Equilibrium Selection with Risk Dominance in a Multiple-unit Unit Price Auction

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Abstract

This paper uses an adapted version of the linear tracing procedure, suggested by Harsanyi and Selten (1988), in order to discriminate between two types of multiple Nash equilibria. Equilibria of the same type are pay-off equivalent in the analysed multiple-unit unit price auction where two sellers compete in order to serve a fixed demand. The equilibria where the firm with the larger capacity bids the maximum price, serves the residual demand and is undercut by the low capacity firm that sells its total capacity risk dominate the equilibria where the roles are interchanged.

Keywords: Equilibrium Selection, Risk Dominance, Auctions.

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1 Introduction

Lots of meaningful economic applications of game theory generate multiple equilibria. Often they are due to co-ordination problems similar to the by now famous *Battle of the Sexes*. In these sort of games the players have divergent preferences over the different equilibria and therefore pareto dominance and other well established criteria for equilibrium selection cannot discriminate among the different Nash equilibria whereas risk dominance which has been introduced by Harsanyi and Selten (1988) does a good job and is easy to apply in 2×2 games with asymmetric players (see Cabrales *et al.*, 2000, for a more thorough discussion and some endorsing evidence from experiments).

If the players can however choose from more than two strategies applying risk dominance as an equilibrium selection device becomes much more complicated, because then the full tracing procedure, developed by Harsanyi (1975), needs to be applied.[1] This is done, for example, by van Damme and Hurkens (2004) who apply risk dominance to a sequential pricing game with two firms with different constant marginal costs and differentiated products. They show that applying risk dominance only to the reduced game where firms are restricted to the strategies chosen in the two existing Nash equilibria selects the wrong equilibrium in their context. If the criterion is properly applied, taking into account the full range of possible deviations from the equilibrium strategies, then the equilibrium where the more efficient firm acts as a price leader and the less efficient firm as a follower risk dominates the one where the firms take the opposite roles. The latter equilibrium is, however, selected if the criterion of risk dominance is applied to the reduced game where the two players are restricted to their prices as a leader and as a follower.

Here we consider a particular form of a multiple-unit unit price auction which is known for generating multiple equilibria when the firms in the market are sufficiently capacity constrained (see e.g. Moreno and Ubeda, 2006; Crampes and Creti, 2005; Le Coq, 2002; Boom, 2002) and which has been used in order to model wholesale electricity markets. We show that similar to the case considered by van Damme and Hurkens (2004) applying risk dominance only to a reduced version of the game might select the wrong equilibrium. Then we analyse the auction with the full strategy set. Contrary to van Damme and Hurkens (2004), we can only discriminate between the two types of equilibria when we discretize the strategy space and turn the game into a finite game as has been demanded by Harsanyi (1975) and Schanuel *et al.* (1991). In our

[1] In some instances like in Amir and Stepanova (2006) it is nevertheless possible to reduce the game to a 2×2 game and analyse this in the usual way.

setting risk dominance selects the (pay-off equivalent) equilibria where the firm with the larger capacity acts as a monopolist on the residual demand and the firm with the smaller capacity undercuts sufficiently and supplies its whole capacity to the market.

2 The model

Suppose that a demand of x units of an item shall be satisfied via a unit-price auction. There are two potential suppliers A and B which can produce up to their maximum capacity of k_A and k_B. Their marginal cost of production is constant and for the sake of simplicity assumed to be equal to zero. We assume that both firms' capacities are sufficient and necessary to satisfy demand, meaning

$$k_A + k_B > x > \max\{k_A, k_B\} \tag{1}$$

In the auction each firm $i = A, B$ has to announce its reservation price p_i which is the lowest price at which it is willing to produce and supply the product up to its total capacity k_i. The price bids are restricted to $0 \leq p_i \leq \bar{p}$ and to a multiple of the smallest allowable money unit ε with $\varepsilon < \bar{p}/2$. The auctioneer sets the price which balances supply and demand. Since he needs always the capacities of both firms for this, the auction price can be described by

$$p(p_i, p_j) = p_i \text{ if } p_i \geq p_j \text{ with } i, j = A, B \text{ and } i \neq j. \tag{2}$$

The firm that announced a price below the market clearing price delivers its total capacity at the market clearing price, whereas the firm which announced the market clearing price is rationed to the production level necessary to balance supply and demand. With identical announcements both firm share the market. The produced volumes therefore depend on the price bids and are given by

$$y_i(p_i, p_j) = \begin{cases} k_i & \text{if } p_i < p_j, \\ \frac{k_i}{2} + \frac{x - k_j}{2} & \text{if } p_i = p_j, \\ x - k_j & \text{if } p_i > p_j, \end{cases} \tag{3}$$

with $i, j = A, B$ and $i \neq j$. Thus, firm i's pay-off, given the price bids (p_i, p_j), is

$$\pi_i(p_i, p_j) = p(p_i, p_j) y_i(p_i, p_j). \tag{4}$$

For this setting Crampes and Creti (2005) and Le Coq (2002) have already proven the following proposition

Proposition 1 *There are two types of Nash equilibria in pure strategies: one with* $p_A = \bar{p}$ *and* $p_B \leq \bar{p}(x - k_B)/k_A \equiv \underline{p}_B$*, and another with* $p_B = \bar{p}$ *and* $p_A \leq \bar{p}(x - k_A)/k_B \equiv \underline{p}_A$*. The wholesale price is the same* ($p(p_i, p_j) = \bar{p}$) *for both types of equilibria, but the profits in equilibrium differ:*

$$\pi_i(p_i, \bar{p}) = \bar{p}k_i \geq \pi_i(\bar{p}, p_j) = \bar{p}(x - k_j) \text{ with } p_i \leq \underline{p}_i, p_j \leq \underline{p}_j, i, j = A, B \text{ and } i \neq j.$$

Proof: See appendix A of Le Coq (2002) or the proof of proposition 3 in Crampes and Creti (2005), using that the constant marginal cost of generating electricity is identical and equal to zero by assumption. ■

Thus, there are multiple Nash equilibria of each of the two types in this game. Each equilibrium of the same type is pay-off equivalent whereas those belonging to different types are not. Each firm prefers an equilibrium where it itself chooses to undercut its rival and the rival chooses the maximum price. Thus, if this game represents only one intermediate stage in a multistage game, one (type of) equilibrium needs to be selected and pareto dominance as a selection criterion is not an option. In addition even side payments would not change the picture because, no matter which type of Nash equilibrium prevails, the aggregate profits of the two firms are always identical because $\pi_A(\bar{p}, p_B) + \pi_B(\bar{p}, p_B) = \pi_A(p_A, \bar{p}) + \pi_B(p_A, \bar{p}) = \bar{p}x$ with $p_i \leq \underline{p}_i$ and $i = A, B$ holds.

3 Applying Risk Dominance to a Reduced Version of the Game

Obviously, it is not straightforward, how to reduce the considered game to a 2×2 game, especially since there are multiple Nash equilibria of each type. Suppose, however, for the sake of illustration that each firm i is constrained to play either the maximum price $\bar{p}$ or the price $\underline{p}_i$ which ensures that the rival does not want to undercut the firm. This reduced game can then be represented by the normal form given in figure 1. This game has the two Nash equilibria in pure strategies $(p_A, p_B) = (\bar{p}, \underline{p}_B)$ and $(p_A, p_B) = (\underline{p}_A, \bar{p})$ which are each also equilibria in the original game and represent each one type of equilibrium. In addition the strategy $p_A = \underline{p}_A$ weakly dominates $p_A = \bar{p}$ as long as $k_A > k_B$. Applying the criterion of risk dominance to this reduced version of the game yields proposition 2.

Proposition 2 *Risk dominance selects the Nash equilibrium* $(p_A, p_B) = (\underline{p}_A, \bar{p})$ *in the game presented in figure 1 as long as* $k_A > k_B$ *holds.*

		Firm B	
		$\bar{p}$	$\underline{p}_B$
Firm A	$\bar{p}$	$\frac{\bar{p}(x-k_B+k_A)}{2}, \frac{\bar{p}(x-k_A+k_B)}{2}$	$\bar{p}(x-k_B), \bar{p}k_B$
	$\underline{p}_A$	$\bar{p}k_A, \bar{p}(x-k_A)$	$\bar{p}(x-k_B), \frac{\bar{p}(x-k_B)(x-k_A)}{k_A}$

Figure 1: A Reduced Version of the Game for $k_A > k_B$

Proof: The Nash equilibrium $(p_A, p_B) = (\underline{p}_A, \bar{p})$ risk dominates the Nash equilibrium $(p_A, p_B) = (\bar{p}, \underline{p}_B)$ because

$$\begin{aligned}
\Big[\pi_A(\underline{p}_A,\bar{p}) - \pi_A(\bar{p},\bar{p})\Big]\Big[\pi_B(\underline{p}_A,\bar{p}) - \pi_B(\underline{p}_A,\underline{p}_B)\Big] \quad &> \quad \Big[\pi_A(\bar{p},\underline{p}_B) - \pi_A(\underline{p}_A,\underline{p}_B)\Big] \\
&\quad \cdot \Big[\pi_B(\bar{p},\underline{p}_B) - \pi_B(\bar{p},\bar{p})\Big] \\
\Leftrightarrow \frac{\bar{p}^2(3k_A - x - k_B)(x-k_A)(k_A+k_B-x)}{2k_A} \quad &> \quad 0
\end{aligned}$$

holds for $k_A + k_B > x > k_A > k_B$. ■

Proposition 2 hinges on the assumption that the firms are playing the highest undercutting price, if they undercut. Assuming any other undercutting price compatible with each type of Nash equilibrium would change the pay-offs of the two firms in the south-east cell of the normal form in figure 1 and might lead to other conclusions than the one drawn in proposition 2.[2]

4 Applying Risk Dominance to the Full Game

As van Damme and Hurkens (2004) have already pointed out, reducing the strategy space to a 2×2 game can be misleading when selecting a Nash equilibrium on the grounds of risk dominance. This is also true in our case. In addition we do not only have to apply the concept of risk dominance to two equilibria as in their case but to potentially multiple equilibria of each type if ε is sufficiently small to allow for more than just one price below the critical level $\underline{p}_i$ for $i = A, B$.

In order to figure out which Nash equilibrium risk dominates, we need to proceed in two steps. First we have to determine the bi-centric prior of the

[2] This is for example the case if we alternatively restricted each firm's strategies to $p_i = \bar{p}$ and $p_i = 0$ with $i = A, B$.

two players and then we have to apply a tracing procedure (see Harsanyi and Selten, 1988; van Damme and Hurkens, 2004, for the details). Given that we do not have two equilibria but mainly two groups of equilibria between which we want to discriminate we need to adapt the determination of the bi-centric priors.

Following as close as possible the argument in Harsanyi and Selten (1988) and in van Damme and Hurkens (2004) we can arrive at Lemma 1

Lemma 1 *Player i's prior belief about Player j with $i, j = A, B$ and $i \neq j$ is that he is facing the mixed strategy*

$$m_j = b_j(Z_j)$$

defined in (5) where Z_j is a uniformly distributed random variable on $[0, 1]$.

Proof: Suppose, firm j believes that firm i plays $\bar{p}$ with probability z_j and with probability $1 - z_j$ a uniform distribution on $[0, \underline{p}_i]$, since all $p_i \leq \underline{p}_i$ generate the same profit in all the Nash equilibria where firm j chooses $p_j = \bar{p}$. Then firm j's profit is

$$\tilde{\pi}_j(p_i, p_j, z_j) = \begin{cases} z_j \frac{\bar{p}(k_j + x - k_i)}{2} + (1 - z_j)\bar{p}(x - k_i) & \text{for } p_j = \bar{p}, \\ z_j \bar{p} k_j + (1 - z_j) p_j (x - k_i) & \text{for } \underline{p}_i \leq p_j < \bar{p}, \\ z_j \bar{p} k_j + (1 - z_j) \left[\frac{p_j^2 k_j (2x - 2k_i - k_j)}{2\bar{p}(x - k_i)} + \frac{\bar{p}(x - k_i)}{2} \right] & \text{for } 0 \leq p_j < \underline{p}_i. \end{cases}$$

From firm j's profit maximization firm j's best response b_j can be derived

$$b_j(z_j) = \begin{cases} \bar{p} & \text{if } 0 \leq z_j < \tilde{z}_j(\varepsilon) \\ \bar{p} \text{ and } \bar{p} - \varepsilon \text{ each} & \\ \text{with probability } \frac{1}{2} & \text{if } z_j = \tilde{z}_j(\varepsilon) \\ \bar{p} - \varepsilon & \text{if } \tilde{z}_j(\varepsilon) < z_j \leq 1, \end{cases} \tag{5}$$

where the following definition is used:

$$\tilde{z}_j(\varepsilon) \equiv \frac{2(x - k_i)\varepsilon}{2(x - k_i)\varepsilon + \bar{p}(k_i + k_j - x)}. \tag{6}$$

Player i does not know the subjective probability z_j which player j assigns to i choosing the maximum price $\bar{p}$. Applying the principle of insufficient reason, player i considers z_j as being uniformly distributed on $[0, 1]$. ■

In order to apply the linear tracing procedure we need to look at a game with the same strategies for both players, but where firm i's pay-off $u_i(p_i, p_j, t, m_j)$, $i, j = A, B$ and $i \neq j$, is a linear combination of the pay-off from the original game given in (4) and the pay-off from the original game, given that the opponent chooses always the prior m_j defined in lemma 1:

$$u_i(p_i, p_j, t, m_j) = (1-t)\pi_i(p_i, m_j) + t\pi_i(p_i, p_j) \text{ with } t \in [0,1]. \tag{7}$$

From the analysis of this alternative game for all $t \in [0,1]$ we can derive lemma 2:

Lemma 2 *Suppose $k_i > k_j$ with $i, j = A, B$ and $i \neq j$. Then the pure Nash equilibria with $(p_i, p_j) = (\bar{p}, p_j)$, and $p_j < \underline{p}_j(t)$, as defined in (13), exists for the game with the pay-off function given in (7) if $t \geq \underline{t}_i(\varepsilon)$ with*

$$\begin{aligned}\underline{t}_i(\varepsilon) \equiv \bar{p}(k_i + k_j - x)\left[\bar{p}(k_i + k_j - x) - \varepsilon(2k_i - x)\right] \\ \big[\bar{p}(\bar{p}-\varepsilon)(k_i + k_j - x)^2 + \varepsilon\bar{p}(2x - k_i - k_j)(k_i + k_j - x) \\ +4\varepsilon^2(x - k_i)(x - k_j)\big]^{-1}.\end{aligned} \tag{8}$$

The pure Nash equilibria with $(p_i, p_j) = (p_i, \bar{p})$, and $p_i < \underline{p}_i(t)$, as defined in (16), exists for the game with the pay-off function given in (7) if $t > \bar{t}_j(\varepsilon)$ with

$$\begin{aligned}\bar{t}_j(\varepsilon) \equiv \bar{p}(k_i + k_j - x)\left[(\bar{p}-\varepsilon)(k_i + k_j - x) + 2\varepsilon(k_i - k_j)\right] \\ \big[\bar{p}(\bar{p}-\varepsilon)(k_i + k_j - x)^2 + \varepsilon\bar{p}(2x - 2k_j)(k_i + k_j - x) \\ +4\varepsilon^2(x - k_i)(x - k_j)\big]^{-1}.\end{aligned} \tag{9}$$

Proof: See the Appendix.■

Now we are in the position to derive the main result of our paper, given in proposition 3.

Proposition 3 *If the capacities of the two firms satisfy $k_i > k_j$ with $i, j = A, B$ and $i \neq j$ then the Nash equilibria with $(p_i, p_j) = (\bar{p}, p_j)$ and $p_j < \underline{p}_j$ dominate the equilibria with $(p_i, p_j) = (p_i, \bar{p})$ and $p_i < \underline{p}_i$, where $\underline{p}_i$ and $\underline{p}_j$ are defined in proposition 1, from a risk point of view.*

Proof: Comparing the two critical values for the linear weight t in the alternative game given in lemma 2 we can distinguish three cases

(i) If the two firms capacities satisfy $k_j < x/2 < k_i < x < k_i + k_j$ two situations can occur

$$0 < \underline{t}_i(\varepsilon) < \bar{t}_j(\varepsilon) < 1 \text{ if } 0 < \varepsilon < \frac{\bar{p}(k_i + k_j - x)}{2k_i - x} \text{ and}$$

$$\underline{t}_i(\varepsilon) \leq 0 < \bar{t}_j(\varepsilon) < 1 \text{ if } \frac{\bar{p}(k_i + k_j - x)}{2k_i - x} \leq \varepsilon < \frac{\bar{p}}{2}$$

(ii) If the two firms capacities satisfy $x/2 \leq k_j < k_i < x < k_i + k_j$ the relationship

$$0 < \underline{t}_i(\varepsilon) < \bar{t}_j(\varepsilon) < 1 \text{ holds for all } 0 < \varepsilon < \frac{\bar{p}}{2}.$$

(iii) If the two firms capacities satisfy $x/2 < k_j = k_i < x < k_i + k_j$ then

$$0 < \underline{t}_i(\varepsilon) = \bar{t}_j(\varepsilon) < 1 \text{ holds for all } 0 < \varepsilon < \frac{\bar{p}}{2}.$$

From these comparisons we see that the support of t for which equilibria with $(p_i, p_j) = (\bar{p}, p_j)$ and $p_j < \underline{p}_j(t)$ exist is larger in the relevant range $[0, 1]$ than the support for which the equilibria $(p_i, p_j) = (p_i, \bar{p})$ and $p_i < \underline{p}_i(t)$ exist as long as $k_i > k_j$ holds. Since $\lim_{t\to 1} \underline{p}_i(t) = \underline{p}_i$ and $\lim_{t\to 1} \underline{p}_j(t) = \underline{p}_j$, the conclusion can be drawn.■

Note that for all t always two types of pure Nash equilibria exist, one where the large firm bids high and the small firm bids sufficiently low and the other way round. However, bidding high for $0 < t < \underline{t}_i$ means for both firms to bid $\bar{p} - 2\varepsilon$ and, thus, still below the prices $\bar{p}$ and $\bar{p} - \varepsilon$ which correspond to the price levels on which they have positive priors for their components (see Lemma 1 and the Appendix). For the larger firm loosing on the price when bidding semi-high instead of really high and getting in both cases the residual demand is sooner more important than having a smaller and smaller chance to undercut the rivals high prior price and then to sell more. The large firm's residual demand is always higher than the small firm's residual demand. Therefore the smaller firm does not lose that much on sticking longer on semi-high prices in order to have a certain chance to sell more if the rival sticks to the prior. This intuition is confirmed in the following corollary 1.

Corollary 1 *If the the two firm's capacities are identical, $k_i = k_j$ with $i, j = A, B$ and $i \neq j$, or if the strategy space of the two firms becomes continuous with $\varepsilon \to 0$ then the different types of Nash equilibria cannot be ranked according to the risk dominance criterion because $\underline{t}_i(\varepsilon) = \bar{t}_j(\varepsilon)$.*

With $\varepsilon \to 0$ the loss on the price from slightly undercutting the priors of the rival firm becomes negligible compared to the maximum price. Both firms bid the maximum price instead of slightly undercutting the priors only when the priors are certainly not played any more because $\underline{t}_i = \bar{t}_j = 1$. With identical capacities both firms lose always the same amount from not bidding the top price instead of the semi-top and therefore they switch from semi-high to high prices at the exact same $\underline{t}_i = \bar{t}_j = t \in (0, 1)$.

5 Conclusions

We have shown in this paper how one can adapt the linear tracing procedure in order to rank two groups of Nash equilibria from a risk dominance perspective when the Nash equilibria in each group are pay-off equivalent, but cannot be pareto-ranked across the groups. In the considered two-players multi-unit unit-price auction the larger firm in terms of capacity bids high and sells the residual demand whereas the smaller firm bids sufficiently low and sells its total capacity.

Considering only a restricted game where the strategy space is reduced to a 2×2-game leads easily to wrong conclusions, mainly because strategies which are not equilibrium strategies in the original game are excluded from the analysis although they may play a major role (as our discussion of high versus semi-high prices in the last section shows).

If we either introduce symmetric players (here firms with identical capacities) or a continuous strategy space a ranking of the two types of Nash equilibria from a risk perspective is not possible anymore.

Appendix

Assume $k_i \geq k_j$. The alternative game's pay-off function given in (7) for $t \to 0$ implies that $\lim_{t \to 0} u_i(p_i, p_j, t, m_j) = \pi_i(p_i, m_j)$ with

$$\pi_i(p_i, m_j) = \begin{cases} \int_0^{\tilde{z}_j(\varepsilon)} \bar{p}\frac{x-k_j+k_i}{2} dz + \int_{\tilde{z}_j(\varepsilon)}^1 \bar{p}(x-k_j) dz & \text{if } p_i = \bar{p}, \\ \int_0^{\tilde{z}_j(\varepsilon)} \bar{p} k_i dz + \int_{\tilde{z}_j(\varepsilon)}^1 (\bar{p}-\varepsilon)\frac{x-k_j+k_i}{2} & \text{if } p_i = \bar{p}-\varepsilon, \\ \int_0^{\tilde{z}_j(\varepsilon)} \bar{p} k_i dz + \int_{\tilde{z}_j(\varepsilon)}^1 (\bar{p}-\varepsilon) k_i dz & \text{if } p_i < \bar{p}-\varepsilon, \end{cases} \quad (10)$$

and $i, j = A, B$ and $i \neq j$. From the analysis of this profit function it becomes obvious that firm i's best response is given by

$$b_i \begin{cases} < \bar{p} - \varepsilon & \text{if } \varepsilon < \frac{\bar{p}(k_i+k_j-x)}{2k_i-x} \\ = \bar{p} \text{ or } p_i < \bar{p} - \varepsilon & \text{if } \varepsilon = \frac{\bar{p}(k_i+k_j-x)}{2k_i-x} \\ = \bar{p} & \text{if } \varepsilon > \frac{\bar{p}(k_i+k_j-x)}{2k_i-x} \end{cases}$$

Confronting the two firms' best response functions and taking into account that

$$\frac{\bar{p}(k_i + k_j - x)}{2k_j - x} > \frac{\bar{p}(k_i + k_j - x)}{2k_i - x} > \frac{\bar{p}}{2} \text{ with } k_i + k_j > x > k_i > k_j > \frac{x}{2} \text{ and}$$

$$\frac{\bar{p}}{2} > \frac{\bar{p}(k_i + k_j - x)}{2k_i - x} > 0 > \frac{\bar{p}(k_i + k_j - x)}{2k_j - x} \text{ with } k_i + k_j > x > k_i > \frac{x}{2} > k_j$$

yields the following two lemmas 3 and 4

Lemma 3 *With $k_i + k_j > x > k_i > \frac{x}{2} > k_j$ it depends on the size of the discrete unit ε in which the price bids can be given which Nash equilibria exist for the alternative game with the pay-off function given in (7) if $t \to 0$:*

(i) With $\varepsilon < \frac{\bar{p}(k_i+k_j-x)}{2k_i-x}$ multiple Nash equilibria in pure strategies exist with (p_i, p_j) chosen such that $p_i < \bar{p} - \varepsilon$ and $p_j < \bar{p} - \varepsilon$.

(ii) With $\varepsilon = \frac{\bar{p}(k_i+k_j-x)}{2k_i-x}$ multiple Nash equilibria in pure strategies exist with (p_i, p_j) chosen such that either $p_i < \bar{p} - \varepsilon$ or $p_i = \bar{p}$ and $p_j < \bar{p} - \varepsilon$.

(iii) With $\varepsilon > \frac{\bar{p}(k_i+k_j-x)}{2k_i-x}$ multiple Nash equilibria in pure strategies exist with (p_i, p_j) chosen such that $p_i = \bar{p}$ and $p_j < \bar{p} - \varepsilon$.

Lemma 4 *With $k_i + k_j > x > k_i > k_j > \frac{x}{2}$ there exist multiple Nash equilibria in pure strategies for the alternative game with the pay-off function given in (7) and $t \to 0$ for all $\varepsilon < \bar{p}/2$ which imply that (p_i, p_j) are chosen such that $p_i < \bar{p} - \varepsilon$ and $p_j < \bar{p} - \varepsilon$.*

Now suppose $t \in (0, 1)$, then the pay-off of firm i with $i, j = A, B$ and $j \neq i$ for the alternative game given in (7) depends on the price bid chosen by the rival j. The pay-off is given by

$$u_i(p_i, \bar{p}, t, m_j) = \begin{cases} (1-t)\pi_i(p_i, m_j) + tp_i(x - k_j) & \text{if } p_j < p_i \leq \bar{p}, \\ (1-t)\pi_i(p_i, m_j) + t\frac{p_j(x-k_j+k_i)}{2} & \text{if } p_i = p_j \\ (1-t)\pi_i(p_i, m_j) + tp_jk_i & \text{if } p_i < p_j. \end{cases} \quad (11)$$

where $\pi_i(p_i, m_j)$ is defined in (10) and $i, j = A, B$, $i \neq j$. Analysing firm i's pay-off function for all potential p_j and assuming $k_i > k_j$ yields firm i's best response function. One can distinguish two cases:

(i) With $k_i + k_j > x > k_i > \frac{x}{2} > k_j$ and $\varepsilon > \frac{\bar{p}(k_i+k_j-x)}{2k_i - x}$ firm i's best response for all $t \in (0, 1)$ is

$$b_i(p_j) \begin{cases} < p_j & \text{if } p_j \geq \underline{p}_j(t), \\ = \bar{p} & \text{if } p_j \leq \underline{p}_j(t), \end{cases} \tag{12}$$

where $\underline{p}_j(t)$ is defined as

$$\begin{aligned} \underline{p}_j(t) &= \bar{p}\,\{\bar{p}(k_i + k_j - x)\,[x - k_j - k_i(1-t)] \\ &\quad + \varepsilon\left[2k_i^2(1-t) + (1+t)(x - k_j)x + k_i(2k_j - (3-t)x)\right]\} \\ &\quad \cdot \frac{1}{k_i t\,[\bar{p}(k_i + k_j - x) + 2\varepsilon(x - k_i)]}. \end{aligned} \tag{13}$$

Note that $\underline{p}_j(1) = \underline{p}_j$ as defined in proposition 1.

(ii) With either $k_i + k_j > x > k_i > k_j > \frac{x}{2}$ or $k_i + k_j > x > k_i > \frac{x}{2} > k_j$ and $\varepsilon \leq \frac{\bar{p}(k_i+k_j-x)}{2k_i - x}$ firm i's best response function depends on the level of t. For $0 < t \leq \underline{t}_i(\varepsilon)$ where $\underline{t}_i(\varepsilon)$ is defined in (8) it is given by

$$b_i(p_j) \begin{cases} < p_j & \text{if } p_j \geq \frac{(\bar{p}-2\varepsilon)(x-k_j)}{k_i}, \\ = \bar{p} - 2\varepsilon & \text{if } p_j \leq \frac{(\bar{p}-2\varepsilon)(x-k_j)}{k_i}. \end{cases}$$

For $t \geq \underline{t}_i(\varepsilon)$ firm $i's$ best response is again represented by (12).

Firm j's best response depends always on the level of t and is given by

$$b_j(p_i) \begin{cases} < p_i & \text{if } p_i \geq \frac{(\bar{p}-2\varepsilon)(x-k_i)}{k_j}, \\ = \bar{p} - 2\varepsilon & \text{if } p_i \leq \frac{(\bar{p}-2\varepsilon)(x-k_i)}{k_j}, \end{cases}$$

if $0 < t < \tilde{t}_j(\varepsilon)$ with

$$\begin{aligned} \tilde{t}_j(\varepsilon) &= \bar{p}(\bar{p} - \varepsilon)(k_i + k_j - x)^2 \\ &\quad \cdot \frac{1}{\bar{p}^2(k_i + k_j - x)^2 + 2\varepsilon\bar{p}(x - k_i)(k_i + k_j - x) + 4\varepsilon^2(x - k_i)(x - k_j)}. \end{aligned} \tag{14}$$

For $\tilde{t}_j(\varepsilon) \leq t \leq \bar{t}_j(\varepsilon)$ where $\bar{t}_j(\varepsilon)$ is given by (9) firm j's best response function is

$$b_j(p_i) \begin{cases} < p_i & \text{if } p_i \geq \hat{p}_i(t), \\ = \bar{p} - \varepsilon & \text{if } p_i \leq \hat{p}_i(t), \end{cases}$$

where $\hat{p}_i(t)$ is given by

$$\begin{aligned}\hat{p}_i(t) &= (\bar{p}-\varepsilon)\left\{\bar{p}(k_i+k_j-x)\left[(x-k_i)(1+t)-k_j(1-t)\right]\right. \\ &\quad \left.+4\varepsilon t(x-k_i)(x-k_j)\right\} \\ &\quad \cdot\frac{1}{2k_j t\left[\bar{p}(k_i+k_j-x)+2\varepsilon\right]}.\end{aligned} \tag{15}$$

Finally, for $\bar{t}_j(\varepsilon) \leq t \leq 1$, firm j's best response function is given by

$$b_j(p_i)\begin{cases} < p_i & \text{if } p_i \geq \underline{p}_i(t), \\ = \bar{p} & \text{if } p_i \leq \underline{p}_i(t), \end{cases}$$

where $\underline{p}_i(t)$ from the perspective of firm j is the equivalent to $\underline{p}_j(t)$ from the perspective of firm i, meaning

$$\begin{aligned}\underline{p}_i(t) &= \bar{p}\left\{\bar{p}(k_i+k_j-x)\left[x-k_i-k_j(1-t)\right]\right. \\ &\quad \left.+\varepsilon\left[2k_j^2(1-t)+(1+t)(x-k_i)x+k_j(2k_i-(3-t)x)\right]\right\} \\ &\quad \cdot\frac{1}{k_j t\left[\bar{p}(k_i+k_j-x)+2\varepsilon(x-k_j)\right]}.\end{aligned} \tag{16}$$

Again $\underline{p}_i(1) = \underline{p}_i$ holds, where $\underline{p}_i$ is defined in proposition 1. From the analysis of firm i's and firm j's best responses we can derive lemma 5 and lemma 6.

Lemma 5 *The Nash equilibria with $(p_i, p_j) = (\bar{p}, p_j)$ and $p_j < \underline{p}_j(t)$ exist*

(i) for all $t \in [0,1]$ if $k_i + k_j > x > k_i > \frac{x}{2} > k_j$ and $\varepsilon \geq \frac{\bar{p}(k_i+k_j-x)}{2k_i-x}$ and

(ii) for all $t \in [\underline{t}_i, 1]$, where $\underline{t}_i$ is defined in (8), if either $k_i + k_j > x > k_i > \frac{x}{2} > k_j$ and $\varepsilon < \frac{\bar{p}(k_i+k_j-x)}{2k_i-x}$ or if $k_i + k_j > x > k_i > k_j \geq \frac{x}{2}$ and $0 < \varepsilon < \bar{p}/2$.

Lemma 6 *The Nash equilibria with $(p_i, p_j) = (p_i, \bar{p})$ and $p_i < \underline{p}_i(t)$ exist for all $t \in [\bar{t}_j(\epsilon), 1]$, where $\bar{t}_j(\epsilon)$ is defined in (9), and for all $k_j \leq k_i < x < k_i + k_j$ and $0 < \varepsilon < \bar{p}/2$.*

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