Certification and Exchange in Vertically Concentrated Markets

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Abstract

Drawing from a case study on upstream supply procurement in the automotive industry, we model the interaction between a monopolistic supplier and a monopolistic buyer when the quality of the product procured is unknown to the buyer. If procured by the seller a certifier can credibly signal quality to the buyer. If procured by the buyer, certification induces the buyer not to pay too much for a bad product. We show that seller–induced certification maximizes the certifier's profit and welfare, and that this result is reflected in our case study example.

1 Introduction

In many, if not most markets, the seller of a good knows its quality better than its buyer. This informational asymmetry yields a demand for an independent certifier who reduces the degree of asymmetric information by evaluating the commodity's quality and credibly communicating its result (e.g., Biglaiser 1993, Lizerri 1999). More specifically, a demand for certification exists from

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both sides of the market: The seller of a high quality commodity has a demand for certification because it allows him to sell his high quality at higher prices. Alternatively, the buyer has a demand for certification to check the producer's claims about the quality and thereby prevent himself from buying a commodity at a price that *ex post* is not justified by its quality.

The question then arises as to whom – to the seller or to the buyer – a profit–maximizing certifier will address himself and whether the certifier's choice is efficient. The answers to these two straightforward questions are not obvious. In particular, it is unclear to whom certification is more valuable, and thus from whom the certifier can extract more rents. Moreover, it is unclear whether this rent extraction leads to a socially desirable outcome.

Drawing from a case study on upstream supply procurement in the automotive industry, this paper gives theoretical answers and insights to these two questions. The theoretical model we use is motivated by a case study example taken from a large scale study on upstream relations in the automotive industry. The model's main economic insight is that the exact economic role of certification depends on whether the buyer or the seller demands it. Because the buyer's demand for certification originates from his desire to check the seller's (implicit) claim about quality, certification plays the role of a costly *inspection device* when the buyer demands it. In contrast, certification plays the role of a *signalling device* with seller–induced certification because the seller's demand for certification originates from his desire to prove its quality to the buyer.

We show that these two different roles of certification lead to two fundamentally different economic games. When certification is an inspection device, the buyer and the seller play an inspection game with a typical mixed strategy equilibrium. For this game, the optimal pricing behavior of the certifier is to pick a price of certification that maximizes its revenue in the mixed equilibrium of the subsequent inspection game. Instead, when the certifier addresses himself to the seller, the buyer and the seller play a signalling game. Hence, the optimal pricing behavior of the certifier is to pick a price of certification that maximizes revenue but still ensures that certification is an effective signalling device and separates high quality sellers from the low quality ones. Hence, the certifier must ensure that the price of certification is low enough such that the high quality producer wants to signal its high quality but, at the same time, high enough such that the low quality seller does not find it worthwhile to buy certification and mimic the high quality seller. Note that the equilibrium outcome in the signalling game tends to be more efficient than the equilibrium outcome in the inspection game because the mixed equilibria of the inspection game yield the inefficiency that the good is not always traded. Hence, in the inspection game not all benefits of trade are realized, whereas in the signaling game they are. We demonstrate that this inefficiency makes seller–induced certification more social desirable.

The rest of the paper is organized as follows. In the next Section 2, we elaborate in detail on the automotive industry case, that motivates our theoretical model. In Section 3, we describe the model. In Section 4, we derive the results for buyer–induced certification. Section 5 contains the results for seller–induced certification as well as the comparison between the two from the point of view of the certifier. In Section 6 we evaluate that outcome from a welfare point of view. Section 7 contains a literature review and Section 8 concluding remarks on possible extensions of the model. All proofs are relegated to the Appendix.

2 A Case Study Example

In this section, we study the upstream supply relationships in the automotive industry. It serves to motivate the exact theoretical model that we use to study our questions. The certifier we focus on is EDAG, a German engineering company centering its activities on the development and prototype–construction of cars. One of its central divisions includes the independent certification of car modules and systems. In this function it serves all major car producers world wide.[1]

At first sight, one would expect that, for a highly capital intensive in-

[1]See http://www.edag.de/produkte/prueftechnik/automotive/index_html

dustry such as the automotive one, the development and production pattern of car parts is dominated by economies of scope that potentially arise from the use of the same parts within different car models.[2] Surprisingly, this is *not* the case. At least when it comes to German cars, most, if not all parts that are assembled for a particular car model are specific to this very model. In fact, the parts are not even shared on different car models within the same firm.[3] Also, most parts, and in particular all complex components or modules, are supplied by *one* upstream supplier.

We extract from these observations two building blocks for our theoretical model. First, in most cases there is exactly one supplier and one buyer of the product; the buyer–seller relationship is a bilateral monopoly. Second, at the time the procurement decision is taken, the quality of the good is unknown by the downstream manufacturer; there is asymmetric information between the buyer and the seller.

We now turn to the certifier's position. An interview with the head of EDAG's testing and certification division revealed that while there are six to seven competitors in the market for certification worldwide, EDAG offers the widest portfolio of components to be examined and certified, and holds a dominant, if not exclusive, market position in a number of components. Hence at least in some markets EDAG as a certifier holds a quasi monopoly. We, therefore, model the certifier as a monopolist who determines the terms of certification. In particular, it can choose whether it serves the demand from the seller or the buyer and at which price.

[2]The evidence is taken from Mueller et al. (2007), and from a large scale study conducted in 2007/08 by Konrad Stahl et al. for the German Association of Automotive Manufacturers (VDA) on Upstream Relationships in the Automotive Industry. Survey participants are car producers and their upstream suppliers. All German car producers were questioned as to their upstream procurement relationships (differentiated by part category and degree of perceived competitiveness of the part specific upstream industry), and 13 important upstream suppliers were questioned as to their relationships (differentiated by part category) with each one of the German automotive suppliers. In addition, in depth interviews were conducted with corporate representatives involved in these relationships.

[3]This does not preclude the development of norms ("platforms") to enhance the efficiency of development procedures and parts production.

Typically, EDAG performs tests on either the prototype of a car module or on that module as part of the so–called null–series, which directly precedes the start of production. One of the key test criteria is the fulfilment of safety norms. Importantly, our interview revealed that the testing of car modules and systems is predominantly performed on the request of the upstream supplier rather than the down–stream firm. Moreover, the buyer conditions his actual purchase on the quality certification.

3 Model

Consider a seller offering a good at price p whose quality, before certification, is revealed only to him and is unobservable to a buyer. From the buyer's point of view, the seller's quality is high, q_h, with probability λ and low, $q_l > 0$, with probability $1 - \lambda$, where $\Delta q \equiv q_h - q_l > 0$. The good's quality is identified with the buyer's willingness to pay. The risk neutral buyer is therefore willing to pay up to a price that equals expected quality $\bar{q} \equiv \lambda q_h + (1 - \lambda)q_l$. The seller of high quality has a production cost $c_h > 0$ and the low quality seller has a production cost $c_l = 0$. If the seller does not produce, he receives his reservation payoff of zero.

We assume that the high quality good delivers higher economic rents: $q_h - c_h > q_l - c_l = q_l > 0$. Moreover, the production costs of a high quality product exceeds the average quality, $c_h > \bar{q}$. This assumption creates a lemon's problem and leads to adverse selection. Without certification, a high quality seller would, therefore, not offer his good to the market and the market outcome with informational asymmetries is inefficient. Without the informational asymmetry, however, the high quality seller could sell his good for a price $q_h > c_h$. Consequently, the high quality seller has demand for a certifier who reveals the good's true quality to the buyer. Clearly, the high quality seller is willing to pay at most $q_h - c_h$ for the certification.

On the other hand, also the buyer is willing to pay a positive price for certification. Whenever the seller is unwilling to sell the good at the market price, he signals that his good has high quality. In this case, the buyer may engage the services of a certifier to ascertain the good has indeed high quality and buy it at a higher price.

This reasoning implies that both the buyer and the seller have a demand for certification. For a monopolistic certifier this raises the question as to whom he should offer his services. As motivated in the previous section, we consider a monopolistic certifier. The certifier has the technology to perfectly detect the seller's quality at a cost $c_c \in [0, q_h - c_h)$ and to announce it publicly.

The certifier's problem is as follows. In an initial stage, he has to decide whether to offer his services to the buyer or the seller. After this decision, he sets a price p_c at which the buyer or the seller, respectively, can obtain certification. In accordance with most literature on certification, we focus on honest certification where the certifier cannot be bribed. Our research question is whether the monopolistic certifier is better off offering his services to the uninformed buyer or to the informed seller. In addition, we are interested in the welfare properties of his decision. In order to answer these questions, we separately study "buyer–induced", and "seller–induced" certification, and contrast their outcomes from both the certifier's and a social welfare point of view.

4 Buyer–Induced Certification

In this section we consider the certification problem when the buyer decides about, and pays for the certification services. Before analyzing the formal model, it is helpful to develop some intuition about this setup by first discussing the role of certification intuitively.

Intuitively, buyer–induced certification enables the buyer to check the seller's claim about the quality of his product, and to assure herself of the quality of the good. In particular, certification offers the buyer protection against a low quality seller who claims or pretends to have high quality. From the buyer's perspective, therefore, certification is an *inspection device* to detect low quality sellers.

The economic game underlying buyer–induced certification, therefore, resembles an inspection game with its typical mixed strategy equilibrium. Indeed, a pure strategy equilibrium in which the buyer never buys certification cannot exist, because it would give the low quality seller an incentive to claim high quality – yet against this claim the buyer would have a strong incentive to buy certification. Likewise, an equilibrium in which the buyer always buys certification cannot exist either, because it keeps the low quality seller from claiming high quality – yet against such behavior certification is only wasteful for the buyer. Consequently, we typically have a mixed strategy equilibrium, where the low quality seller cheats with some probability and claims to offer high quality, and the buyer certifies with some probability when the seller claims to have high quality.

Hence, buyer–induced certification plays the role of reducing cheating. The buyer's demand for certification will therefore be high when the problem of cheating is large. This reasoning suggest that a monopolistic certifier, who targets his services towards the buyer, will choose a certification price that maximizes the buyer's cheating problem.

A closer look reveals that the buyer's cheating problem depends on two factors: the buyer's uncertainty and the seller's price of the good. First, the buyer's cheating problem is bigger when she is more uncertain about the true quality offered by the seller. Second, checking true quality through certification is especially worthwhile for intermediate prices of the good. Indeed, for a low price the buyer would not lose much from simply buying the good uncertified. By contrast, when the price is high, the buyer would not lose much from not buying the good at all. Hence, the buyer's willingness to pay for certification is largest for intermediate prices that are neither too low nor too high.

Our intuitive reasoning therefore suggests that under buyer–induced certification a monopolistic certifier will choose his price for certification, p_c, so that it induces a relatively high uncertainty for the buyer and an intermediate price for the good. We now present the formal analysis of our framework to confirm that this intuition is correct but nontrivial.

With buyer–induced certification, the parties play the following game:

t=1 The certifier sets a price p_c for his service.

t=2 Nature selects the quality $q_i, i \in \{l, h\}$, of the good offered by the seller.

t=3 The seller offering the good of quality q_i and cost c_i decides about the price p at which he offers the good.

t=4 The buyer decides whether or not to demand certification of the good.

t=5 The buyer decides whether or not to buy the good.

Note that we assume that if the seller q_i sets a price in stage 3, he incurs the production cost c_i for sure, even though the buyer may decide not to buy the good in stage 5. This assumption is natural under different interpretations of certification.

First, certification may mean that the certifier inspects the actual good the buyer is interested in. In this case, the good must already be produced in order for the certifier to inspect it, and the seller must therefore have incurred the production cost even if the buyer decides not to acquire it. A second possibility is that the certifier determines the seller's product quality by inspecting his production facility, and certifying his production technology. In this case, the production cost c_h may be interpreted as a fixed cost that differs between high and low quality sellers. Under both interpretations, the seller incurs the cost even if the buyer, in the end, does not buy the product.

We focus on the Perfect Bayesian Equilibria (PBE) of the game described above. Note that after the certifier has set its price p_c, a proper subgame, $\Gamma(p_c)$, starts with nature's decision about the quality of the seller's product. The subgame $\Gamma(p_c)$ is a signalling game where the seller's price p may or may not reveal his private information about the quality of the good.

In the subsequent analysis, we first consider the PBE of the subgame $\Gamma(p_c)$. A PBE specifies three components: First, the seller's pricing strategy as a function of the good's type q_i; second, the buyer's belief $\mu(p)$ after observing the price p; third, the buyer's behavior; in particular whether or not to buy certification and the actual good.

We allow the seller to randomize over an infinite but countable number of prices. Consequently, we denote the strategy of the seller of quality q_i by the function $\sigma_i : R_+ \to [0,1]$ with the interpretation that $\sigma_i(p_j)$ denotes the probability that the seller with quality q_i chooses the price p_j. Thus, for both $i \in \{h, l\}$,

$$\sum_j \sigma_i(p_j) = 1.$$

The buyer's decisions are based on his belief specified as a function $\mu : \mathbb{R}_+ \to [0,1]$ with the interpretation that, after observing price p, the buyer believes that the seller is of type q_h with probability $\mu(p)$.

We can express the buyer's behavior after observing the price p and possessing some belief μ by the following six actions:

1. Action s_{nn}: The buyer does not buy certification nor buy the good. This action yields the payoff

$$U(s_{nn}|p, \mu) = 0.$$

2. Action s_{nb}: The buyer does not buy certification, but buys the product. This action yields the expected payoff

$$U(s_{nb}|p, \mu) = \mu q_h + (1 - \mu)q_l - p.$$

3. Action s_{ch}: The buyer buys certification and buys the product only when the certifier reveals high quality. This action yields the expected payoff

$$U(s_{ch}|p, \mu) = \mu(q_h - p) - p_c.$$

4. Action s_{cb}: The buyer buys certification and buys the product irrespective of the outcome of certification. This action yields the expected payoff

$$U(s_{cb}|p, \mu) = \mu(q_h - p) + (1 - \mu)(q_l - p) - p_c.$$

Clearly, $U(s_{cb}|p,\mu) < U(s_{nb}|p,\mu)$ for any $p_c > 0$ so that the action s_{nb} dominates the action s_{cb}.

5. Action s_{cl}: The buyer buys certification and buys the product only when the certifier reveals low quality. This action yields the expected payoff

$$U(s_{cl}|p,\mu) = (1-\mu)(q_l - p) - p_c.$$

   Clearly, $U(s_{cl}|p,\mu) \leq U(s_{nb}|p,\mu)$ for $p \leq q_h$ and $U(s_{cl}|p,\mu) \leq U(s_{nn}|p,\mu)$ for $p > q_h$. Hence, also the action s_{cl} is weakly dominated.

6. Action s_{cn}: The buyer demands certification and does not buy the product. This action yields the expected payoff

$$U(s_{cn}|p,\mu) = -p_c.$$

   Clearly, $U(s_{cn}|p,\mu) < U(s_{nn}|p,\mu)$ for any $p_c > 0$ so that the action s_{cn} is dominated.

To summarize, only the first three actions s_{nn}, s_{nb}, s_{ch} are not (weakly) dominated for some combination (p,μ). The intuition is straightforward: the role of certification is to enable the buyer to discriminate between high and low quality. It is therefore only worthwhile to buy certification when the buyer uses it to screen out bad quality.

In the following, we delete the weakly dominated actions from the buyer's action space. Consequently, we take the buyer's action space as $S \equiv \{s_{nn}, s_{nb}, s_{ch}\}$. Since we want to allow the buyer to use a mixed strategy, we let $\sigma(s|p,\mu) \in [0,1]$ represent the probability that the buyer takes action $s \in S = \{s_{nn}, s_{nb}, s_{ch}\}$ given price p and belief μ. Thus

$$\sum_{s \in S} \sigma(s|p,\mu) = 1.$$

A PBE in our subgame $\Gamma(p_c)$ can now be described more specifically: it is a tuple of functions $\{\sigma_l, \sigma_h, \mu, \sigma\}$ satisfying the following three equilibrium conditions. First, seller type i's pricing strategy σ_i must be optimal with respect to the buyer's strategy σ. Second, the buyer's belief μ has to be consistent with the sellers' pricing strategy, whenever possible. Third, the buyer's strategy σ must be a best response given the observed price p and her beliefs μ.

We start our analysis of the Perfect Bayesian Equilibria of $\Gamma(p_c)$ by studying the third requirement: the optimality of the buyer's strategy given a price p and beliefs μ.

Fix a price $\bar{p}$ and a belief $\bar{\mu}$. Then the pure strategy s_{nn} is a best response whenever $U(s_{nn}|\bar{p},\bar{\mu}) \geq U(s_{nb}|\bar{p},\bar{\mu})$ and $U(s_{nn}|\bar{p},\bar{\mu}) \geq U(s_{ch}|\bar{p},\bar{\mu})$. It follows that the strategy s_{nn} is a best response whenever

$$(\bar{p},\bar{\mu}) \in S(s_{nn}|p_c) \equiv \{(p,\mu)|p \geq \mu q_h + (1-\mu)q_l \ \wedge \ p_c \geq \mu(q_h - p)\}.$$

Likewise, the pure strategy s_{nb} is (weakly) preferred whenever $U(s_{nb}|\bar{p},\bar{\mu},p_c) \geq U(s_{nn}|\bar{p},\bar{\mu},p_c)$ and $U(s_{nb}|\bar{p},\bar{\mu},p_c) \geq U(s_{ch}|\bar{p},\bar{\mu},p_c)$. It follows that the strategy s_{nb} is a best response whenever

$$(\bar{p},\bar{\mu}) \in S(s_{nb}|p_c) \equiv \{(p,\mu)|p \leq \mu q_h + (1-\mu)q_l \ \wedge \ p_c \geq (1-\mu)(p - q_l)\}.$$

Finally, the pure strategy s_{ch} is (weakly) preferred whenever $U(s_{ch}|\bar{p},\bar{\mu},p_c) \geq U(s_{nn}|\bar{p},\bar{\mu},p_c)$ and $U(s_{ch}|\bar{p},\bar{\mu},p_c) \geq U(s_{nb}|\bar{p},\bar{\mu},p_c)$. It follows that the strategy s_{ch} is a best response whenever

$$(\bar{p},\bar{\mu}) \in S(s_{ch}|p_c) \equiv \{(p,\mu)|p_c \leq \mu(q_h - p) \wedge p_c \leq (1-\mu)(p - q_l)\}.$$

Since a mixed strategy is only optimal if it randomizes among those pure strategies that are a best response, we arrive at the following result:

Lemma 1 *In any Perfect Bayesian Equilibrium* $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ *of the subgame* $\Gamma(p_c)$ *we have for any* $s \in S = \{s_{nn}, s_{nb}, s_{ch}\}$,

$$\sigma^*(s|p,\mu) > 0 \Rightarrow (p,\mu^*(p)) \in S(s|p_c). \tag{1}$$

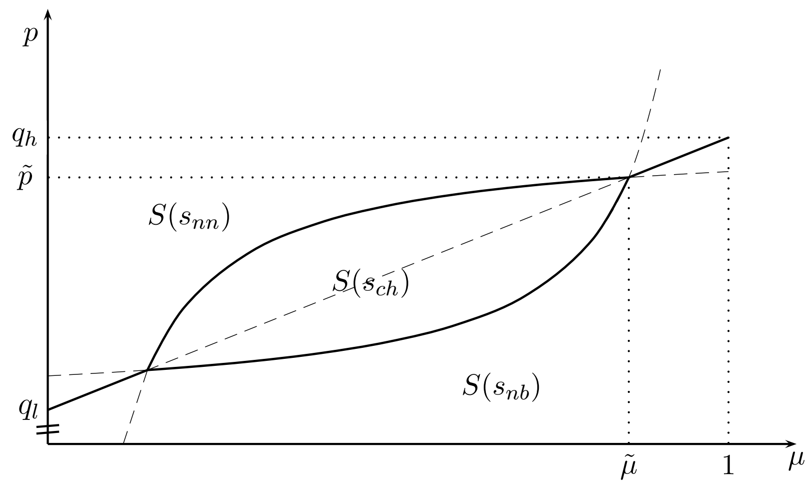

Figure 1: Buyer's buying behavior for given $p_c < \Delta q/4$.

Figure 1 illustrates the buyer's behavior for a given certification price p_c. For low prices p the buyer buys the good uncertified, $(p, \mu) \in S(s_{nb})$, whereas for high prices p the buyer refrains from buying, $(p, \mu) \in S(s_{nn})$. As long as $p_c < \Delta q/4$ there is an intermediate range of prices p and beliefs μ such that the buyer demands certification, i.e. $(p, \mu) \in S(s_{ch})$. In this case, the buyer only buys the product when certification reveals that it has high quality. Intuitively, the buyer demands certification to ensure that the highly priced product is indeed of high quality. Note that apart from points on the thick, dividing lines, the buyer's optimal buying behavior of both certification services and the product is uniquely determined, and mixing does not take place.

For future reference we define

$$\tilde{p} \equiv \left(q_h + q_l + \sqrt{\Delta q(\Delta q - 4p_c)}\right)/2$$

and

$$\tilde{\mu} \equiv \left(1 + \sqrt{1 - 4p_c/\Delta q}\right)/2.$$

Note that if the seller prices at $\tilde{p}$ and the buyer has beliefs $\tilde{\mu}$, the buyer is indifferent between all three decisions namely not to buy the good, s_{nn}, to buy the good uncertified, s_{nb}, or to buy the good only after it has been certified as high quality, s_{ch}.

We previously argued that the monopolistic certifier benefits from high buyer uncertainty and an intermediate price of the good. We can give precision to this statement. The buyer's willingness to pay for certification is the difference between her payoff from certification and the next best alternative, namely either to buy the good uncertified, or to not buy the good at all. More precisely, given her beliefs are μ, the difference in the buyer's expected payoffs between buying the high quality good when certified and buying any good uncertified is

$$\Delta U^1 \equiv \mu(q_h - p) - (\bar{q} - p).$$

Similarly, the difference in the buyer's expected payoffs between buying the good only when certified and buying the good not at all is

$$\Delta U^2 = \mu(q_h - p).$$

Hence, the buyer's willingness to pay for certification is maximized for a price $\hat{p}$ and a belief $\hat{\mu}$ that solves

$$\max_{p,\mu} \ \min\{\Delta U^1, \Delta U^2\}.$$

The solution is $\hat{\mu} = 1/2$ and $\hat{p} = (q_h + q_l)/2$. We later demonstrate that, with buyer–induced certification, the certifier chooses a price p_c for certification to induce this outcome as closely as possible.

Next, we address the optimality of type i seller's strategy $\sigma_i(p)$. For a given strategy σ of the buyer and a fixed belief μ, a seller with quality q_h expects the following payoff from setting a price p:

$$\Pi_h(p, \mu|\sigma) = [\sigma(s_{nb}|p, \mu) + \sigma(s_{ch}|p, \mu)]p - c_h.$$

A specific strategy σ_h yields seller q_h, therefore, an expected profit of

$$\bar{\Pi}_h(\sigma_h) = \sum_i \sigma_h(p_i)\Pi_h(p_i, \mu(p_i)|\sigma).$$

Likewise, a seller with quality q_l obtains the payoff

$$\Pi_l(p, \mu|\sigma) = \sigma(s_{nb}|p, \mu)p$$

and any strategy σ_l yields

$$\bar{\Pi}_l(\sigma_l) = \sum_i \sigma_l(p_i)\Pi_l(p_i, \mu(p_i)|\sigma).$$

It follows that in a PBE $(\sigma_h^*, \sigma_l^*, \mu^*, \sigma^*)$ the high quality seller q_h and the low quality seller q_l's payoffs, respectively, are

$$\Pi_h^* = \sum_i \sigma_h^*(p_i)\Pi_h(p_i, \mu^*(p_i)|\sigma^*) \text{ and } \Pi_l^* = \sum_i \sigma_l^*(p_i)\Pi_l(p_i, \mu^*(p_i)|\sigma^*),$$

respectively.

The next lemma makes precise the intuitive result that the seller's expected profits increase when the buyer has more positive beliefs about the good's quality.

Lemma 2 *In any PBE $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ of the subgame $\Gamma(p_c)$ with $p_c > 0$ the payoffs $\Pi_h(p, \mu|\sigma^*)$ and $\Pi_l(p, \mu|\sigma^*)$ are non–decreasing in μ.*

Seller type i's pricing strategy σ_i is an optimal response to the buyer's behavior (σ^*, μ^*) exactly if, for any p', we have

$$\sigma_i^*(p) > 0 \;\Rightarrow\; \Pi_i(p, \mu^*(p)|\sigma^*) \geq \Pi_i(p', \mu^*(p')|\sigma^*). \tag{2}$$

Because the buyer's beliefs depend on the observed price p, it affects the buyer's behavior and, therefore, the belief function μ^* plays a role in condition (2).

Finally, a PBE demands that the buyer's beliefs μ^* have to be consistent with equilibrium play. In particular, they must follow Bayes' rule:

$$\sigma_i^*(p) > 0 \;\Rightarrow\; \mu^*(p) = \frac{\lambda\sigma_h^*(p)}{\lambda\sigma_h^*(p) + (1-\lambda)\sigma_l^*(p)}. \tag{3}$$

The next lemma shows some intuitive implications on PBE's that are due to Bayes' rule. In particular, it shows that the seller, no matter his type, never sets a price below q_l, and the low quality seller never sets a price above q_h. The lemma also shows that, in equilibrium, the low quality seller never loses from the presence of asymmetric information, since he can always guarantee himself the payoff q_l that he obtains with observable quality. By contrast, the high quality seller loses from the presence of asymmetric information; his payoff is strictly smaller than $q_h - c_h$.

Lemma 3 *In any PBE $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ of the subgame $\Gamma(p_c)$ we have i) $\sigma_l^*(p) = 0$ for all $p \notin [q_l, q_h]$ and $\sigma_h^*(p) = 0$ for all $p < q_l$; ii) $\Pi_l^* \geq q_l$; iii) $\Pi_h^* < q_h - c_h$.*

As is well known, the concept of Perfect Bayesian Equilibrium places only very weak restrictions on admissible beliefs. In particular, it does not place any restrictions on the buyer's beliefs for prices that are not played in equilibrium; any out–of–equilibrium belief is allowed. However, as is typical for signalling games, without any restrictions on out–of–equilibrium beliefs we cannot pin down behavior in the subgame $\Gamma(p_c)$ to a specific equilibrium. Especially by the use of pessimistic out–of–equilibrium beliefs, one can sustain many pricing strategies in equilibrium.

Therefore, to reduce the arbitrariness of equilibrium play, it is necessary to strengthen the solution concept of PBE by introducing more plausible restrictions on out–of–equilibrium beliefs. Bester and Ritzberger (2001) demonstrate that the following extension of the intuitive criterium of Cho–Kreps suffices to pin down equilibrium play.

Belief restriction (B.R.): A Perfect Bayesian Equilibrium $(\sigma_h^*, \sigma_l^*, \mu^*, \sigma^*)$ satisfies the Belief Restriction if, for any $\mu \in [0, 1]$ and any out–of–equilibrium price p, we have

$$\Pi_l(p, \mu) < \Pi_l^* \wedge \Pi_h(p, \mu) > \Pi_h^* \;\Rightarrow\; \mu^*(p) \geq \mu.$$

The belief restriction contains the intuitive criterion of Cho–Kreps as the special case for $\mu = 1$. Indeed, the underlying idea of the restriction is to extend the idea behind the Cho–Kreps criterion to a situation where a deviation to p is profitable only for the q_h seller when the buyer believes that the deviation originates from the q_h seller with probability μ. As we may have $\mu < 1$, the restriction considers more pessimistic beliefs than the Cho–Kreps criterium. If the pessimistic belief μ gives only the q_h seller an incentive to deviate, then the restriction requires that the buyer's actual belief should not be even more pessimistic than μ.

The next Lemma establishes characteristics of the equilibrium that are due to the belief refinement (B.R.). It shows that the belief restriction implies that the high quality seller can sell his product for a price of at least $\tilde{p}$.

Lemma 4 *Any Perfect Bayesian Equilibrium* $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ *of the subgame* $\Gamma(p_c)$ *that satisfies B.R. exhibits i)* $\sigma_h^*(p) = 0$ *for all* $p < \tilde{p}$ *and ii)* $\Pi_h^* \geq \tilde{p} - c_h$.

By combining the previous two lemmata we are now able to characterize the equilibrium outcome.

Proposition 1 *Consider a PBE* $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ *of the subgame* $\Gamma(p_c)$ *that satisfies B.R. Then*

i) for $\lambda < \tilde{\mu}$ *and* $c_h < \tilde{p}$ *it exhibits unique pricing behavior by the seller and unique buying behavior by the buyer. In particular, the high quality seller sets the price* $\tilde{p}$ *with certainty and the low quality seller randomizes between the price* $\tilde{p}$ *and* q_l*. Observing the price* $\tilde{p}$ *the buyer buys certification with positive probability. The certifier's equilibrium profit equals*

$$\Pi_c(p_c) = \frac{\lambda(\tilde{p} - q_l)}{\tilde{\mu}\tilde{p}}(p_c - c_c). \tag{4}$$

ii) For $\lambda > \tilde{\mu}$ *or* $c_h > \tilde{p}$ *we have in any equilibrium* $\Pi_c(p_c) = 0$.

iii) For $\lambda \leq \tilde{\mu}$ *and* $c_h \leq \tilde{p}$ *there exists an equilibrium outcome, in which the certifier's profits equal expression (4).*

The Proposition shows that the buyer and the low quality seller play the mixed strategies that reflect the typical outcome of an inspection game. Indeed, by choosing the low price q_l a low quality seller honestly signals his low quality. In contrast, we may interpret a low quality seller, who sets a high price $\tilde{p}$, as trying to cheat. Hence, whenever the buyer observes the price $\tilde{p}$, she is uncertain whether the good is supplied by the high quality or the low quality seller. She, therefore, inspects the good by buying certification with positive probability. Through inspection, the buyer tries to dissuade the low quality seller to set the "cheating" price $\tilde{p}$. Yet, as in an inspection game, the buyer has only an incentive to buy certification and inspect when the low quality cheats "often enough". This gives rise to the use of mixed strategies. As in an inspection game the buyer's certification probability is such that the low quality seller is indifferent between cheating, i.e., setting the high price $\tilde{p}$, and honestly signaling his low quality by setting the price q_l. On the other hand, the probability with which the low quality seller chooses the high price $\tilde{p}$ is such that the buyer is indifferent between buying the good uncertified and asking for certification.

Proposition 1 also describes the certifier's profits in the subgame $\Gamma(p_c)$. The certifier anticipates this outcome when choosing its price p_c for certifying the good's quality. When the certifier maximizes its profits Π_c with respect to the certification price p_c, it must take into account that $\tilde{\mu}$ depends on p_c itself and the certifier therefore anticipates that the very case distinction $\lambda \lesseqgtr \tilde{\mu}$ and $c_h \gtreqless \tilde{p}$ depends on its choice of p_c. The following proposition shows that expression (4) is increasing in p_c. Hence, the certifier picks the largest price such that $\lambda \leq \tilde{\mu}$ and $c_h \leq \tilde{p}$.

Proposition 2 *Consider the full game with buyer–induced certification.*

i.) Suppose that $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$. Then the certifier sets a price $p_c^b = \Delta q/4$ and obtains a profit of

$$\Pi_c^b = \frac{\lambda \Delta q}{2(q_h + q_l)}(\Delta q - 4c_c).$$

ii.) Suppose that $\lambda > 1/2$ or $c_h > (q_h + q_l)/2$. Then the certifier sets the price $p_c^b = (q_h - c_h)(c_h - q_l)/\Delta q$ and obtains a profit of

$$\Pi_c^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h}.$$

We argued that the monopolistic certifier benefits from a relatively high uncertainty for the buyer and an intermediate price of the good; we also showed that the buyer's willingness to pay for certification is maximized for $\hat{\mu} = 1/2$ and $\hat{p} = (q_h + q_l)/2$. A comparison demonstrates that, for the parameter constellation $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$, the equilibrium induces exactly this outcome. Indeed, the certifier's optimal price $p_c = \Delta q/4$ leads to a price $p = (q_h + q_l)/2$ and a belief $\mu = 1/2$ and maximizes the expression

$$\min\{\Delta U^1, \Delta U^2\}.$$

For $c_h > (q_h + q_l)/2$, the price $p = (q_h + q_l)/2$ would imply a loss to the high quality seller and, intuitively, the certifier cannot induce this maximum degree of uncertainty. For $\lambda > 1/2$, the ex ante belief of the buyer about the product exceeds $1/2$. Consequently, the certifier is unable to induce the belief $\mu = 1/2$. Instead, the certifier is restricted and maximizes the expression $\min\{\Delta U^1, \Delta U^2\}$ under a feasibility constraint. That is, the certifier's price maximizes the buyer's uncertainty about the seller's quality and, thereby, her willingness to pay.

5 Seller Induced Certification

In this section we consider the case where the seller instead of the buyer may buy certification. Here certification plays a different role. Rather than giving the buyer the possibility to protect herself from bad quality, it enables a high quality seller to ascertain the quality of his product to the buyer. Although the distinction seems small, it has a major impact on the equilibrium outcome, primarily because only the high quality seller is prepared to demand certification. Because of this, we can show that seller–induced certification is simpler and easier to control by the certifier.

Under seller–induced certification the parties play the following game:

t=1 The certifier sets a price p_c.

t=2 Nature selects the quality $q_i, i \in \{l, h\}$ of the good offered by the seller.

t=3 The seller offering the good at quality q_i and cost c_i decides about the price p at which he offers the good.

t=4 The seller decides whether or not to demand certification for his good.

t=5 The buyer decides whether or not to buy the good.

Thus, in comparison to the model described in the previous section, we only change stage four, by letting the seller decide about certification. Clearly, the sequence of stages 3 and 4 is immaterial. Our setting where the seller first chooses his price and then decides about certification is strategically equivalent to the situation where he simultaneously takes both decisions, or reverses their order.

We again focus on Perfect Bayesian Equilibria of this game. Note again that after the certifier has set his price p_c a proper subgame, $\Gamma(p_c)$, starts with nature's decision about the quality offered by the seller. The subgame $\Gamma(p_c)$ is a pure signalling game if the seller does not buy certification in stage 4. In contrast, if the seller does decide to certify, the quality is revealed to the buyer, and there is no asymmetric information. In the subsequent subgame, the q_h seller sells his good at price $p = q_h$, whence the low quality seller sells his good at a price $p = q_l$.

In order to capture the seller's option to certify, we expand the actions open to the seller by an action c that represents the seller's option to certify and to charge the maximum price q_i. Hence, the seller's payoff associated with the action c are $\Pi_h(c) = q_h - c_h$ and $\Pi_l(c) = q_l$ for a high and low quality seller, respectively. Let $\sigma_i(c)$ denote the probability that the q_i seller buys certification. We further adopt the notation of the previous section. Then we may express a mixed strategy of the seller q_i over certification and a, possibly, infinite but countable number of prices by probabilities $\sigma_i(p_j)$ such that

$$\sigma_i(c) + \sum_j \sigma_i(p_j) = 1. \tag{5}$$

In contrast to the previous section, the buyer can no longer decide to buy certification so that her actions are now constrained to s_{nn} and s_{nb}. As before let $\mu(p)$ represent the buyer's belief upon observing a non–certified good priced at p. Consequently, s_{nb} is individually rational whenever

$$\mu(p)\Delta q + q_l \geq p$$

and s_{nn} is individually rational whenever

$$\mu(p)\Delta q + q_l \leq p.$$

Proposition 3 *For any price of certification $p_c < q_h - c_h$, the equilibrium outcome in the subgame $\Gamma(p_c)$ is unique. The high quality seller certifies with probability 1 and obtains the profit $\Pi_h^* = q_h - c_h - p_c > 0$, whereas the low quality seller does not certify and obtains the payoff $\Pi_l^* = q_l$. For any price $p_c > q_h - c_h$, any equilibrium outcome of the subgame $\Gamma(p_c)$ involves no certification. For $p_c = q_h - c_h$, the subgame $\Gamma(p_c)$ has an equilibrium in which high quality seller certifies with probability 1 and obtains the profit $\Pi_h^* = 0$, whereas the low quality seller does not certify and obtains the payoff $\Pi_l^* = q_l$.*

The proposition characterizes the equilibrium outcome of the subgame $\Gamma(p_c)$. From this characterization, we can derive the equilibrium of the overall game of seller–induced certification.

Proposition 4 *The full game with seller–induced certification has the unique equilibrium outcome $p_c = q_h - c_h$ with equilibrium payoffs $\Pi_c^s = \lambda(q_h - c_h - c_c)$, $\Pi_h^* = 0$, and $\Pi_l^* = q_l$.*

Comparing the outcome of seller–induced certification with the outcome under buyer–induced certification we get the following result.

Proposition 5 *The certifier obtains a higher profit under seller–induced than under buyer–induced certification: $\Pi_c^s > \Pi_c^b$.*

The proposition shows that the certifier is better off when it sells certification to the seller. The intuition behind this result is that if the buyer decides whether or not to certify, the decision to certify cannot be made contingent on the actual quality. This is different from when the seller has the right to decide about certification. Clearly, a seller with low quality q_l will never demand certification. In contrast, we showed that, in any equilibrium, the seller q_h always certifies. The intuition is that if seller q_h does not certify at a price p_c quoted by the certifier, then the certifier gets zero profits from the seller. It, therefore, does strictly better by lowering the certification price to a level where it is worthwhile for the seller to demand certification.

6 Welfare

Certification enables the high quality seller to sell his good. This yields an increase in social efficiency. This positive effect obtains both under buyer– and seller–induced certification. From an efficiency perspective, the differences between the two regimes relate to differences in the probability at which the low quality good is sold, and differences in the probability at which certification costs arise.

First, under seller–induced certification the low quality good is always sold, if offered at all. This is different under buyer–induced certification, where the good is not sold when the low quality seller picks the high price $\tilde{p}$ and the buyer certifies. This happens with probability

$$\omega = \sigma_l^*(\tilde{p})\sigma^*(s_{ch}|\tilde{p}, \mu^*(\tilde{p})).$$

Thus, under buyer–induced certification an efficiency loss of q_l occurs with probability $(1 - \lambda)\omega$.

Second, the different regimes may lead to different intensities of certification and therefore differences in expected certification costs. In particular, the probability of certification under buyer–induced certification is

$$x^b = [\lambda + (1 - \lambda)\sigma_l^*(\tilde{p})]\sigma(s_{ch}|\tilde{p}, \mu^*(\tilde{p})).$$

Remember that the buyer demands certification only if the seller quotes a high price. Now, the cornered bracket contains the probabilities at which the seller quotes that high price, which include the probability λ at which he sells the high quality product, and the probability $(1-\lambda)\sigma_l^*(\tilde{p})$ by which he has low quality product but quotes the high price.

By comparison, under seller–induced certification the probability of certification is

$$x^s = \lambda.$$

Let $WF^i, i = b, s$ denote social welfare under buyer and seller–induced certification, respectively. As usual, it is defined as the sum of consumer and producer surplus. Then, social welfare under buyer–induced certification is

$$WF^b = \lambda(q_h - c_h) + (1-\lambda)(1-\omega)q_l - x^b c_c,$$

whereas social welfare under seller–induced certification equals

$$WF^s = \lambda(q_h - c_h) + (1-\lambda)q_l - x^s c_c.$$

Consequently, the difference in social welfare between the two regimes is

$$\Delta WF = WF^s - WF^b = (1-\lambda)\omega q_l + (x^b - x^s)c_c,$$

In Proposition 5 we have established that the profits of a monopolistic certifier are larger under seller certification. The certifier will therefore have a preference for seller–induced certification. We now check whether these preferences are aligned with social efficiency. Clearly, when certification costs are zero, this follows immediately. The more interesting case is therefore when the cost of certification, c_c, is strictly positive. In this case, the certifier's preferences are still in line with social efficiency, when the probability of certification is smaller under seller–induced certification. In the next lemma we compare the probabilities of certification in both regimes.

Lemma 5 *For $\lambda > 1/2$ or $c_h > (q_h + q_l)/2$ the probability of certification under seller–induced certification, x^s, is lower than under buyer–induced certification, x^b. For $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$ the probability of certification under seller–induced certification, x^s, is higher than under buyer–induced certification, x^b, if and only if $q_h < 3q_l$.*

The lemma identifies a case where the probability of certification is higher under seller–induced certification than under buyer–induced certification. This leaves open the possibility that the decision of a monopolistic certifier to offer its services to the seller rather than the buyer is not in the interest of social efficiency. In particular, if certification costs, c_c, are large, the certifier's decision may be suboptimal. Yet, the following proposition shows that this possibility does not arise. Whenever the certifier's profit under buyer–induced certification is non–negative, social welfare is larger under seller–induced certification, in spite of a possibly higher probability of certification.

Proposition 6 *Social welfare is higher under seller–induced certification than under buyer–induced certification.*

Appendix

The appendix contains all formal proofs to our Lemmata and Propositions.

Proof of Lemma 1: Follows directly from the text. Q.E.D.

Proof of Lemma 2: To show that $\Pi_h(p, \mu|\sigma^*)$ is non–decreasing in μ we first establish that, in any PBE, $\sigma^*(s_{nn}|p, \mu)$ is weakly decreasing in μ. Suppose not, then we may find $\mu_1 < \mu_2$ such that $0 \leq \sigma^*(s_{nn}|p, \mu_1) < \sigma^*(s_{nn}|p, \mu_2) \leq 1$. Lemma 1 implies that $(p, \mu_2) \in S(s_{nn}|p_c)$. That is,

$$p \geq \mu_2 q_h + (1 - \mu_2) q_l \tag{6}$$

and

$$p_c \geq \mu_2 (q_h - p). \tag{7}$$

Now since $\sigma^*(s_{nn}|p, \mu_1) < 1$ we have either $\sigma^*(s_{nb}|p, \mu_1) > 0$ or $\sigma^*(s_{ch}|p, \mu_1) > 0$. Suppose first $\sigma^*(s_{nb}|p, \mu_1) > 0$, then by Lemma 1 we have $p \leq \mu_1 q_h + (1 - \mu_1) q_l$. But from $\mu_2 > \mu_1$ and $q_h > q_l$ it then follows that $\mu_2 q_h + (1 - \mu_2) q_l > p$, which contradicts (6). Suppose therefore that $\sigma^*(s_{ch}|p, \mu_1) > 0$, then by

Lemma 1 we have $\mu_1(q_h - p) \geq p_c > 0$. This requires $q_h > p$. But then, due to $\mu_2 > \mu_1$, we get $\mu_2(q_h - p) > p_c$, which contradicts (7).

Hence, we establish that $\sigma^*(s_{nn}|p, \mu)$ is weakly decreasing in μ and therefore $\sigma^*(s_{nb}|p, \mu) + \sigma^*(s_{ch}|p, \mu)$ must be weakly increasing in μ. Consequently, $\Pi_h(p, \mu|\sigma^*)$ is weakly increasing in μ.

Next we show that in any PBE $\sigma^*(s_{nb}|p, \mu)$ is weakly increasing in μ. Suppose not, then we may find $\mu_1 < \mu_2$ such that $1 \geq \sigma^*(s_{nb}|p, \mu_1) > \sigma^*(s_{nb}|p, \mu_2) \geq 0$. Since $\sigma^*(s_{nb}|p, \mu_1) > 0$, Lemma 1 implies that $(p, \mu_1) \in S(s_{nb}|p_c)$. That is,

$$p \leq \mu_1 q_h + (1 - \mu_1) q_l \tag{8}$$

and

$$p_c \geq (1 - \mu_1)(p - q_l). \tag{9}$$

Now since $\sigma^*(s_{nb}|p, \mu_2) < 1$ we have either $\sigma^*(s_{nn}|p, \mu_2) > 0$ or $\sigma^*(s_{ch}|p, \mu_2) > 0$. Suppose first $\sigma^*(s_{nn}|p, \mu_2) > 0$, then by Lemma 1 this implies $p \geq \mu_2 q_h + (1 - \mu_2) q_l$. But due to $\mu_2 > \mu_1$ and $q_h > q_l$ we get $p > \mu_1 q_h + (1 - \mu_1) q_l$. This contradicts (8). Suppose therefore that $\sigma^*(s_{ch}|p, \mu_2) > 0$, then by Lemma 1 we have $(1 - \mu_2)(p - q_l) \geq p_c > 0$. This requires $p > q_l$. But then, due to $\mu_2 > \mu_1$, we get $(1 - \mu_1)(p - q_l) > p_c$. This contradicts (9). Hence, $\sigma^*(s_{nb}|p, \mu)$ must be weakly increasing in μ. Consequently, $\Pi_l(p, \mu|\sigma^*)$ is weakly increasing in μ. Q.E.D.

Proof of Lemma 3: i) For any $\bar{p} < q_l$, $\mu \in [0, 1]$ we have $(\bar{p}, \mu) \notin S(s_{nn})$, $(\bar{p}, \mu) \notin S(s_{ch})$ and $(\bar{p}, \mu) \in S(s_{nb})$. Hence, $\sigma^*(s_{nb}|\bar{p}, \mu^*(barp)) = 1$. Now suppose for some $\bar{p} < q_l$ we have $\sigma_i^*(\bar{p}) > 0$. This would violate (2), because instead of charging $\bar{p}$ seller q_i could have raised profits by $\varepsilon\sigma_i(\bar{p})$ by charging the higher price $\bar{p} + \varepsilon < q_l$ with $\varepsilon \in (0, (q_l - \bar{p}))$. At $\bar{p} + \varepsilon < q_l$ the buyer always buys, because, as established, $\sigma^*(s_{nb}|\bar{p} + \varepsilon, \mu) = 1$ for all μ and in particular for $\mu = \mu^*(\bar{p} + \varepsilon)$.

For any $\bar{p} > q_h$, $\mu \in [0, 1]$ we have $(\bar{p}, \mu) \in S(s_{nn})$, $(\bar{p}, \mu) \notin S(s_{ch})$ and $(\bar{p}, \mu) \notin S(s_{nb})$. Hence, $\sigma^*(s_{nn}|\bar{p}, \mu^*(\bar{p})) = 1$. Now suppose we have $\sigma_l(\bar{p}) > 0$. This would violate (2), because instead of charging $\bar{p}$ seller q_l could have raised profits by $(q_l - \varepsilon)\sigma_l(\bar{p})$ by charging the price $q_l - \varepsilon$.

ii) Suppose $q_l - \Pi_l^* = \delta > 0$. Now consider a price $p' = q_l - \varepsilon$ with $\varepsilon \in (0,\delta)$ then for any $\mu' \in [0,1]$ we have $(p',\mu') \in S(s_{nb})$ and $(p',\mu') \notin S(s_{nn}) \cup S(s_{ch})$ so that, by (1), we have $\sigma^*(s_{nb}|p'^*,\mu^*(p')) = 1$ and, therefore, $\Pi_l(p'^*,\mu^*(p')|\sigma^*) = p' > \Pi_l^*$. This contradicts (2).

iii) For any p such that $\sigma_h^*(p) > 0$, we have $\Pi_h^* = \Pi_h(p,\mu^*(p)|\sigma^*) = [\sigma^*(s_{nb}|p,\mu^*(p)) + \sigma^*(s_{ch}|p,\mu^*(p))]p - c_h$. As argued in i), we have $\sigma^*(s_{nn}|p,\mu) = 1$ for all $p > q_h$ and $\mu \in [0,1]$. Hence, $\Pi_h(p,\mu|\sigma^*) = 0$ whenever $p > q_h$. But for any price $p \leq q_h$ we have $\Pi_h(p,\mu|\sigma^*) \leq q_h - c_h$. Hence, it follows that $\Pi_h^* \leq q_h - c_h$. Now suppose $\Pi_h^* = q_h - c_h$. Then we must have $\sigma_h^*(q_h) = 1$ and $\sigma^*(s_{nb}|q_h,\mu^*(q_h)) + \sigma^*(s_{ch}|q_h,\mu^*(q_h)) = 1$. But, due to $\mu^*(q_h)(q_h - q_h) = 0 < p_c$, we have $(q_h,\mu^*(q_h)) \notin S(s_{ch}|q_h)$ so that $\sigma^*(s_{ch}|q_h,\mu^*(q_h)) = 0$. Hence, we must have $\sigma^*(s_{nb}|q_h,\mu^*(q_h)) = 1$. This requires $(q_h,\mu^*(q_h)) \in S(s_{nb}|p_c)$ so that we must have $\mu^*(q_h) = 1$. By (3), this requires $\sigma_l^*(q_h) = 0$. But since $\Pi_l(q_h,1|\sigma^*) = \sigma^*(s_{nb}|q_h,\mu^*(q_h))q_h = q_h$ we must, by (2), have $\Pi_l^* \geq q_h$. Together with $\sigma_l^*(q_h) = 0$, it would require $\sigma_l^*(p) > 0$ for some $p > q_h$ and leads to a contradiction with i). Q.E.D.

Proof of Lemma 4: We first prove ii): Suppose to the contrary that $\delta \equiv \tilde{p} - c_h - \Pi_h^* > 0$. Then, due to the countable number of equilibrium prices, we can find an out–of–equilibrium price $p' = \tilde{p} - \varepsilon$ for some $\varepsilon \in (0,\delta)$. Then for any belief $\mu' \in (p_c/(q_h - p'), 1 - p_c/(p' - q_l)) \neq \emptyset$[4] we have $(p',\mu') \in S(\sigma_{ch})$ and $(p',\mu') \notin S(\sigma_{nn}) \cup S(\sigma_{nb})$. Consequently, $\sigma^*(s_{ch}|p',\mu') = 1$. Hence, $\Pi_h(p',\mu') = p' - c_h = \tilde{p} - c_h - \varepsilon > \tilde{p} - c_h - \delta = \Pi_h^*$ and $\Pi_l(p',\mu') = 0 < q_l \leq \Pi_l^*$. Therefore, by B.R. the buyer's equilibrium belief must satisfy $\mu^*(p') \geq \mu'$. By Lemma 2 it follows $\Pi_h(p',\mu^*(p')) \geq \Pi_h(p',\mu') = \tilde{p} - c_h - \varepsilon > \Pi_h^*$. This contradicts (2). Consequently, we must have $\Pi_h^* \geq \tilde{p} - c_h$. To show i) note that for all $p < \tilde{p}$ and $\mu \in [0,1]$ we have $\Pi_h(p,\mu|\sigma) \leq p - c_h < \tilde{p} - c_h \leq \Pi_h^*$ so that $\sigma_h(p) > 0$ would violate (2). Q.E.D.

Proof of Proposition 1: i): First we show that for $\lambda < \tilde{\mu}$ and $c_h < \tilde{p}$ there exists no pooling, i.e., there exists no price $\bar{p}$ such that $\sigma_h^*(\bar{p}) = \sigma_l^*(\bar{p}) >$

[4]Let $l(p) \equiv p_c/(q_h - p)$ and $h(p) \equiv 1 - p_c/(p - q_l)$. Then by the definition of $\tilde{p}$ we have $l(\tilde{p}) = h(\tilde{p})$. Moreover, for $q_l < p < q_h$ we have $l'(p) = p_c/(q_h - p)^2 > h'(p) = p_c/(p - q_l)^2 > 0$. Hence, $l(\tilde{p} - \varepsilon) < h(\tilde{p} - \varepsilon)$ for $\varepsilon > 0$ so that $\tilde{p} - \varepsilon > q_l$ and, therefore, $l(p') < h(p')$.

0. For suppose there does. Then, by Lemma 4.i, we have $\bar{p} \geq \tilde{p}$ and, by Lemma 3.i, we have $\bar{p} \leq q_h$. Yet, due to (3) we have $\mu^*(\bar{p}) = \lambda < \tilde{\mu}$ so that $q_l + \mu^*(\bar{p})\Delta q - \bar{p} < q_l + \tilde{\mu}\Delta q - \tilde{p} = 0$. Moreover, $\mu^*(\bar{p})(q_h - \bar{p}) < \tilde{\mu}(q_h - \tilde{p}) = p_c$. Therefore, $\sigma^*(s_{nn}|\bar{p}, \mu^*(\bar{p})) = 1$ and $\Pi_h(\bar{p}, \mu^*(\bar{p})) = 0$. As a result, $\sigma_h^*(\bar{p}) > 0$ contradicts (2), because, by Lemma 4.ii, $\Pi_h^* \geq \tilde{p} - c_h > 0 = \Pi_h(\bar{p}, \mu^*(\bar{p}))$.

Second, suppose that for some $\bar{p} > \tilde{p}$ we have $\sigma_h^*(\bar{p}) > 0$ then, by definition of $\tilde{p}$, we have $(\bar{p}, \mu) \notin S(s_{ch})$ for any $\mu \in [0, 1]$. Hence, $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) = 0$ so that $\Pi_l(\bar{p}, \mu^*(\bar{p})) = \Pi_h(\bar{p}, \mu^*(\bar{p})) + c_h$. From Lemma 4.ii it then follows $\Pi_l(\bar{p}, \mu^*(\bar{p})) \geq \tilde{p}$ and, therefore, $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$. From $\bar{p} > \tilde{p}$ and $\tilde{\mu} > \lambda$ it follows $\lambda\Delta q + q_l - \bar{p} < \tilde{\mu}\Delta q + q_l - \tilde{p} = 0$ so that $\lambda\Delta q + q_l < \bar{p}$. Now take a $\bar{p} > \tilde{p}$ with $\sigma_l(\bar{p}) > 0$ then, by Lemma 3.ii and (2), $0 < q_l \leq \Pi_l^* = \Pi_l(\bar{p}, \mu^*(\bar{p})|\sigma^*) = \sigma(s_{nb}|\bar{p}, \mu^*(\bar{p}))\bar{p}$. This requires $\sigma(s_{nb}|\bar{p}, \mu^*(\bar{p})) > 0$ and therefore $(\bar{p}, \mu^*(\bar{p})) \in S(s_{nb}|p_c)$ and, hence, $\mu^*(\bar{p})\Delta q + q_l \geq \bar{p}$. Combining the latter inequality with our observation that $\lambda\Delta q + q_l < \bar{p}$ and using (3), it follows

$$\lambda\Delta q + q_l < \frac{\lambda\sigma_h^*(\bar{p})}{\lambda\sigma_h^*(\bar{p}) + (1-\lambda)\sigma_l^*(\bar{p})}\Delta q + q_l,$$

which is equivalent to $\sigma_h^*(\bar{p}) > \sigma_l^*(\bar{p})$. Summing over all $p \geq \tilde{p}$ and using $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$ yields the contradiction $\sum_{p \geq \tilde{p}} \sigma_h^*(p) > 1$. Hence, we must have $\sigma_l^*(\bar{p}) = 0$ for any $\bar{p} > \tilde{p}$. But this contradicts $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$ and, therefore, we must have $\sigma_h^*(\bar{p}) = 0$ for all $\bar{p} > \tilde{p}$. Hence, if an equilibrium for $\lambda < \tilde{\mu}$ and $\tilde{p} > c_h$ exists then, by Lemma 4, it exhibits $\sigma_h^*(\tilde{p}) = 1$, $\Pi_h^* = \tilde{p} - c_h$ and $\sigma^*(s_{ch}|\tilde{p}, \tilde{\mu}) + \sigma^*(s_{nb}|\tilde{p}, \tilde{\mu}) = 1$.

We now show existence of such an equilibrium and demonstrate that any such equilibrium has a unique equilibrium outcome. If $\sigma_h^*(\tilde{p}) = 1$ then (3) implies that $\mu^*(\tilde{p}) = \tilde{\mu}$ whenever

$$\sigma_l^*(\tilde{p}) = \frac{\lambda(1-\tilde{\mu})}{\tilde{\mu}(1-\lambda)},$$

which is smaller than one exactly when $\lambda < \tilde{\mu}$. By definition, $(\tilde{p}, \tilde{\mu}) \in S(s_{ch}) \cap S(s_{nb})$ so that any buying behavior with $\sigma^*(s_{ch}|\tilde{p}, \tilde{\mu}) + \sigma^*(s_{nb}|\tilde{p}, \tilde{\mu}) = 1$ is consistent in equilibrium. In particular, $\sigma^*(s_{nb}|\tilde{p}, \tilde{\mu}) = q_l/\tilde{p} < 1$ is consistent in equilibrium. Only for this buying behavior we have $\Pi_l(q_l, 0) = q_l = \Pi_l(\tilde{p}, \tilde{\mu})$ so that seller q_l is indifferent between price $\tilde{p}$ and q_l. The equilibrium therefore prescribes $\sigma_l^*(q_l) = 1 - \sigma_l^*(\tilde{p})$. Finally, let $\mu^*(q_l) = 0$ and $\sigma^*(s_{nb}|q_l, \mu^*(q_l)) = 1$

and $\mu^*(p) = 0$ for any price p larger than q_l and unequal to $\tilde{p}$. This out–of–equilibrium beliefs satisfies B.R.. Hence, the expected profit to the certifier is

$$\Pi_c(p_c) = (\lambda + (1-\lambda)\sigma_l^*(\tilde{p}))\, \sigma^*(s_{ch}|\tilde{p}, \tilde{\mu})(p_c - c_c) = \frac{\lambda(\tilde{p} - q_l)}{\tilde{\mu}\tilde{p}}(p_c - c_c).$$

ii) In order to show that, in any equilibrium of $\Gamma(p_c)$, we have $\Pi_c(p_c) = 0$ whenever $\lambda > \tilde{\mu}$, we prove that for any $\bar{p}$ such that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$, it must hold $\sigma_h^*(\bar{p}) = \sigma_l^*(\bar{p}) = 0$. Suppose we have $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$, then $(\bar{p}, \mu^*(\bar{p})) \in S(s_{ch})$ and, necessarily, $\bar{p} \leq \tilde{p}$. But by Lemma 4.i, $\sigma_h^*(\bar{p}) > 0$ also implies $\bar{p} \geq \tilde{p}$. Therefore, we must have $\bar{p} = \tilde{p}$. But $(\tilde{p}, \mu) \in S(s_{ch})$ only if $\mu = \tilde{\mu}$. Hence, we must have $\mu^*(\tilde{p}) = \tilde{\mu}$. By (3) it therefore must hold

$$\tilde{\mu} = \mu^*(\tilde{p}) = \frac{\lambda\sigma_h^*(\tilde{p})}{\lambda\sigma_h^*(\tilde{p}) + (1-\lambda)\sigma_l^*(\tilde{p})}.$$

For $\lambda > \tilde{\mu}$ this requires $\sigma_h^*(\tilde{p}) < \sigma_l^*(\tilde{p}) \leq 1$ and therefore there is some other $p' > \tilde{p}$ such that $\sigma_h^*(p') > 0$. But if also p' is an equilibrium price, then $\Pi_h(\tilde{p}, \mu^*(\tilde{p})|\sigma^*) = \Pi_h(p', \mu^*(p')|\sigma^*)$. Yet, for any $p' > \tilde{p}$ it holds $(p', \mu) \notin S(s_{ch}|p_c)$ for any $\mu \in [0,1]$ so that $\Pi_l(p', \mu|\sigma^*) = \Pi_h(p', \mu|\sigma^*) + c_h$ and, together with our assumption $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$ yields $\Pi_l(\bar{p}, \mu^*(\bar{p})|\sigma^*) < \Pi_h(\bar{p}, \mu^*(\bar{p})|\sigma^*) + c_h = \Pi_h(p', \mu^*(p')|\sigma^*) + c_h = \Pi_l(p', \mu^*(p')|\sigma^*)$ so that, by (2), $\sigma_l^*(\bar{p}) = 0$. Since $\bar{p} = \tilde{p}$, this violates $\sigma_l^*(\tilde{p}) > \sigma_h^*(\tilde{p}) \geq 0$. As a result, $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$ implies $\sigma_h^*(\bar{p}) = 0$.

In order to show that we must also have $\sigma_l^*(\bar{p}) = 0$, assume again that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$. We have shown that his implies $\sigma_h^*(\bar{p}) = 0$. Now if $\sigma_l^*(\bar{p}) > 0$ then, by (3), it follows $\mu^*(\bar{p}) = 0$. But then $q_l + \mu^*(\bar{p})\Delta q - \bar{p} - p_c = q_l - \bar{p} - p_c < q_l - \bar{p}$ so that $(\bar{p}, \mu^*(\bar{p})) \notin S(s_{ch})$, which contradicts $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$.

In order to show that $\tilde{p} < c_h$ implies $\Pi_c(p_c) = 0$ suppose, on the contrary that, $\Pi_c(p_c) > 0$. This requires that there exists some $\bar{p}$ such that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$ and $\sigma_i^*(\bar{p}) > 0$ for some $i \in \{l, h\}$. First note that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$ implies $\bar{p} \leq \tilde{p}$. Now suppose $\sigma_h^*(\bar{p}) > 0$ then $\Pi_h(\bar{p}, \mu^*(\bar{p})|\sigma^*) = (\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) + \sigma^*(s_{nb}|\bar{p}, \mu^*(\bar{p})))\bar{p} - c_h < 0$ so that the high quality seller would make a loss and, thus, violates (2). Therefore, we have $\sigma_h^*(\bar{p}) = 0$. Now if $\sigma_l^*(\bar{p}) > 0$ then (3) implies $\mu^*(\bar{p}) = 0$ so that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) = 0$, which contradicts $\Pi_c(p_c) > 0$. Q.E.D.

Proof of Proposition 2: In order to express the dependence of $\tilde{\mu}$ and $\tilde{p}$ on p_c explicitly, we write $\tilde{\mu}(p_c)$ and $\tilde{p}(p_c)$, respectively. We maximize expression (4) with respect to p_c over the relevant domain

$$P = \{p_c | p_c \leq \Delta q/4 \wedge \tilde{\mu}(p_c) \geq \lambda \wedge \tilde{p}(p_c) \geq c_h\}.$$

First, we show that (4) is increasing in p_c. Define

$$\alpha(p_c) \equiv \frac{\lambda(\tilde{p}(p_c) - q_l)}{\tilde{\mu}(p_c)\tilde{p}(p_c)}$$

so that $\Pi_c(p_c) = \alpha(p_c)(p_c - c_c)$. We have

$$\alpha'(p_c) = \frac{4\lambda\Delta q^2}{\sqrt{\Delta q(\Delta q - 4p_c)}\left(q_h + q_l + \sqrt{\Delta q(\Delta q - 4p_c)}\right)^2} > 0$$

so that $\alpha(p_c)$ is increasing in p_c and, hence, $\Pi_c(p_c)$ is increasing in p_c and maximized for $\max P$.

We distinguish two cases. First, for $\lambda \leq 1/2$, it follows $\tilde{\mu}(p_c) \geq 1/2 \geq \lambda$. Therefore,

$$P = \{p_c | p_c \leq \Delta q/4 \wedge \tilde{p}(p_c) \geq c_h\}.$$

Hence, $\max P$ is either $p_c = \Delta q/4$ or such that $\tilde{p}(p_c) = c_h$. Because $\tilde{p}(\Delta q/4) = (q_h + q_l)/2$, it follows that for $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$, the maximum obtains for $p_c = \Delta q/4$ with

$$\Pi_c^b = \frac{\lambda\Delta q}{2(q_h + q_l)}(\Delta q - 4c_c).$$

For $\lambda \leq 1/2$ and $c_h > (q_h + q_l)/2$ the maximum obtains for p_c such that $\tilde{p}(p_c) = c_h$, which yields $p_c = (q_h - c_h)(c_h - q_l)/\Delta q$ with

$$\Pi_c^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h};$$

Second, for $\lambda > 1/2$ we have

$$\tilde{\mu}(p_c) \geq \lambda \Leftrightarrow p_c \leq \lambda(1-\lambda)\Delta q.$$

Since $\lambda(1-\lambda) \leq 1/4$ the requirement $p_c < \lambda(1-\lambda)\Delta q$ automatically implies $p_c \leq \Delta q/4$. Hence for $\lambda > 1/2$ we have

$$P = \{p_c | p_c \leq \lambda(1-\lambda)\Delta q \wedge \tilde{p}(p_c) \geq c_h\}.$$

Because, $\tilde{p}(\lambda(1-\lambda)\Delta q) = \lambda q_h + (1-\lambda)q_l$, which by assumption is smaller than c_h, we have $\max P = (q_h - c_h)(c_h - q_l)/\Delta q$. Note that $c_h > \lambda q_h + (1-\lambda)q_l$ and $\lambda > 1/2$ implies that $c_h > (q_h + q_l)/2$. It follows $\tilde{\mu} = (c_h - q_l)/\Delta q$ and

$$\Pi_c^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h};$$

Q.E.D.

Proof of Proposition 3 Fix some $p_c < q_h - c_h$. By certifying, seller q_h guarantees himself the payoff $\Pi_h(c) = q_h - c_h - p_c > 0$. Hence, in any equilibrium of the subgame $\Gamma(p_c)$ seller q_h must obtain a payoff of at least $\Pi_h(c) > 0$.

Now suppose that there exists some equilibrium in which $\sigma_h(c) < 1$. Then, by (5) there exists some price $\tilde{p}$ such that $\sigma_h(\tilde{p}) > 0$. For $\tilde{p}$ to be optimal, it is required that $\Pi_h(\tilde{p}, \mu^*(\tilde{p})|\sigma^*) = \tilde{p}\sigma(s_{nb}|\tilde{p}, \mu^*(\tilde{p})) - c_h \geq \Pi_h(c) > 0$. This implies $\Pi_l(\tilde{p}, \mu^*(\tilde{p})|\sigma^*) = \tilde{p}\sigma(s_{nb}|\tilde{p}, \mu^*(\tilde{p})) > c_h$ so that the equilibrium payoff of seller q_l is $\Pi_l^* > c_h > \bar{q}$. Consequently, $\sigma_l^*(p) = 0$ for any $p < \bar{q}$ and therefore

$$\sum_{p \geq \bar{q}} \sigma_l^*(p) = 1. \tag{10}$$

But if $\sigma_l^*(p) > 0$ then we must have $p\sigma(s_{nb}|p, \mu^*(p)) > c_h$. This requires $\sigma(s_{nb}|p, \mu^*(p)) > 0$. Therefore, s_{nb} must be an optimal response given price p and belief $\mu^*(p)$. Hence, $\mu^*(p)\Delta q + q_l \geq p > c_h > \lambda \Delta q + q_l$. As a result, $\mu^*(p) > \lambda$ and, due to (3), it holds $\sigma_h^*(p) > \sigma_l^*(p)$ for any $\sigma_l^*(p) > 0$. Together with (10) we arrive at the contradiction

$$\sum_{p \geq \bar{q}} \sigma_h^*(p) > \sum_{p \geq \bar{q}} \sigma_l^*(p) = 1. \tag{11}$$

It is straightforward to verify that for $p_c \leq q_h - c_h$, the strategies $\sigma_h(c) = 1$, $\sigma_l(q_l) = 1$, $\sigma^*(s_{nn}|p, \mu) = 1$ whenever $\mu \Delta q + q_l \geq p$ and zero otherwise together with $\mu^*(p) = q_l$ constitute an equilibrium that sustains the equilibrium outcome.

For $p_c > q_h - c_h$, certification would yield seller q_h a negative payoff: $\Pi_h(c) = q_h - c_h - p_c < 0$. Certification would yield seller q_l a payoff $\Pi_l(c) =$

$q_l - p_c < q_l$, whereas seller q_l could guarantee himself the payoff q_l by not certifying. Q.E.D.

Proof of Proposition 4: First, suppose there exists an equilibrium in which the payoff of the certifier, Π_c^*, is strictly smaller than $\lambda(q_h - c_h - c_c)$. That is, $\delta = \lambda(q_h - c_h - c_c) - \Pi_c^* > 0$. Now note that the price $p_c = q_h - c_h - \delta/2 < q_h - c_h$ yields the certifier a payoff $\lambda(q_h - c_h + \delta/2) > \Pi_c^*$, because Proposition 3 shows that its subgame $\Gamma(p_c)$ has the unique outcome that seller q_h always certifies and seller q_l does not. Second, note that the certifier cannot obtain a profit that exceeds $\lambda(q_h - c_h - c_c)$, because it would require that the price of certification exceeds $q_h - c_h$ or that the low quality seller certifies with a strictly positive probability. Hence, in any equilibrium the certifier obtains the payoff $\lambda(q_h - c_h - c_c)$. According to Proposition 3 the certifier may become this payoff only for $p_c = q_h - c_h$ with $\sigma_h(c) = 1$. Q.E.D.

Proof of Proposition 5: For $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$ we have $\Pi_c^s = \lambda(q_h - c_h - c_c) \geq \lambda(q_h - c_h - c_c)\frac{q_h - q_l}{q_h + q_l} \geq \lambda(q_h - (q_h + q_l)/2 - c_c)\frac{q_h - q_l}{q_h + q_l} = \lambda(q_h - q_l - 2c_c)\frac{q_h - q_l}{2(q_h + q_l)} \geq \lambda(q_h - q_l - 4c_c)\frac{q_h - q_l}{2(q_h + q_l)} = \Pi_s^b$, where the second inequality uses $c_h \leq (q_h + q_l)/2$.

For $\lambda > 1/2$ or $c_h > (q_h + q_l)/2$ it follows that $\Pi_c^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h} < \frac{\lambda[(q_h - c_h)(c_h - q_l) - (c_h - q_l)c_c]}{c_h} = \lambda(q_h - c_h - c_c)\frac{c_h - q_l}{c_h} \leq \lambda(q_h - c_h - c_c) = \Pi_b^s$, where the first inequality uses $q_h > c_h$. Q.E.D.

Proof of Lemma 5: For $\lambda > 1/2$ or $c_h > (q_h + q_l)/2$, it follows

$$x_c^b = (\lambda + (1 - \lambda)\sigma_l^*(\tilde{p}))\sigma(s_{ch}|\tilde{p}, \mu_h^*) = \lambda\frac{\Delta q}{c_h} \leq \lambda = x_c^s,$$

where the inequality obtains from $q_h - c_h - c_c > q_l \;\Rightarrow\; \Delta q < c_h + c_c < c_h$.

For $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$, it follows

$$x_c^b = (\lambda + (1 - \lambda)\sigma_l^*(\tilde{p}))\sigma(s_{ch}|\tilde{p}, \mu_h^*) = \lambda\frac{2\Delta q}{q_h + q_l}.$$

Hence, $x_c^b < x_c^s$ if and only if $2\Delta q < q_h + q_l$. This yields the condition $q_h < 3q_l$. Q.E.D.

Proof of Proposition 6: Due to Lemma 5 we need only check for the case $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$ and $q_h < 3q_l$. According to Proposition 2 the certifier in this case makes non–negative profits exactly when $p_c^b = \Delta q/4 \geq c_c$. The differences in social welfare for this case is

$$\begin{aligned} \Delta WF &= \lambda \frac{\Delta q}{q_h + q_l} q_l + \lambda \left(\frac{2\Delta q}{q_h + q_l} - 1 \right) c_c && (12) \\ &= \frac{\lambda}{q_h + q_l} \left(\Delta q q_l - (3q_l - q_h) c_c \right) && (13) \\ &\geq \frac{\lambda}{q_h + q_l} \left(\Delta q q_l - (3q_l - q_h) \Delta q/4 \right) && (14) \\ &= \lambda \Delta q/4 > 0. && (15) \end{aligned}$$

Q.E.D.

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