g_i}{\partial \alpha_i}$ contains all strategic effects that a change of the incentive parameter induces. That includes the answer of the competitor and the trade-off between the utility raising effect of the bonus for additional production quantity and the profit reducing effect because of the quadratic cost term. The slope is positive in a wide range of α_i (but is always positive for $\alpha_i \leq 0$) and becomes negative as α_i gets too large. This term is the driving force for a manager to produce extra goods. The slope of the correction term rightmost of (27) is positive as well but with the negative sign the effect is negative.

Concluding our findings we get

$$\underbrace{-\frac{\partial E_i}{\partial \alpha_i} p_0\Big|_{\alpha_i=0}}_{>0} + \underbrace{\frac{\partial g_i}{\partial \alpha_i}\Big|_{\alpha_i=0}}_{>0} > 0 \tag{29}$$

$$-\frac{\partial}{\partial \alpha_i}\left(\int_0^\infty \alpha_i q_i(t)dt\right)\Big|_{\alpha_i=0} < 0. \tag{30}$$

Equation (29) shows that a manager would prefer a positive value of α_i but comparing (29) and (30) using the absolute values we get the inequality

$$\left| -\frac{\partial E_i}{\partial \alpha_i} p_0\Big|_{\alpha_i=0} + \frac{\partial g_i}{\partial \alpha_i}\Big|_{\alpha_i=0} \right| < \left| -\frac{\partial}{\partial \alpha_i}\left(\int_0^\infty \alpha_i q_i(t)dt\right)\Big|_{\alpha_i=0} \right|. \tag{31}$$

Thus, owners always choose $\alpha_i < 0$. ■

In fact the equilibrium strategy of the owners is to choose a negative α_i. As the term is very long we show here only a simplified version with $r = 0$. The optimal value is then given by

$$\alpha^* = -\tfrac{(Ks-1)(7Ks-12)+a(2Ks-3)(2Ks(Ks-3)+3)}{2(3-2Ks)(Ks-2)(Ks-1)} < 0 \tag{32}$$

By hiring agents that are compensated with this kind of contracts firms are able to implement some kind of collusive behavior, that would not be possible, because not credible, without committment.

This finding stands in complete contrast to the static quantity games introduced by VFJS. In a dynamic framework with sticky prices entrepreneurs are not interested in making their managers aggressive, whereas in the static version making managers aggressive is the essence for acheiving competitive advantage.

Int the next section we give an explanation why, although our solutions are quite different, they do not contradict VFJS.

4 The corresponding static game

In order to better understand the economic intuition of our results derived in the preceding section, we now investigate how our findings correspond to the settings of a static delegation model. Using the insights of Dockner (1992) we relate the closed loop equilibrium of the dynamic quantity game to a conjectural variations equilibrium of a corresponding static game with constant and negative conjectures. This analogy then helps us to motivate the equilibrium contracts.

4.1 Equivalence of dynamic closed loop strategies and static conjectural variations

In Dockner (19929 the relationship between dynamic oligopolistic competition and static conjectural variations equilibria is explored. Using an infinite horizon adjustment cost model he demonstrates that any steady state closed-loop (subgame-perfect) equilibrium coincides with a conjectural variations equilibrium. In the case of linear demand and quadratic costs the dynamic conjectures consistent with closed-loop steady state equilibria are negative, constant, symmetric, and vary continuously with the discount rate (and the adjustment cost parameter) in an interval between the static consistent conjectures and zero (Cournot). We will show that these findings also hold with a small adaption in the sticky price model.

The existence proof already reveals important properties of the closed-loop equilibrium for the dynamic Cournot model with sticky prices. In particular, the linear decision rules (14) and (15) are of interest. They summarize the reactions of firm i to an action taken by the rival. With the decision rules (14) and (15) each firm increases its output with an increase in the market price. This relationship is the driving force behind the long-run qualitative characteristics of the equilibrium.

Proposition 3 *The steady state closed-loop (subgame-perfect) equilibrium of the dynamic game with sticky prices coincides with a conjectural variations equilibrium of the corresponding static game with constant conjectures equal to* $\chi = -1 + \frac{s}{r+s+s(1-sK)}$. *The conjectures are symmetric and satisfy*

$$-1 < -1 + \frac{s}{r+s+s(1-sK)} < \frac{-3+\sqrt{5}}{2}. \tag{33}$$

Proof A closed-loop equilibrium has to satisfy Pontryagin's maximum principle. We have to keep in mind that the strategies of players i and j depend on the state variable p. Hence, the adjoint equations become

$$\dot{\lambda} = (r+s)\lambda_i - q_i + s\frac{\partial q_i}{\partial p}. \tag{34}$$

From the decision rule (24) we know that $\frac{\partial q_i}{\partial p} = (1-sK)$ so that the adjoint equations become

$$\dot{\lambda} = (r+s+s(1-sK))\,\lambda_i - q_i. \tag{35}$$

A steady state equilibrium is characterized by the set of equations

$$q_i = p + \alpha_i - s\lambda_i \tag{36}$$
$$p = a - q_i - q_j \tag{37}$$
$$\lambda_i = \frac{q_i}{r + s + s(1 - sK)} \tag{38}$$

which result in

$$q_i \left(2 + \frac{s}{r + s + s(1 - sK)} \right) + q_j = a + \alpha_i. \tag{39}$$

If we consider the corresponding static game with linear demand $p = a - q_i - q_j$ and quadratic costs $\frac{1}{2} q_i^2$ and derive the first order conditions for a conjectural variations equilibrium with constant conjectures given by $\frac{dq_j}{dq_i} = \chi$, we get

$$q_i (3 + \chi) + q_j = a + \alpha_i. \tag{40}$$

Comparing systems (39) and (40) reveals that the steady state closed-loop equilibrium can be viewed as a conjectural variations equilibrium with constant conjectures equal to

$$\chi = -1 + \frac{s}{r + s + s(1 - sK)}. \tag{41}$$

These conjectures are continuous in (s, r) and they do satisfy the following conditions. Let us define

$$f(s, r) = \frac{s}{r + s + s(1 - sK)}. \tag{42}$$

Then we find for a given $r > 0$

$$f(0, r) = 0, \;\; f(\infty, r) = \frac{1}{1 + \sqrt{\frac{2}{3}}} \tag{43}$$

and

$$\frac{\partial f}{\partial s} = f_s = \frac{r + s^2 \frac{\partial sk}{\partial s}}{(r + s + s(1 - sK))^2}. \tag{44}$$

It is now easily checked that $\frac{\partial sk}{\partial s} > 0$ so that $f_s > 0$ for all $r, s > 0$. On the contrary we find for a given constant $s > 0$ that

$$f(s,\infty)=0,\ f(s,0)=\frac{1}{1+\sqrt{\frac{2}{3}}} \tag{45}$$

and that $f_r<0$. Hence, we can conclude that $f(s,r)<\frac{1}{1+\sqrt{\frac{2}{3}}}$ for all (s,r). Therefore we get

$$-1<-1+\frac{s}{r+s+s(1-sK)}<-1+\frac{1}{1+\sqrt{\frac{2}{3}}}<\frac{-3+\sqrt{5}}{2} \tag{46}$$

■

As mentioned above the key to understanding the results of Proposition 3 is the feedback decision rule adopted by each player when selecting equilibrium actions. With this rule each firm increases output with an increase in the market price. Let us explore what the consequences of this behavior are. Consider the case where one firm ignores this decision rule (i.e. the rival's reaction to an increase in market price) and simply makes the Cournot assumption that its rival's output will remain at its present level. In this setting the firm makes its decision on the basis of the residual demand curve it faces. If it does take into account the rival's reaction, it knows that as it contracts its output and market price increases the rival will expand its output. Therefore, the movement up the residual demand curve is offset partly by an inward shift of the residual demand curve as the rival increases its output. This behavior can be interpreted as a conjectural variations equilibrium with negative conjectures. Consequently, prices and quantities are more competitive than Cournot. This result corresponds to the model with adjustment costs, see for instance Dockner (1992). What is different is the numerical value of the dynamic conjecture. Contrary to the dynamic Cournot game with adjustment costs conjectures are now below the level of consistent ones in the sense of Bresnahan (1981).

Corollary 1 *The dynamic conjectures $\chi=\chi(s,r)$ of the Cournot game with sticky prices vary continuously with (s,r). In the limit as the discount rate becomes very large, i.e. $r\to\infty$, or the speed of adjustment is small $s\to 0$, the dynamic conjectures χ converge to -1. In the limit as the speed of adjustment becomes very large $(s\to\infty)$, or the discount rate is small $(r=0)$ the conjectures χ converge to $-1+\frac{1}{1+\sqrt{\frac{2}{3}}}$.*

Together with inequality (46), Corollary 1 demonstrates that oligopolistic competition with sticky prices is consistent with dynamic conjectures from the interval $[-1,\kappa]$ ($\kappa=\max_{r,k}\chi(r,k)=-1+\frac{1}{1+\sqrt{\frac{2}{3}}}$) which is a subset of $[-1,\frac{-3+\sqrt{5}}{2}]$. Thus, according to this differential game model, not every conjectural variations equilibrium is consistent with dynamic competition played with linear differentiable closed-loop strategies. The result that the long run closed-loop equilibrium corresponds variations equilibrium with negative conjectures is comparable to the dynamic model with adjustment costs. But there is one important difference. While the dynamic conjectures of the adjustment cost model are always above the consistent conjecture in the sense of Bresnahan (1981) the dynamic conjectures for the model with sticky prices are always below it. In a sense dynamic competition the sticky price model played with closed-loop strategies results in an equilibrium price that is closer to the competitive price than the corresponding price in the adjustment cost model. For both scenarios, however, strategic interactions result in more competitive behavior than Cournot.

That prices in the sticky price model are below that found for consistent conjectural variations and those found for the adjustment cost model can be given the following interpretation. Sticky prices imply that the market price reacts only slowly to a change in equilibrium quantities. Hence, firms have an incentive to expand output while price remains unchanged and profits rise. This explains why firms will produce more in this setting than in the model with adjustment costs where prices immediately jump to their level on the demand curve.

4.2 Incentives for managers

Since the equilibrium of our dynamic Cournot game coincides with a conjectural variations equilibrium with constant and negative conjectures we can solve the incentive game for by simply using a conjectural variations equilibrium approach.

Suppose now the static duopoly game with the identical firms 1 and 2 in a homogenous market. The inverse demand function is again given by

$$p = a - q_1 - q_2 \tag{47}$$

and the cost functions

$$C_i = \frac{1}{2} q_i^2 \tag{48}$$

The profit functions are

$$\pi_i = (1 - q_i - q_j)\, q_i - \frac{1}{2} q_i^2 \tag{49}$$

The owners delegate their output decision to manager who are compensated according to their contract and maximize their utility function (8).

We consider a conjectural variations framework with the conjectural variations parameter λ_i. So FOC for the managers look as follows

$$\frac{dU_i}{dq_i} = 1 - q_i(3 + \lambda_i) - q_j + \alpha_i \tag{50}$$

Solving the FOC (50) gives the reaction functions

$$q_i^R = \frac{1 - q_j}{\lambda_i + 3} \tag{51}$$

and the equilibrium quantities dependent of α_k.

$$q_i(\alpha_i, \alpha_j) = -\frac{\lambda_j c - 3\alpha_i + \alpha_j - \alpha_i \lambda_j - \lambda_j - 2}{\lambda_j \lambda_i + 3\lambda_i + 3\lambda_j + 8} \tag{52}$$

To get the corresponding game of the above introduced dynamic game we substitute the static conjectural variations with the dynamic conjectures χ of equation (41) instead of the static conjectures λ_i. This yields

$$q_i(\alpha_i, \alpha_j) = \frac{(a + \alpha_i)(\chi + 2) + \alpha_i - \alpha_j}{(3 + \chi)^2 - 1}. \tag{53}$$

If no one can use delegation ($\alpha_k = 0$) the in equilibrium functions are

$$q_i^O = \frac{1}{\chi + 4} \tag{54}$$

$$\pi_i^O = \frac{(2\chi + 3)}{2(\chi + 4)^2}. \tag{55}$$

The Index O means owner managed firms. As we have shown that dynamic conjectures are negative quantities in dynamic Cournot competition with sticky prices are higher and profits lower and therefore dynamic competition is more intense than static quantity competition ($\chi = 0$).

Looking at the reaction function in (52) we see that positive α_i makes firm i aggressive and harms the rival. In classical static Cournot competition and following the findings of VFJS we could expect $\alpha > 0$. But surprisingly, the optimal contract is different from the classical Cournot game.

Proposition 4 *The optimal contract corresponding to a steady state closed-loop equilibrium in the model with sticky prices contains a negative value of the incentive parameter α for the manager and enforces a less aggressive behavior compared to the equilibrium with owner managed firms.*

Proof Given the closed-loop steady state equilibrium quantities from (52) the objective functions of the respective owners become

$$\Pi_i(\alpha_i, \alpha_j) = (a - q_i(\alpha_i, \alpha_j) - q_j(\alpha_i, \alpha_j))\, q_i(\alpha_i, \alpha_j) - \frac{1}{2}\left(q_i(\alpha_i, \alpha_j)\right)^2 \qquad (56)$$

The first order conditions for a Nash equilibrium of the game played by the owners are

$$\frac{\partial \Pi_i}{\partial \alpha_i} = 0 \qquad (57)$$

These conditions result in a linear system of equations with a unique symmetric solution given by

$$\alpha^* = \frac{(\chi(\chi + 3) + 1)}{4\chi + 11} \qquad (58)$$

It is now easily seen that α^* is negative whenever $(\chi(\chi + 3) + 1) < 0$. This last inequality, however is equivalent to

$$\chi < \frac{-3 + \sqrt{5}}{2}. \qquad (59)$$

Inequality (59) simply expresses that the conjecture χ is below Bresnahan's consistent one. This, however has been demonstrated in Theorem 3. ■

This last result is very interesting. Contrary to the standard static duopoly models by VFJS optimal incentive contracts in the quantity game with firms employing dynamic strategies is unambiguously determined and given by reducing competitive pressure. The interpretation of this is the following. As it is common knowledge in industrial organization literature the firm owners have an incentive to introduce strategies that make their respective firms a Stackelberg leader in the output game. With no prevenient strategic actions firms in the sticky price model overexpand output (dynamic conjectures are close to -1, the competitive outcome) so that a shift towards a Stackelberg leader position requires the introduction of of less aggressive behavior. This commitment increases firms' profits since with lowered outputs firms are able to charge higher prices while marginal costs declined (convex cost curve).

As a last result we can strengthen the findings of Fudenberg and Tirole (1984) that strategic behavior in the sense of whether commitments are used as over- or underinvestment is not only driven by the fact that competitive variables are strategic substitute or complements. The at least equally important ingredient is the conjectured reaction of the competitor. In their seminal paper Fudenberg and Tirole (1984) call this effect the effect that makes a firm tough or weak. It is now possible to render that statement more precisely.

Proposition 5 *If competition is in quantities (strategic substitutes) strategic behavior makes a firm aggressive as long as conjectural variations are above the consistent ones. If conjectural variations are below the consistent conjectures strategic instruments are used to reduce competition.*

Proof In the market stage the managers select the quantity such that $U_i = \pi_i + \alpha_i q_i$ is maximized. The first order condition reads

$$\frac{dU_i}{dq_i} = \frac{\partial \pi_i}{\partial q_i} + \alpha_i = 0. \tag{60}$$

This condition yields the optimal production quantity of a firm determined by its manager, $q_i^*(\alpha_1, \alpha_2)$. To determine the delegation incentives of the owners, we write the FOC at the delegation stage as

$$\frac{d\pi_i}{d\alpha_i} = \frac{\partial \pi_i}{\partial \alpha_i} + \frac{\partial \pi_i}{\partial q_i}\frac{\partial q_i^*}{\partial \alpha_i} + \frac{\partial \pi_i}{\partial q_j}\frac{\partial q_j^*}{\partial \alpha_i}. \tag{61}$$

Since α_i has no direct effect on π_i, the first term on the right side of equation (61) is zero. Taking (60) into account and the derivative $\frac{\partial q_i^*}{\partial \alpha_i} > 0$ we get for the middle term (the strategic effect on own behavior) of (61)

$$\underbrace{\frac{\partial \pi_i}{\partial q_i}}_{-\alpha_i}\underbrace{\frac{\partial q_i^*}{\partial \alpha_i}}_{>0}. \tag{62}$$

Obviously any value of α_i different from zero has negative effects on profits. This is the reason why a decision maker would choose always $\alpha_i = 0$ in the absence of competition. That means that the only left part that determines the investment strategy is the last term of (61). This strategic effect on the rival's behavior is two folded. The first ingeredient is the conjecture of firm i about the quanity reaction of firm j. On the other hand this conjecture is not necessarily true and the effective reaction is different from the conjectured one. So we get two equations out of the last term of (61). The one that includes the conjectures of i about j's reaction

$$\frac{\partial \pi_i}{\partial q_j}\frac{\partial q_j^*}{\partial \alpha_i} = \frac{\partial \pi_i}{\partial q_j}\chi\frac{\partial q_i^*}{\partial \alpha_i} \tag{63}$$

and the real reaction

$$\frac{\partial \pi_i}{\partial q_j}\frac{\partial q_j^*}{\partial \alpha_i} = \frac{\partial \pi_i}{\partial q_j}R_j\frac{\partial q_i^*}{\partial \alpha_i}. \tag{64}$$

So the overall strategic effect results in

$$\frac{\partial \pi_i}{\partial q_j}\frac{\partial q_j^*}{\partial \alpha_i} = \frac{\partial \pi_i}{\partial q_j}(R_j - \chi)\frac{\partial q_i^*}{\partial \alpha_i}. \tag{65}$$

In (65) we see that the strategic effect vanishes if the change of profit evoked by a change of the rivals output can be perfectly predicted. This optimal answer is only possible if firm i has consistent conjectures about the competitor. Are conjectures above (below) consistent conjectures (65) becomes negative (positive). Summarizing we get for the whole derivative

$$\frac{d\pi_i}{d\alpha_i} = \underbrace{\frac{\partial \pi_i}{\partial \alpha_i}}_{=0} + \underbrace{\frac{\partial \pi_i}{\partial q_i}}_{-\alpha_i}\underbrace{\frac{\partial q_i^*}{\partial \alpha_i}}_{>0} + \underbrace{\frac{\partial \pi_i}{\partial q_j}}_{<0}\underbrace{(R_j - \chi)}_{\lesseqgtr 0}\underbrace{\frac{\partial q_i^*}{\partial \alpha_i}}_{>0}. \tag{66}$$

So for $\alpha_i = 0$ an owner has an incentive to raise (decrease) the value of α_i depending on whether conjectures are above (below) consistent conjectures. ■

This finding is in line with Hwang and May (1995), where they analyze a similar model. These results show that it is possible to compare dynamic competition with it's static counterpart. Furthermore Cournot competition does not always cause overinvestment behavior by the competitors. This result is very sensitive to whether the commitment makes a firm tough or soft ant thus, whether conjectures are above or below the consistent ones. In the dynamic setup this is crucual to the kind of dynamics of the market. In the sticky price model firms tend to act very aggressively and offer a high amount of goods because they want to exploit the advantages of the sticky consumer behavior. Managers are hired to decrease the resulting intensity of competition. In a model with different structural dynamics e.g. the model with adjustment costs [5] the effects are quite different. Although it is not explicitely considered in this paper we want to note that in this case the contract for managers is written such that the incentive parameter α is positive.

[5] This model is used by Driskill and McAfferty (1989) and Dockner (1992).

5 Conclusion

In this paper we have analyzed strategic incentives under the assumption that managers play a dynamic output game while owners choose their incentives on the basis of a single parameter. It turns out that profit maximization is not the equilibrium incentive for managers. Dynamic competition played with Markovin (state dependent) decision rules causes managers to set pro competitive quantities (quantities that are higher than corresponding Cournot quantities) which ultimately result in lower than optimal profits. Hence, equilibrium incentives penalize this pro competitive behavior and introduce an additional cost for managers which causes them to reduce output and hence increase the present value of profits. We provide a detailed explanation for these incentives and show that the dynamic incentives can be replicated within a static game when managers have to play a conjectural variations equilibrium.

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