Sequential Merger Review*

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Abstract

We analyze the optimal dynamic policy of an antitrust authority towards horizontal mergers when merger proposals are endogenous and occur over time. Approving a currently proposed merger will affect the profitability and welfare effects of potential future mergers, the characteristics of which may not yet be known to the antitrust authority. We show that, in many cases, this apparently difficult problem has a simple resolution: an antitrust authority can maximize discounted consumer surplus by using a completely myopic merger review policy that approves a merger today if and only if it does not lower consumer surplus given the current market structure.

1 Introduction

The traditional approach to the review of horizontal mergers stresses the tradeoff between market power and efficiencies. Mergers, which cause firms to internalize pricing externalities among former rivals, increase the exercise of market power, and therefore tend to reduce social welfare. On the other hand, since they can create efficiencies, horizontal mergers may instead increase welfare. This tradeoff was first articulated by Williamson [1968], using a diagram like Figure 1. In the diagram, a competitive industry merges to become a monopolist, but lowers its cost of production from c to c'. Whether aggregate surplus increases or not

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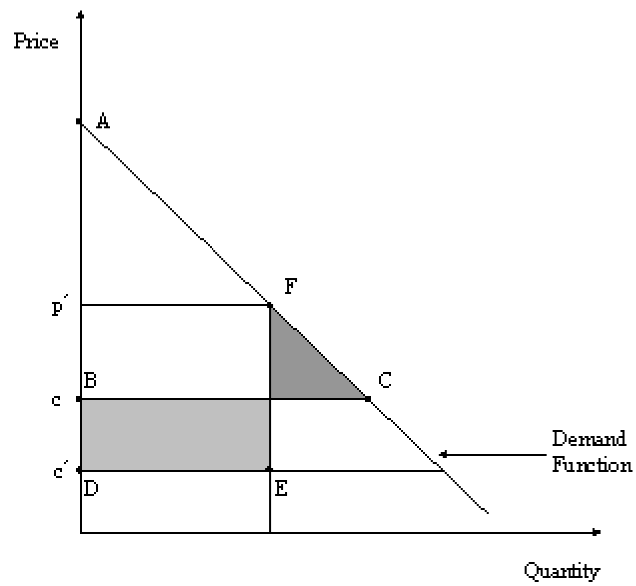

Figure 1: The Williamson tradeoff of merger approval: deadweight loss of market power (dark-shaded triangle) *vs.* efficiency gain (light-shaded rectangle).

depends on whether the dark-grey deadweight loss triangle exceeds the light-grey efficiency gain. More recently, Farrell and Shapiro [1990] (see also McAfee and Williams [1992]) have provided a more complete and formal analysis of this tradeoff for the context of Cournot competition. Farrell and Shapiro provide a necessary and sufficient condition for a merger to increase consumer surplus, as well as a sufficient condition for a merger to increase aggregate surplus.

With few exceptions, however, the literature on merger review has focused on the review of a single merger, while in reality mergers are usually not one-time events.[1] That is, one merger in an industry may be followed by other proposed mergers. In that case, approval of a merger today based on current conditions, as in the Farrell and Shapiro test, appears inappropriate. Rather, an antitrust authority would seem to need to forecast the welfare effect of the current merger given the potential for future merger approvals, and given the fact that approval of the merger today may change the set of mergers that are later proposed.

[1] Motta and Vasconcelos [2005] is the one paper we are aware of that considers merger review in a dynamic context. Nilssen and Sorgard [1998], Gowisankaran [1999], Fauli-Oller [2000], and Pesendorfer [2005] are among the articles that study equilibrium merger decisions in dynamic settings without considering merger policy (and, usually, without allowing for efficiencies).

Surprisingly, perhaps, we show that in many cases this apparently difficult problem has a very simple resolution: an antitrust authority who wants to maximize consumer surplus can accomplish this objective by using a completely myopic merger review policy that approves a merger today if and only if it does not lower consumer surplus given the current market structure.[2]

We begin in Section 2 by describing our basic model and establishing some preliminary characterizations of consumer surplus-enhancing mergers and their interactions. Our central results focus on a model of Cournot competition with constant returns to scale. Most importantly, we show in this section that there is a form of complementarity among consumer surplus-enhancing mergers in this setting.

Section 3 contains our main result. There we imbed our Cournot competition framework in a dynamic model in which merger opportunities arise, and may be proposed, over time. We show that if the set of possible mergers is disjoint, then a completely myopic consumer surplus-based approval policy maximizes discounted consumer surplus for every possible realization of the set of feasible mergers.

In Section 4 we discuss the robustness of this result, considering other models of competition (homogeneous and differentiated product price competition), nonconstant returns to scale, nondisjointness of mergers, and the use of an aggregate surplus criterion.

2 Mergers in the Cournot Model

2.1 Cournot Oligopoly

Consider an industry with n firms producing a homogeneous good and competing in quantities. Let $N \equiv \{1, 2, ..., n\}$ denote the set of firms. Firm i's cost of producing q_i units of output is given by $C_i(q_i) = c_i q_i$. Thus, for now, we restrict attention to firms producing under constant returns to scale. The inverse market demand is given by the twice differentiable function $P(Q)$, where $Q \equiv \sum_{i \in N} q_i$ is industry output. We make the following (standard) assumption on demand.

Assumption 1 *For any $Q > 0$ such that $P(Q) > 0$:*

(i) $P'(Q) < 0$;

(ii) $P'(Q) + QP''(Q) < 0$;

(iii) $\lim_{Q \to \infty} P(Q) = 0$;

(iv) $\lim_{Q \to 0} P(Q) > \min_i c_i$.

[2] Note that in both the U.S. and the EU, the legal standard of merger policy is close to being a consumer surplus standard rather than an aggregate surplus standard.

Part (i) of the assumption says that demand is downward-sloping, part (ii) implies that quantities are strategic substitutes, part (iii) says that price converges to zero as industry output grows without bound, while part (iv) ensures that industry output is positive.

Let $Q_{-i} \equiv \sum_{j \neq i} q_j$ denote the aggregate output of all firms other than i. Firm i's best-response function, $b(Q_{-i}; c_i)$, is the unique solution in b_i to the first-order condition

$$P(Q_{-i} + b_i) - c_i + b_i P'(Q_{-i} + b_i) = 0. \tag{1}$$

Let Q^* and q_i^* denote industry output and firm i's output in equilibrium. From (1),

$$q_i^* = -\frac{P(Q^*) - c_i}{P'(Q^*)} \tag{2}$$

if $c_i < P(Q^*)$, and $q_i^* = 0$ otherwise. As is well known (see e.g., Farrell and Shapiro [1990]), Assumption 1 implies that each firm's best-response function $b(\cdot; c_i)$ satisfies $\partial b(Q_{-i}; c_i)/\partial Q_{-i} \in (-1, 0)$ at all Q_{-i} such that $b(Q_{-i}; c_i) > 0$. In turn, this implies that there exists a unique Nash equilibrium. These assumptions also imply that the equilibrium is "stable," so that comparative statics are "well behaved." In what follows, we will make extensive use of two comparative statics properties. First, a reduction in a firm's marginal cost increases its equilibrium output and profit, reduces the output of each of its rivals, and increases aggregate output. Second, following any change in the incentives of a subset of firms, the equilibrium aggregate output increases [decreases] if and only if the equilibrium output of that set of firms increases [decreases].[3]

2.2 The CS-Effect of Mergers

Consider a merger between a subset $M \subseteq N$ of firms. The post-merger marginal cost is denoted $\overline{c}_M$. Aggregate output before the merger is Q^*, and after is $\overline{Q}^*$. We are interested in the effect of the merger on consumer surplus, $CS(\overline{Q}^*) - CS(Q^*)$, where

$$CS(Q) = \int_0^Q [P(s) - P(Q)]\, ds.$$

Since $CS'(Q) = -QP'(Q)$, a merger raises consumer surplus if and only if it induces an increase in industry output. We will say that the merger is *CS-neutral* if the merger does not affect consumer surplus. Similarly, we will say that a merger is *CS-increasing* [*CS-decreasing*], if consumer surplus following the merger is higher [lower] than before. Finally, a merger is *CS-nondecreasing* [*CS-nonincreasing*] if it is not CS-decreasing [CS-increasing].

The following result catalogs some useful properties of CS-neutral mergers among active firms (who produce positive outputs prior to the merger).

Lemma 1 *If a merger among active firms is CS-neutral then*

[3] See Farrell and Shapiro [1990]'s Lemma, p. 110.

1. *it causes no changes in the output of any nonmerging firm nor the total output of the merging firms;*

2. *the merging firm's margin at the pre-merger price $P(Q^*)$ equals the sum of their premerger margins:*

$$P(Q^*) - \overline{c}_M = \sum_{i \in M} [P(Q^*) - c_i]\,;$$

3. *the merger reduces the merging firm's marginal cost below the marginal cost of the most efficient merger partner: $\overline{c}_M < \min_{i \in M}\{c_i\}$;*

4. *the merger is profitable (it raises the joint profit of the merging firms).*

Proof. To see Property 1, observe that under Assumption 1 the merger must leave unchanged the joint output of the merging firms (otherwise aggregate output would change). In turn, this implies that all other firms must also have unchanged outputs (since their original outputs are still best responses). For Property 2, a central feature in Farrell and Shapiro [1990]'s analysis, note that the merged firm's first-order condition must satisfy

$$P(Q^*) - \overline{c}_M + \left(\sum_{i \in M} q_i^*\right) P'(Q^*) = 0. \tag{3}$$

Summing up the pre-merger first-order conditions of the merger partners yields

$$\sum_{i \in M} [P(Q^*) - c_i] + \sum_{i \in M} q_i^* P'(Q^*) = 0. \tag{4}$$

Combining equations (3) and (4), yields Property 2:

$$P(Q^*) - \overline{c}_M = \sum_{i \in M} [P(Q^*) - c_i]\,. \tag{5}$$

Property 3 follows directly from Property 2. Property 4 holds since the merging firms' joint output has not changed (Property 1), but its margin has increased (Property 2). ■

The following useful corollary follows from Properties 2 and 4 of Lemma 1 plus the fact that the post-merger aggregate output, $\overline{Q}^*$, and the profit of the merged firm are both decreasing in the merged firm's marginal cost, $\overline{c}_M$:

Corollary 1 *A merger among active firms is CS-neutral if*

$$\overline{c}_M = \widehat{c}_M(Q^*) \equiv P(Q^*) - \sum_{i \in M} [P(Q^*) - c_i]\,,$$

CS-increasing if $\overline{c}_M < \widehat{c}_M(Q^*)$, and CS-decreasing if $\overline{c}_M > \widehat{c}_M(Q^*)$. Moreover, any CS-nondecreasing merger is profitable for the merging firms.

Thus, an antitrust authority concerned with maximizing consumer surplus and confronted with a single merger between the firms in set M would want to approve the merger if and only if $\overline{c}_M < \widehat{c}_M(Q^*)$. Moreover, any merger the antitrust authority would want to approve will be profitable for the merging parties and hence proposed. Observe also that the threshold $\widehat{c}_M(Q^*)$ is increasing in Q^*; the larger is Q^* (and the lower is the current price), the more likely it is that a merger is CS-nondecreasing. Intuitively, when the price is lower, the increase in the incentive to raise price that arises from the merger is smaller. This fact will play an central role in the next subsection when we look at interactions among mergers.

Figure 2 illustrates the cases of CS-neutral, CS-increasing, and CS-decreasing mergers. The figure considers a merger involving the firms in set M_1. The complementary set of firms is denoted $M_2 \equiv N \backslash M_1$. The axes in the figure measure the joint outputs of the two sets of firms. The blue curves depict what we call the "group-reaction functions" of each set of firms prior to the merger. Specifically, M_i's pre-merger group-reaction function $r_{M_i}(q_{M_j})$ gives the joint pre-merger Nash-equilibrium output of the firms in M_i conditional on the firms in M_j jointly producing q_{M_j}. It is routine to verify that these group-reaction functions satisfy $-1 < r'_{M_i}(q_{M_j}) < 0$.

The equilibrium before the merger is point A, the intersection of the two pre-merger group-reaction curves. With a CS-neutral merger, the post-merger best-response curve of the merged firm, $b(q_{M_j}; \widehat{c}_{M_1}(Q^*))$, intersects group M_2's group-reaction curve, $r_{M_1}(\cdot)$, at point A. (The fact that it lies to the right [left] of the premerger group-reaction curve of M_1 for $q_{M_2} > [<] q^*_{M_2}$ can be shown using logic similar to that leading to Corollary 1.[4]) With a CS-increasing merger, the merged firm's marginal cost is less than $\widehat{c}_{M_1}(Q^*)$, so its best-response curve lies further to the right, shifting the equilibrium to point B. In contrast, with a CS-decreasing merger, the merged firm's marginal cost is greater than $\widehat{c}_{M_1}(Q^*)$, so its best-response curve lies further to the left, shifting the equilibrium to point C.

2.3 Interactions between Mergers

We now turn to the interactions between mergers. In this subsection, we consider two potential disjoint mergers, involving firms in sets M_1 and M_2 with $M_1 \cap M_2 = \emptyset$. We'll refer to these simply as merger M_1 and merger M_2. The set of firms not involved in either merger is $N^c \equiv N \backslash (M_1 \cup M_2)$.

[4] Specifically, any post-merger best-response curve for the merged firm must cross the merged firms' pre-merger group-reaction curve once at the non-merging firms' joint output $\overline{q}_{M_2}$ such that $\widehat{c}_{M_1}(\overline{q}_{M_2} + r_{M_1}(\overline{q}_{M_2}) = \overline{c}_{M_1}$. Moreover, this crossing must be from above: at $q_{M_2} > [<] \overline{q}_{M_2}$, we have

$$\begin{aligned} b(q_{M_2}; \overline{c}_{M_1}) &= b(q_{M_2}; \widehat{c}_{M_1}(\overline{q}_{M_2} + r_{M_1}(\overline{q}_{M_2}))) \\ &> [<] b(q_{M_2}; \widehat{c}_{M_1}(q_{M_2} + r_{M_1}(q_{M_2}))) \\ &= r_{M_1}(q_{M_2}). \end{aligned}$$

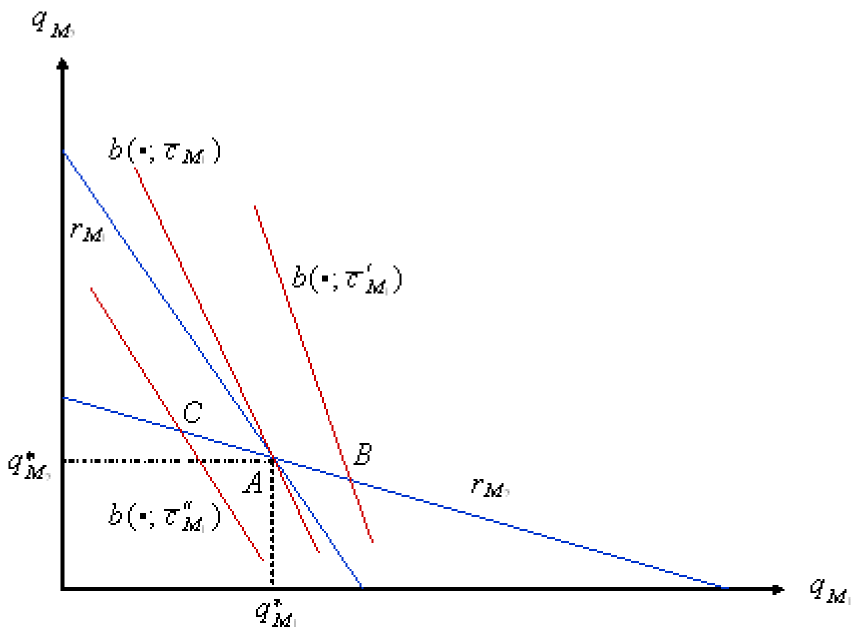

Figure 2: A merger involving the firms in M_1. Depending on the merged entity's marginal cost, the merger is CS-neutral, CS-increasing, or CS-decreasing. In the figure, $\overline{c}''_{M_1} > \overline{c}_{M_1} > \overline{c}'_{M_1}$.

Our first result establishes a certain complementarity between mergers that change consumer surplus in the same direction:

Proposition 1 *The CS-effect of mergers is complementary:*

(i) *If a merger is CS-nondecreasing (and hence profitable) in isolation, it remains CS-nondecreasing (and hence profitable) if another merger that is CS-nondecreasing in isolation takes place.*

(ii) *If a merger is CS-decreasing in isolation, it remains CS-decreasing if another merger that is CS-decreasing in isolation takes place.*

Proof. For part (i), suppose that mergers M_1 and M_2 are both CS-nondecreasing in isolation. Let Q^* denote aggregate output in the absence of either merger and let $\overline{Q}_i^*$ denote aggregate output if only merger i takes place. So $\overline{Q}_i^* \geq Q^*$ for $i = 1, 2$. From Corollary 1 we know that $\overline{c}_{M_i} \leq \widehat{c}_{M_i}(Q^*)$ for $i = 1, 2$. Since the threshold $\widehat{c}_{M_i}(Q)$ is strictly increasing in Q, we have $\overline{c}_{M_i} \leq \widehat{c}_{M_i}(\overline{Q}_j^*)$ for $i, j = 1, 2$, $i \neq j$. That is, each merger is also CS-nondecreasing once the other merger has taken place. The argument for part (ii) follows similar lines. ■

Figure 3 illustrates the complementarity between two mergers that are CS-increasing in isolation when no other firms exist ($N^c = \emptyset$). In isolation, merger M_1 moves the equilibrium from point A to point B, while merger M_2 moves the equilibrium from point A to point C. But, conditional on merger M_1 taking place, merger M_2 moves the equilibrium from point B to point D along $b(\cdot; \overline{c}_{M_1})$. Since $b'(\cdot; \overline{c}_{M_1}) \in (-1, 0)$, aggregate output must increase. That is, conditional on merger M_1 taking place, merger M_2 remains CS-increasing. Moreover, we know from Corollary 1 it also remains profitable. Using the same type of argument, the reverse is also true: conditional on merger M_2 taking place, the merger M_1 remains CS-increasing and profitable.

Figure 4 illustrates the case where both mergers are CS-decreasing in isolation. Conditional on the other merger taking place, each merger remains CS-decreasing. For instance, conditional on merger M_1 taking place, merger M_2 moves the equilibrium from point B to point D along $b(\cdot; \overline{c}_{M_1})$. Since $b'(\cdot; \overline{c}_{M_1}) \in (-1, 0)$, it follows that the merger reduces industry output.

We now turn to the interaction between mergers which have opposite effects on consumer surplus if implemented in isolation. Specifically, suppose that merger M_1 is CS-increasing (and therefore profitable) in isolation, while merger M_2 is CS-decreasing in isolation. Figure 5 illustrates that merger M_2 can become CS-increasing (and therefore profitable) conditional on merger M_1 occurring. In isolation, merger M_2 moves the equilibrium from point A to point C along $r_{M_1}(\cdot)$, and thus decreases industry output. But conditional on merger M_1 taking place, merger M_2 moves the equilibrium from point B to point D along $b(\cdot; \overline{c}_{M_1})$, and thus increases industry output.

When this occurs, we can say the following:

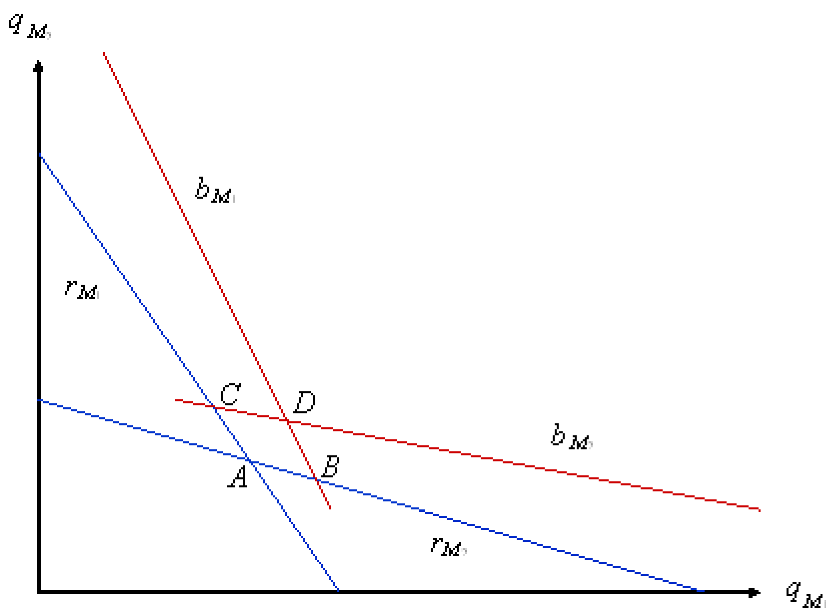

Figure 3: Each merger is CS-increasing in isolation and remains so if the other merger takes place.

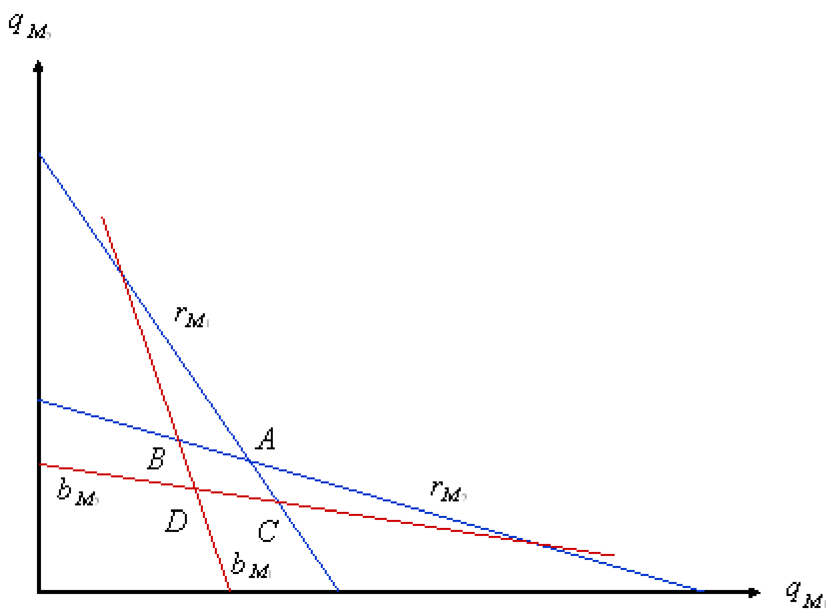

Figure 4: Each merger is CS-decreasing in isolation and remains so if the other takes place.

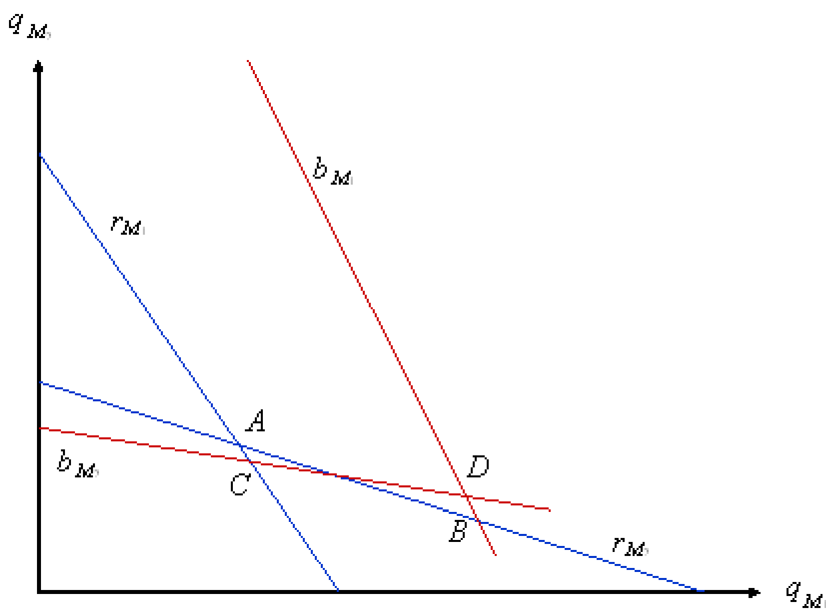

Figure 5: A CS-decreasing merger becomes CS-increasing after a CS-increasing merger takes place.

Proposition 2 *Suppose that merger M_1 is CS-increasing in isolation, while merger M_2 is CS-decreasing in isolation but CS-nondecreasing once merger M_1 has taken place. Then:*

(i) *Merger M_1 is CS-increasing (and therefore profitable) conditional on merger M_2 taking place.*

(ii) *The joint profits of the firms involved in merger M_1 are higher if both mergers take place than if neither merger takes place.*

Proof. Consider implementing merger M_1 first followed by merger M_2. By hypothesis, consumer surplus weakly increases at each step, so the combined effect of the two mergers is weakly positive. Suppose we now reverse the order and implement merger M_2 first. Since merger M_2 is CS-decreasing in isolation, consumer surplus strictly decreases. But since the combined effect of the two mergers on consumer surplus is weakly positive, consumer surplus must strictly increase when the merger M_1 is implemented following merger M_2. Hence, part (i) must hold: merger M_1 is CS-increasing (and therefore profitable) conditional on merger M_2 taking place. To see that part (ii) holds, suppose the merger M_2 is implemented first. Since merger M_2 is CS-decreasing in isolation, the joint output of the firms involved in this merger decreases and aggregate output falls. But this means that the M_2 merger must increase the profit of each firm $i \in M_1$ [the output of all firms other than i must decrease, otherwise the fact that $b'(\cdot; c_i) \in (-1, 0)$ would imply that aggregate output increases]. Since merger M_1 is profitable given merger M_2, the sequence of mergers must increase the joint profit of the firms in M_1. ■

The result is illustrated in Figure 5, where merger M_1 is CS-increasing (and hence profitable) in isolation and remains so conditional on merger M_2 taking place: it moves the equilibrium from point C to point D along $b(\cdot; \overline{c}_{M_2})$.

3 CS-Maximizing Merger Review

In this section, we consider the optimal merger approval policy for an antitrust authority concerned with maximizing (discounted) consumer surplus when multiple disjoint mergers may be proposed over time. We will show that, in the Cournot model of Section 2, a consumer surplus-oriented antitrust authority can achieve its optimal outcome using a myopic policy that approves mergers if and only if they are CS-nondecreasing at the time of approval.

As before, we denote the set of firms by N. The set of possible mergers are those in set $\mathbb{A} \equiv \{M_1, ..., M_K\}$, where $M_k \subseteq N$ is a set of firms that may merge. We assume that the set of possible mergers are disjoint; that is, $M_j \cap M_k = \emptyset$ for $j \neq k$. Thus, no firm has the possibility of being part of more than one merger. The merger process lasts for T periods. At the start of each period t, merger M_k may become feasible with probability $p_{kt} \in [0, 1]$, where $\sum_t p_{kt} \leq 1$.[5] Conditional on merger M_k becoming feasible in period t, the firms in M_k receive

[5] Since a merger that results in a post-merger marginal cost above the marginal cost of the

a random draw of their post-merger cost $\bar{c}_{M_k}$ according to the distribution F_{kt}. This formulation embodies another form of disjointness in merger possibilities: merger M_k receives at most one efficiency realization throughout the merger process.

Within each period t there are n stages at which the antitrust authority can approve a merger. For simplicity, at most one merger can be approved at each stage. At the start of each period, all firms with feasible mergers decide whether to propose them or not. The antitrust authority observes that a particular merger is feasible and its efficiency (post-merger marginal cost) only once it is proposed. Firms observe their own merger possibility when it becomes feasible but, like the antitrust authority, observe the possibilities of other firms only when those mergers are proposed. Previously proposed but rejected mergers can be either withdrawn or proposed again.[6] Payoffs in each period depend only on the set of mergers approved at the end of the period. The antitrust authority and the firms discount future payoffs (consumer surplus or profit) according to the discount factor $\delta \leq 1$.

In Section 2, we examined the interaction between two mergers. For the purposes of this section, we start by noting two useful properties of the interactions among more than two mergers:

Lemma 2 ***(Incremental Gain Property)*** *Suppose that a set of mergers $\mathfrak{M} \equiv \{M_1, ..., M_{J_1}\}$ has the property that every merger $M \in \mathfrak{M}$ is CS-nondecreasing if all of the other mergers in $\mathfrak{M}$ (those in the set $\mathfrak{M} \backslash M$) have taken place. Then, for any strict subset $Y \subset \mathfrak{M}$, there exists an $M' \in \mathfrak{M} \backslash Y$ which is CS-nondecreasing if all of the mergers in Y have taken place.*

Proof. Suppose the result is not true, so that every $M' \in \mathfrak{M} \backslash Y$ is CS-decreasing if all of the mergers in Y have taken place. Proposition 1(ii) implies that, taking the mergers in Y as given, for any sequencing of the mergers in the set $\mathfrak{M} \backslash Y$ the merger implemented at each step, including the last step, is CS-decreasing. But this contradicts the hypothesis that the last merger is CS-nondecreasing if all of the other mergers in the set $\mathfrak{M} \backslash M$ have occurred. ■

Lemma 3 *Suppose that two distinct sets of mergers $\mathfrak{M}_1 \equiv \{M_1, ..., M_{J_1}\}$ and $\mathfrak{M}_2 \equiv \{M_1, ..., M_{J_2}\}$ with $\mathfrak{M}_1 \nsubseteq \mathfrak{M}_2$, not necessarily disjoint, each have the property that every merger $M \in \mathfrak{M}_i$ is CS-nondecreasing if all of the other mergers in $\mathfrak{M}_i$ (those in the set $\mathfrak{M}_i \backslash M$) have taken place. Then, if all of the mergers in $\mathfrak{M}_2$ have taken place, there exists a sequencing of the mergers in $\mathfrak{M}_1 \backslash (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-nondecreasing at each step.*

Proof. The Incremental Gain Property implies that there exists a merger $M' \in \mathfrak{M}_1 \backslash (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-nondecreasing given that all of the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ have taken place. We will show below that the set $(\mathfrak{M}_2 \cup M')$ has the

most efficient merging firm will never be approved, this "feasibility" may be viewed as the event of receiving a cost draw below that level.

[6] In the model, we do not allow previously approved mergers to be dissolved. However, it follows from our arguments that no merged firm would want to do so.

property that every merger $M \in (\mathfrak{M}_2 \cup M')$ is CS-nondecreasing if all of the other mergers in $(\mathfrak{M}_2 \cup M')$ [those in the set $(\mathfrak{M}_2 \cup M')\backslash M$] have taken place. This implies, first, that M' is CS-nondecreasing given that all of the mergers in $\mathfrak{M}_2$ have taken place. It also lets us replace the set $\mathfrak{M}_2$ with the set $\mathfrak{M}_2 \cup M'$ and apply the argument again, finding a merger M'' that is CS-nondecreasing given that all of the mergers in $\mathfrak{M}_2 \cup M'$ have taken place. Continuing iteratively we identify at each step the next CS-nondecreasing merger in $\mathfrak{M}_1\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ for the sequence.

We now show that the set $(\mathfrak{M}_2 \cup M')$ has the property that every merger $M \in (\mathfrak{M}_2 \cup M')$ is CS-nondecreasing if all of the other mergers in $(\mathfrak{M}_2 \cup M')$ have taken place. The Incremental Gain Property implies that there is a sequencing of the mergers in $\mathfrak{M}_2\backslash(\mathfrak{M}_1 \cap \mathfrak{M}_2)$, say $(M_{21}, M_{22}, ..., M_{2J_2})$, that is CS-nondecreasing at each step given that the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2)$ have taken place. We first argue that M' is CS-nondecreasing given that all of the mergers in $\mathfrak{M}_2$ have taken place. Let $E_j \equiv (\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup M_{21} \cup \cdots \cup M_{2j}$. We know M' is CS-nondecreasing given that the mergers in $E_0 = (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ have taken place. Now suppose M' is CS-nondecreasing given that all of the mergers in E_j have taken place, for $j \leq S-1$. Consider E_S. Since M' and M_{2S} are both CS-nondecreasing given that all of the mergers in E_{S-1} have taken place, Proposition 1(i) implies that M' is CS-nondecreasing given that all of the mergers in E_S have taken place. Continuing iteratively, establishes that M' is CS-nondecreasing given that all of the mergers in $\mathfrak{M}_2$ have taken place. Next, we argue that each merger $M'' \in \mathfrak{M}_2$ is CS-nondecreasing if all of the mergers in $(\mathfrak{M}_2 \cup M')\backslash M''$ have taken place. Recall that each $M'' \neq M'$ is CS-nondecreasing given that all of the mergers in $\mathfrak{M}_2\backslash M''$ have taken place. If M' is CS-nondecreasing given that all of the mergers in $\mathfrak{M}_2\backslash M''$ have taken place, Proposition 1(i) implies the claim. If, instead, M' is CS-decreasing given that all of the mergers in $\mathfrak{M}_2\backslash M''$ have taken place, Proposition 2(i) implies the claim. This establishes the result. ■

3.1 Static Merger Review

It will be useful to first consider merger review in a static setting, corresponding to the case in which $T = 1$. We first define what we mean by a myopic merger review policy:[7]

Definition 1 *A* ***myopic CS-based merger policy*** *approves at each stage a proposed merger that is CS-nondecreasing given the market structure at the start of that stage, if any such mergers exist.*

In the static (one-period) setting, the only thing that matters for the antitrust authority's payoff is the set of approved mergers at the end of the period. This, of course, depends on the realization of the set of feasible mergers (including their cost realizations). The antitrust authority does not know which mergers are feasible. It only sees proposed mergers. Nonetheless, we will see that any myopic CS-based rule maximizes consumer surplus for each realization

[7] Note that, for simplicity, we resolve indifference in favor of approval.

of the set of feasible mergers. That is, the antitrust authority does as well as if it knew the set of feasible mergers and could implement whichever ones it wants. To this end, we introduce the following notion of ex-post optimality, defined relative to a particular realization of the set of feasible mergers.

Definition 2 *Let $\mathfrak{F}$ denote the set of feasible mergers (including their cost realizations). A set of approved mergers $\mathfrak{M}$ is* ***CS-maximizing for*** $\mathfrak{F}$ *if it maximizes consumer surplus given $\mathfrak{F}$. It is a* ***largest CS-maximizing set for*** $\mathfrak{F}$ *if it is not contained in any other set that is CS-maximizing for $\mathfrak{F}$.*

The properties of merger interactions we have identified imply that the largest CS-maximizing set is unique and increasing in the set of feasible mergers:

Lemma 4 *For each set of feasible mergers $\mathfrak{F}$ there is a unique largest CS-maximizing set $\mathfrak{M}^*(\mathfrak{F})$. Moreover, if $\mathfrak{F} \subset \mathfrak{F}'$ then $\mathfrak{M}^*(\mathfrak{F}) \subseteq \mathfrak{M}^*(\mathfrak{F}')$.*

Proof. Suppose that $\mathfrak{M}$ and $\mathfrak{M}'$ are both largest CS-maximizing sets for $\mathfrak{F}$. Then the sets $\mathfrak{M}$ and $\mathfrak{M}'$ must satisfy the hypothesis of Lemma 3. That lemma implies that there exists a sequencing of the mergers in $\mathfrak{M}\backslash(\mathfrak{M} \cap \mathfrak{M}')$ starting with the market structure in which all of the mergers in $\mathfrak{M}'$ have occurred that is CS-nondecreasing at each step. Hence, $(\mathfrak{M} \cup \mathfrak{M}')$ is CS-maximizing as well, contradicting the assumption that $\mathfrak{M}$ and $\mathfrak{M}'$ are largest CS-maximizing sets for $\mathfrak{F}$.

For the second claim, suppose $\mathfrak{M}^*(\mathfrak{F}) \nsubseteq \mathfrak{M}^*(\mathfrak{F}')$, so that $\mathfrak{M}^*(\mathfrak{F})\backslash(\mathfrak{M}^*(\mathfrak{F}) \cap \mathfrak{M}^*(\mathfrak{F}'))$ is nonempty. The sets $\mathfrak{M}^*(\mathfrak{F})$ and $\mathfrak{M}^*(\mathfrak{F}')$ satisfy the hypothesis of Lemma 3. Given that the mergers in $\mathfrak{M}^*(\mathfrak{F}')$ have taken place, there is therefore a sequencing of the mergers in $\mathfrak{M}^*(\mathfrak{F})\backslash(\mathfrak{M}^*(\mathfrak{F}) \cap \mathfrak{M}^*(\mathfrak{F}'))$ that is CS-nondecreasing at each step. Since all of those mergers are in $\mathfrak{F}'$, this would mean that $\mathfrak{M}^*(\mathfrak{F}')$ is not the largest CS-maximizing set for $\mathfrak{F}'$, a contradiction. ■

We next relate the outcome of a myopic CS-based approval policy to largest CS-maximizing sets:

Lemma 5 *Suppose the antitrust authority follows a myopic CS-based merger policy and $T = 1$. Then if $\widehat{\mathfrak{F}}$ is the set of proposed mergers, the set of approved mergers is $\mathfrak{M}^*(\widehat{\mathfrak{F}})$.*

Proof. Suppose that the set of proposed mergers is $\widehat{\mathfrak{F}}$. Let $\mathfrak{M} \subseteq \widehat{\mathfrak{F}}$ denote the set of mergers resulting from a sequence of merger approvals from $\widehat{\mathfrak{F}}$ that is (a) CS-nondecreasing at each stage and (b) that ends with all nonapproved mergers being CS-decreasing given the final market structure. Observe that, by Proposition 1, $\mathfrak{M}$ has the property that every merger $M \in \mathfrak{M}$ is CS-nondecreasing if all of the other mergers in $\mathfrak{M}$ (those in the set $\mathfrak{M}\backslash M$) have taken place. In addition, by definition, $\mathfrak{M}^*(\widehat{\mathfrak{F}})$ also has this property (it is a CS-maximizing set for $\widehat{\mathfrak{F}}$). Thus, if $\mathfrak{M} \nsubseteq \mathfrak{M}^*(\widehat{\mathfrak{F}})$, Lemma 3 implies that, given that the mergers in $\mathfrak{M}^*(\widehat{\mathfrak{F}})$ have occurred, there exists a sequencing of the mergers in $\mathfrak{M}\backslash(\mathfrak{M} \cap \mathfrak{M}^*(\widehat{\mathfrak{F}}))$ that is CS-nondecreasing at each step, contradicting $\mathfrak{M}^*(\widehat{\mathfrak{F}})$ being the largest

CS-maximizing set given $\widehat{\mathfrak{F}}$. Hence, $\mathfrak{M} \subseteq \mathfrak{M}^*(\widehat{\mathfrak{F}})$. If $\mathfrak{M} \subset \mathfrak{M}^*(\widehat{\mathfrak{F}})$, Lemma 3 implies that, given that the mergers in $\mathfrak{M}$ have occurred, there exists a sequencing of the mergers in $\mathfrak{M}^*(\widehat{\mathfrak{F}})\backslash(\mathfrak{M} \cap \mathfrak{M}^*(\widehat{\mathfrak{F}}))$ which is CS-nondecreasing at each step, contradicting $\mathfrak{M}$ satisfying property (b). Thus, $\mathfrak{M} = \mathfrak{M}^*(\widehat{\mathfrak{F}})$. Since the outcome of any myopic CS-based merger approval process satisfies properties (a) and (b), it must therefore equal $\mathfrak{M}^*(\widehat{\mathfrak{F}})$. ■

Lemma 5 implies that, when $T = 1$, if firms propose all feasible mergers then the outcome of any myopic CS-based merger policy will be CS-maximizing given the realized set of feasible mergers. The remaining issue is whether firms have an incentive to do so. The following proposition establishes that they do:

Proposition 3 *Suppose the antitrust authority follows a myopic CS-based merger policy and $T = 1$. It is then a weakly dominant strategy for firms with a feasible merger to propose it. Thus, when firms adopt this weakly dominant proposal strategy, the set of approved mergers is $\mathfrak{M}^*(\mathfrak{F})$, for every $\mathfrak{F}$.*

Proof. Given the discussion in the text, we need only show that regardless of the proposal strategies being followed by other firms, the firms in merger M_k maximize their expected payoff by proposing their merger given that the antitrust authority uses a myopic CS-maximizing policy. Let $\widehat{\mathfrak{F}}$ denote a realization of the set of proposed mergers if merger M_k is proposed (firms in other mergers may be using mixed strategies) and let $\widehat{\mathfrak{F}}_{-k} \equiv \widehat{\mathfrak{F}}\backslash M_k$ denote that realization without merger M_k included. By Lemma 4, $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k}) \subseteq \mathfrak{M}^*(\widehat{\mathfrak{F}})$. If $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k}) = \mathfrak{M}^*(\widehat{\mathfrak{F}})$, then the payoffs of firms in M_k are independent of whether they propose. Suppose instead that $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k}) \subset \mathfrak{M}^*(\widehat{\mathfrak{F}})$. By Lemma 3 there is a sequencing of the mergers in $\mathfrak{M}^*(\widehat{\mathfrak{F}})\backslash\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$ that is CS-nondecreasing at each step. Since any mergers in this set other than M_k must be CS-decreasing given that the mergers in $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$ have taken place [otherwise they would have been in $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$], the first merger in that sequence must be M_k. This implies, by the same logic as in Proposition 2(ii) [and using Proposition 1(i)] that the firms in M_k have greater profit when all of these mergers are approved than when they are not. Hence, it is more profitable in this case for these firms to propose their merger. ■

3.2 Dynamic Merger Review

We now extend the analysis to the dynamic setting, where $T > 1$. A realization of feasible mergers is now a sequence $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$ where $\mathfrak{F}_t \subseteq \mathfrak{F}_{t'}$ for $t' > t$. An outcome of the merger review process is a sequence of all (cumulatively) allowed mergers $\mathfrak{M} = (\mathfrak{M}_1, ..., \mathfrak{M}_T)$ where $\mathfrak{M}_t \subseteq \mathfrak{M}_{t'}$ for $t' > t$ and $\mathfrak{M}_t \subseteq \mathfrak{F}_t$ for all t. We begin with a result characterizing approval sequences that maximize discounted consumer surplus for a given realized feasible sequence:

Lemma 6 *Given a realization of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$, the approval sequence $\mathfrak{M} = (\mathfrak{M}^*(\mathfrak{F}_1), ..., \mathfrak{M}^*(\mathfrak{F}_T))$ maximizes discounted consumer surplus.*

Proof. Given the realized sequence of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$, consider the problem of maximizing discounted consumer surplus. If we ignore the constraint that the set of allowed mergers cannot shrink over time, we can choose the allowed set of mergers in any period independently from the mergers allowed in every other period. It is evident that in that case the approval sequence $\mathfrak{M} = (\mathfrak{M}^*(\mathfrak{F}_1), ..., \mathfrak{M}^*(\mathfrak{F}_T))$ is optimal since it maximizes consumer surplus in every period. However, since Lemma 4 implies that $\mathfrak{M}^*(\mathfrak{F}_1) \subseteq ... \subseteq \mathfrak{M}^*(\mathfrak{F}_T)$, the constraint is satisfied, so this is a solution to the constrained problem. ■

Lemma 6 shows that if all feasible mergers are proposed immediately and the antitrust authority approves the mergers in set $\mathfrak{M}^*(\mathfrak{F}_t) \backslash \mathfrak{M}^*(\mathfrak{F}_{t-1})$ in each period t, then the outcome will maximize discounted consumer surplus given the actual realization of feasible mergers, even though the antitrust authority has no knowledge in any period of future feasible mergers.

Finally, we will argue that proposing a feasible merger is a weakly dominant strategy of a sort. Specifically, we consider subgame perfect equilibria for the firms. We call a profile of strategies for the firms a *weakly dominant subgame perfect equilibrium* if in every period each player is playing a weakly dominant strategy in the one-period game induced by (weakly dominant subgame perfect) continuation play in future periods. The main result of this section is then:

Proposition 4 *Suppose the antitrust authority follows a myopic CS-based merger policy. Then all feasible mergers being proposed in each period after any history is a weakly dominant subgame perfect equilibrium. In this equilibrium, the outcome maximizes discounted consumer surplus given the actual realized sequence of feasible mergers* $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$.

Proof. We proceed by backward induction. Proposition 3 shows that the result is true in period T for any set of previously-approved mergers, $\mathfrak{M}_{T-1}$. Suppose it is true in every period $t > S$, and consider play in period S after the mergers in set $\mathfrak{M}_{S-1}$ have previously been approved. Let $\mathfrak{M}^*(\mathfrak{F}|\mathfrak{M})$ denote the largest CS-maximizing set given that the mergers in $\mathfrak{M}$ have previously been approved and the set of all feasible mergers (including those in $\mathfrak{M}$) is $\mathfrak{F}$. If all feasible mergers are proposed in period S, then the set of approved mergers in period S is $\mathfrak{M}^*(\mathfrak{F}_S|\mathfrak{M}_{S-1})$; if, instead, some are not proposed, then Lemma 4 implies that the set of approved mergers is a subset of $\mathfrak{M}^*(\mathfrak{F}_S|\mathfrak{M}_{S-1})$. Since, for all $t > S$ all feasible mergers are proposed, the set of approved mergers at the end of each period $t > S$ will be $\mathfrak{M}^*(\mathfrak{F}_t|\mathfrak{M}_{S-1})$ [in particular, the approvals in period S do not affect those in any later period since $\mathfrak{M}^*(\mathfrak{F}_S|\mathfrak{M}_{S-1}) \subseteq \mathfrak{M}^*(\mathfrak{F}_t|\mathfrak{M}_{S-1})$ for all $t > S$]. So the strategic considerations reduce to a single-period problem, and firms find proposing any feasible merger to be a weakly dominant strategy in period S for the same reasons as in Proposition 3 when $T = 1$. Applying induction establishes that it is a weakly dominant subgame perfect equilibrium for firms to propose all feasible mergers in every period after any history. Lemma 6 then implies that the outcome of this equilibrium maximizes discounted consumer surplus given the actual realized sequence of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$.
■

4 Robustness

4.1 Mergers in the Bertrand Model

So far, we have assumed that firms compete in quantities. In this subsection, we show that our main results continue to hold when firms compete in prices rather than quantities.

As before, there are n firms producing a homogeneous good at constant returns to scale. Firm i's marginal cost and price are denoted c_i and p_i. Market demand is given by $Q(p)$, where p is the lowest price offered by any firm. Demand is downward-sloping, $Q'(p) < 0$ for any $p > 0$ such that $Q(p) > 0$. Let $\iota(i|N) \in N$ denote the firm with the ith lowest marginal cost when the set of active firms is N, i.e., $c_{\iota(1|N)} \leq c_{\iota(2|N)} \leq ... \leq c_{\iota(n|N)}$. (If a subset of firms have the same marginal cost, then the firms in this subset are ordered arbitrarily.) Assuming $Q(c^{(1)}) > 0$, firm $\iota(i|N)$'s equilibrium price $p_{\iota(i|N)}$ is given by

$$p_{\iota(i|N)} = \begin{cases} c_i & \text{if } 2 \leq i \leq n, \\ \min\{p^M(c_{\iota(1|N)}), c_{\iota(2|N)}\} & \text{if } i = 1, \end{cases} \tag{6}$$

where $p^M(c) \equiv \arg\max_p [p - c]\, Q(p)$ is the (unconstrained) monopoly price of a firm with marginal cost c.[8] In equilibrium, all consumers purchase at price $p_{\iota(1|N)}$, and so consumer surplus is given by

$$CS(p_1, p_2, ..., p_n) = \int_{p_{\iota(1|N)}}^{\infty} Q(p)dp. \tag{7}$$

Note that $CS(p_1, p_2, ..., p_n)$ is independent of $p_{\iota(i|N)}$, $i > 1$, and decreasing in $p_{\iota(1|N)}$.

Consider a merger between a subset $M \subseteq N$ of firms. We will say that the merger is *cost-reducing* if the post-merger marginal cost of the merged entity, $\overline{c}_M$, is below the marginal cost of the most efficient merger partner, i.e., $\overline{c}_M < \min_{i \in M} c_i$. The merger is *cost-increasing* otherwise, i.e., if $\overline{c}_M \geq \min_{i \in M} c_i$. Let $\overline{p}_M$ denote the post-merger price of the merged firm. From (7), the merger is CS-increasing if and only if $\overline{p}_M < p_{\iota(1|N)}$.

The distinction between cost-reducing and cost-increasing mergers turns out to be useful since the marginal cost of a firm that does not sell anything may still affect consumer surplus, depending on the marginal costs of rival firms. As a result, and in contrast to the Cournot model, it is no longer true that every merger that is CS-nondecreasing in isolation remains CS-nondecreasing conditional on another CS-nondecresaing merger taking place. To see this, suppose merger M_1 involves the firm with the lowest marginal cost, $\iota(1|N)$, and that, prior to any merger, firm $\iota(1|N)$ charges a price equal to the marginal cost of the firm the second-lowest marginal cost, $p_{\iota(1|N)} = c_{\iota(2|N)}$. Suppose also that the merger is cost-reducing but CS-neutral in isolatoin, $c_{\iota(2|N)} \geq \overline{c}_{M_1} > c_{\iota(1|N)}$.

[8] While this is the most plausible equilibrium, there are typically other equilibria involving some firm(s) charging price(s) below marginal cost.

On the other hand, merger M_2 is cost-reducing and CS-increasing in isolation: the post-merger marginal cost satisfies $\overline{c}_{M_2} < c_{\iota(1|N)} < p^M(\overline{c}_{M_2})$. Hence, the merged entity M_2 will charge price $c_{\iota(1|N)}$ if M_1 does not take place but price $\min\{p^M(\overline{c}_{M2}), \overline{c}_{M_1}\} > c_{\iota(1|N)}$ if M_1 has taken place. That is, the cost-increasing M_1 is CS-neutral in isolation but becomes CS-decreasing conditional on the CS-increasing M_2 taking place. Note that this issue does not arise in the Cournot model where every cost-increasing merger is CS-decreasing.

The following lemma states some useful properties of mergers.

Lemma 7 *Consider a merger between a subset M of firms.*

1. *If the merger is cost-reducing, it is weakly profitable (in that it weakly increases the joint profit of the firms in M).*
2. *If the merger is CS-increasing, it is a cost-reducing merger (and, therefore, weakly profitable).*
3. *If the merger is cost-reducing and CS-decreasing, it must involve both firms $\iota(1|N)$ and $\iota(2|N)$.*

Proof. To see Property 1, note that, in equilibrium, only firm $\iota(1|N)$ makes any profit before the merger (and only if $c_{\iota(1|N)} < c_{\iota(2|N)}$). Hence, any cost-reducing merger that does not involve firm (1) cannot decrease the joint profit of the merging firms. Suppose now the (cost-reducing) merger involves firm $\iota(1|N)$. In this case, the merger will not affect the equilibrium prices of the firms not involved in the merger, and so the cost-reducing merger must be profitable.

To see Property 2, let $\overline{p}_M$ denote the post-merger price of the merged entity and assume the merger is CS-increasing, i.e., $\overline{p}_M < p_{\iota(1|N)}$. Suppose the merger involves firm $\iota(1|N)$ but not firm $\iota(2|N)$. Then, $\overline{p}_M = \min\{p^M(\overline{c}_M), c_{\iota(2|N)}\}$. Since $p^M(c)$ is strictly increasing in c, $\overline{p}_M < p_{\iota(1|N)} = \min\{p^M(c_{\iota(1|N)}), c_{\iota(1|N)}\}$ implies that the merger is cost-reducing, $\overline{c}_M < c_{\iota(1|N)}$. Suppose now the merger does not involve firm $\iota(1|N)$. Then, since $\overline{p}_M \geq \overline{c}_M$, $\overline{p}_M < p_{\iota(1|N)} = \min\{p^M(c_{\iota(1|N)}), c_{\iota(2|N)}\}$ implies that $\overline{c}_M < c_{\iota(2|N)}$, i.e., the merger is cost-reducing. Suppose finally the merger involves both firms $\iota(1|N)$ and $\iota(2|N)$. Then, $\overline{p}_M = \min\{p^M(\overline{c}_M), c_{\iota(1|N\setminus M)}\}$, where $c_{\iota(1|N\setminus M)}$ is the marginal cost of the most efficient firm not involved in the merger. Since $c_{\iota(1|N)\setminus M} \geq c_{\iota(2|N)}$, $\overline{p}_M = \min\{p^M(\overline{c}_M), c_{\iota(1|N\setminus M)}\} < p_{\iota(1|N)} = \min\{p^M(c_{\iota(1|N)}), c_{\iota(2|N)}\}$ implies that $\overline{c}_M < c_{\iota(1|N)}$, i.e., the merger is cost-reducing.

To see Property 3, assume the merger is CS-decreasing, i.e., $\overline{p}_M > p_{\iota(1|N)}$. Since $p_{\iota(1|N)} = \min\{p^M(c_{\iota(1|N)}), c_{\iota(1|N)}\}$, it follows that the merger has to involve either firm $\iota(1|N)$ or firm $\iota(2|N)$, or both. Suppose the merger involves firm $\iota(1|N)$ but not firm $\iota(2|N)$. If $\overline{c}_M \leq c_{\iota(1|N)}$, then $\overline{p}_M = \min\{p^M(\overline{c}_M), c_{\iota(2|N)}\} \leq p_{\iota(1|N)}$; a contradiction. Hence, $\overline{c}_M > c_{\iota(1|N)}$, i.e., the merger is cost-increasing. Suppose now the merger involves firm $\iota(2|N)$ but not firm $\iota(1|N)$. If $\overline{c}_M \leq c_{\iota(2|N)}$, then $\overline{p}_M = \min\{p^M(\overline{c}_M), p^M(c_{\iota(1|N)}), \overline{c}_M\} \leq p_{\iota(1|N)}$; a contradiction. Hence, $\overline{c}_M > c_{\iota(2|N)}$, i.e., the merger is cost-increasing. It follows that if the merger is cost-reducing and CS-decreasing, it must involve both firms $\iota(1|N)$ and $\iota(2|N)$. ■

We now turn to the interaction between two disjoint mergers, M_1 and M_2, where $M_1 \cap M_2 = \emptyset$. Let N denote the set of firms if neither merger takes place, N_i the set of firms after merger M_i (but not M_j, $j \neq i$) has taken place, and N_{12} the set of firms after both mergers have taken place. Since every CS-increasing merger is cost-reducing (Lemma 7), we will focus on cost-reducing mergers. As in the case of the Cournot model, there is a a certain complementarity between mergers that change consumer surplus in the same direction.

Proposition 5 *The CS-effect of mergers is complementary:*

(i) *If a (cost-reducing) merger is CS-nondecreasing in isolation, it remains CS-nondecreasing if another (cost-reducing) merger that is CS-nondecreasing in isolation takes place.*

(ii) *If a merger is CS-decreasing in isolation, it remains CS-nonincreasing if another merger that is CS-decreasing in isolation takes place. Further, if a merger is cost-reducing and CS-decreasing in isolation, there cannot be another (disjoint) merger that is CS-decreasing in isolation.*

Proof. To see part (i), suppose to the contrary that the merger, say M_2, becomes CS-decreasing if the other merger, say M_1, takes place. Since M_2 is cost-reducing, the merger must involve (after M_1 has taken place) firms $\iota(1|N_1)$ and $\iota(2|N_1)$ (see Lemma 7), and $\min\{p^M(\overline{c}_{M_2}), c_{\iota(1|N_1\setminus M_2)}\} > c_{\iota(2|N_1)}$, where $c_{\iota(1|N_1\setminus M_2)}$ is the marginal cost of the most efficient firm not involved in merger M_2 (after merger M_1 has taken place). But since M_1 is cost-reducing, $\iota(1|N_1) = \iota(1|N)$ and $\iota(2|N_1) = \iota(2|N)$, i.e., M_2 must involve firms $\iota(1|N)$ and $\iota(1|N)$. Further, since M_2 is CS-nondecreasing in isolation, $\min\{p^M(\overline{c}_{M_2}), c_{\iota(1|N\setminus M_2)}\} \leq c_{\iota(2|N)} = c_{\iota(2|N_1)}$, where $c_{\iota(1|N\setminus M_2)}$ is the marginal cost of the most efficient firm not involved in merger M_2 before merger M_1 has taken place. Hence, $c_{\iota(1|N_1\setminus M_2)} > c_{\iota(1|N\setminus M_2)}$. But M_1 is cost-reducing, and so $c_{\iota(1|N_1\setminus M_2)} \leq c_{\iota(1|N\setminus M_2)}$, a contradiction.

To see part (ii), note that since M_i is CS-decreasing in isolation, M_i must involve either firm $\iota(1|N)$ or firm $\iota(2|N)$, or both. This holds for each of the two mergers, and so one of the two mergers, say M_1, must involve firm $\iota(1|N)$, while M_2 involves firm $\iota(2|N)$. Next, note that $p_{\iota(1|N)} = c_{\iota(2|N)}$ and $\overline{c}_{M_2} > c_{\iota(2|N)}$ since, otherwise, if $p_{\iota(1|N)} = p^M(c_{\iota(1|N)})$ or $\overline{c}_{M_2} \leq c_{\iota(2|N)}$, M_2 could not be CS-decreasing. Since M_1 is CS-decreasing in isolation, $\overline{c}_{M_1} > c_{\iota(2|N)} \geq c_{\iota(1|N)}$. Hence, each of the two mergers is cost-increasing. From Lemma 7, it follows that each merger can never become CS-increasing. Finally, note that if merger M_i is cost-reducing and CS-increasing, it must, from Lemma 7, involve firms $\iota(1|N)$ and $\iota(2|N)$. But then the other merger M_j cannot be CS-decreasing. ■

We now turn to the interaction between mergers that, in isolation, affect consumer surplus in opposite directions.

Proposition 6 *Suppose that merger M_1 is CS-increasing (and therefore cost-reducing) in isolation, while merger M_2 is CS-decreasing in isolation but CS-increasing once merger M_1 has taken place. Then:*

(i) *Merger M_2 is a cost-reducing merger.*

(ii) *Merger M_1 is CS-increasing conditional on merger M_2 taking place.*

(iii) *The joint profits of the firms involved in merger M_1 are weakly higher if both mergers take place than if neither merger takes place.*

Proof. To see part (i), suppose otherwise that M_2 is cost-increasing. But then, from Lemma 7, M_2 can never be CS-increasing; a contradiction.

The proof of part (ii) is identical to that of part (i) of Proposition 2.

To see part (iii), note that since M_2 is a cost-reducing merger that is CS-decreasing in isolation, it must involve firms $\iota(1|N)$ and $\iota(2|N)$. Hence, if no merger takes place, the joint profit of the firms in M_1 is zero. ■

We now turn to the optimal merger approval policy for an antitrust authority concerned with maximizing (discounted) consumer surplus when multiple disjoint mergers may be proposed over time, as described in Section 3. Since only the most efficient firm makes any sales in the Bertrand model, a myopic CS-based merger policy needs to distinguish between cost-reducing and cost-increasing mergers. We therefore modify our definition of a myopic CS-based merger policy as follows:

Definition 3 *A* ***myopic CS-based merger policy*** *approves at each stage a proposed merger that is cost-reducing and CS-nondecreasing given the market structure at the start of that stage, if any such mergers exist.*

It can readily be verified that, using this modified definition, Propositions 3 and 4 carry over to the Bertrand model. In particular, in the *weakly dominant subgame perfect equilibrium* firms propose every feasible merger, and so the set of approved mergers in every period t is $\mathfrak{M}^*(\mathfrak{F}_t)$. The outcome maximizes discounted consumer surplus, given the actual realized sequence of feasible mergers.

4.2 Mergers with Differentiated Goods

4.3 Non-Constant Returns to Scale

4.4 Overlapping Mergers

4.5 Aggregate Surplus Standard

While the legal standard of merger policy in the U.S. and the EU is close to being a consumer surplus standard, economists typically focus on aggregate (or total) surplus as the welfare criterion. In this subsection, we investigate whether our main results continue to hold when the antitrust authority adopts an aggregate surplus standard. In doing so, we consider the homogeneous-goods Bertrand model of Section 4.1.

Adopting the notation (and assumptions) of Section 4.1, aggregate (or total) surplus is given by

$$AS(p_1, p_2, ..., p_n; c_1, c_2, ..., c_n) = \int_{p_{\iota(1|N)}}^{\infty} Q(p)dp + [p_{\iota(1|N)} - c_{\iota(1|N)}]Q(p_{\iota(1|N)}), \tag{8}$$

where $p_{\iota(1|N)} = \min\{p^M(c_{\iota(1|N)}), c_{\iota(2|N)}\}$ is the equilibrium price of the most efficient firm. Note that $\partial AS(\cdot)/\partial p_{\iota(1|N)} \leq 0$, $\partial AS(\cdot)/\partial c_{\iota(1|N)} \leq 0$, $dAS(\cdot)/dc_{\iota(1|N)} \leq 0$, $dAS(\cdot)/dc_{\iota(2|N)} \leq 0$, and $dAS(\cdot)/dc_{\iota(k|N)} = 0$ for $k > 2$.

Consider a merger between a subset $M \subseteq N$ of firms. Following our previous notation, we will say that the merger is *AS-neutral* if the merger does not affect aggregate surplus. Similarly, we will say that a merger is *AS-increasing* [*AS-decreasing*], if aggregate surplus following the merger is higher [lower] than before. Finally, a merger is *AS-nondecreasing* [*AS-nonincreasing*] if it is not AS-decreasing [AS-increasing].

The following lemma shows that the set of cost-reducing and AS-nondecreasing mergers is a superset of the set of cost-reducing and CS-nondecreasing mergers, and that every cost-reducing merger that is AS-nondecreasing but CS-decreasing must involve firms $\iota(1|N)$ and $\iota(2|N)$.

Lemma 8 *Consider a merger between a subset M of firms.*

1. *If the merger is AS-increasing, it is cost-reducing.*

2. *If the merger is cost-reducing and CS-nondecreasing, it is AS-nondecreasing.*

3. *If a cost-reducing and AS-nondecreasing merger is CS-decreasing, it involves firms $\iota(1|N)$ and $\iota(2|N)$.*

4. *If a cost-reducing merger is AS-decreasing, it involves firms $\iota(1|N)$ and $\iota(2|N)$.*

5. *If the merger is AS-decreasing, it is CS-nonincreasing. If it is AS-decreasing and CS-neutral, it involves firm $\iota(1|N)$.*

Proof. Property 1 follows immediately from the obervation that $dAS(\cdot)/dc_{\iota(1|N)} \leq 0$, $dAS(\cdot)/dc_{\iota(2|N)} \leq 0$, and $dAS(\cdot)/dc_{\iota(k|N)} = 0$ for $k > 2$.

Property 2 follows from $\partial AS(\cdot)/\partial p_{\iota(1|N)} \leq 0$, $\partial AS(\cdot)/\partial c_{\iota(1|N)} \leq 0$, and $\partial AS(\cdot)/\partial c_{\iota(k|N)} = 0$ for all $k > 1$.

To see Property 3, note that since $p_{\iota(1|N)} = \min\{p^M(c_{\iota(1|N)}), c_{\iota(2|N)}\}$, any cost-reducing merger that does not involve firms $\iota(1|N)$ and $\iota(2|N)$ is CS-nondecreasing (see also Lemma 7).

To see Property 4, suppose otherwise that the merger does not involve firms $\iota(1|N)$ and $\iota(2|N)$. If the marginal cost of the merged entity satisfies $\overline{c}_M \geq c_{\iota(2|N)}$, the merger is AS-neutral; a contradiction. If, on the other hand, $\overline{c}_M < c_{\iota(2|N)}$, the (cost-reducing) merger is CS-nondecreasing. From Property 2, it follows that the merger is AS-nondecreasing; a contradiction.

To see Property 5, note that, from Property 2, a merger that is cost-reducing and AS-decreasing merger must be CS-decreasing. Suppose now the merger is cost-increasing, i.e., $\overline{c}_M \geq c_{\iota(1|M)}$. >From Lemma 7, it must therefore be CS-nonincreasing. Since $dAS(\cdot)/dc_{\iota(k|N)} = 0$ for $k > 2$, the merger must involve either firm $\iota(1|N)$, firm $\iota(2|N)$, or both for it to be AS-decreasing. Suppose the merger involves firm $\iota(2|N)$ but not firm $\iota(1|N)$. Then, since $\partial AS(\cdot)/\partial c_{\iota(k|N)} = 0$ for $k > 1$ and $\partial AS(\cdot)/\partial p_{\iota(1|N)} \leq 0$, the post-merger price of firm $\iota(1|N)$, $\overline{p}_{\iota(1|N)}$, must be higher than its pre-merger price, $\overline{p}_{\iota(1|N)} > p_{\iota(1|N)}$ for the merger to be AS-decreasing. But this means that the merger is CS-decreasing. Hence, if the merger is AS-decreasing and CS-neutral, it must involve firm $\iota(1|N)$. ■

We now turn to the interaction between two disjoint mergers, M_1 and M_2, where $M_1 \cap M_2 = \emptyset$. The following proposition shows that the there is a certain complementarity between mergers that change aggregate surplus in the same direction.

Proposition 7 *The AS-effect of mergers is complementary:*

(i) *If a (cost-reducing) merger is AS-nondecreasing in isolation, it remains AS-nondecreasing if another (cost-reducing) merger that is AS-nondecreasing in isolation takes place.*

(ii) *If a merger is AS-decreasing in isolation, it remains AS-nonincreasing if another merger that is AS-decreasing in isolation takes place. Further, if a merger is cost-reducing and AS-decreasing in isolation, there cannot be another (disjoint) merger that is AS-decreasing in isolation.*

Proof. Consider part (i). If both mergers are CS-nondecreasing in isolation, then, from Proposition 5, each merger remains CS-nondecreasing conditional on the other merger taking place. Lemma 8, Property 2, then implies that each merger remains AS-nondecreasing. Indeed, if neither merger involves both firms $\iota(1|N)$ and $\iota(2|N)$, then, from Lemma 8, Property 3, then both mergers are CS-nondecreasing in isolation. Suppose now instead that one merger, say M_1, involves both firms $\iota(1|N)$ and $\iota(2|N)$ and is CS-decreasing in isolation. This implies that the other merger, M_2, must be CS-nondecreasing in isolation as otherwise it would not be AS-nondecreasing in isolation. Note that, prior to merging, the firms in M_2 do not make any sales, independently of whether or not the merger M_1 takes place. Further, M_2 is cost-reducing, and $dAS(\cdot)/dc_{\iota(k|N)} \leq 0$ for any k. This implies that M_1 must remain AS-nondecreasing conditional on M_2 taking place. Consider now M_2, conditional on M_1 taking place. Since M_2 does not involve both $\iota(1|N_1)$ and $\iota(2|N_1)$ and since it is cost-reducing, Lemma 7 implies that M_2 is CS-nondecreasing conditional on M_1 taking place. From Lemma 8, it follows that M_2 is AS-nondecreasing conditional on M_1 taking place.

Consider now part (ii). Suppose one of the mergers, say M_1, involves both firms $\iota(1|N)$ and $\iota(2|N)$. But then merger M_2 cannot be AS-decreasing. To see this, note that if $\overline{c}_{M_2} \geq c_{\iota(2|N)}$, the merger is AS-neutral. If, on the other hand, $\overline{c}_{M_2} < c_{\iota(2|N)}$, merger M_2 is cost-reducing and CS-nondecreasing. From Lemma

8, Property 2, the merger is thus AS-nondecreasing. Hence, neither merger can involve both firms $\iota(1|N)$ and $\iota(2|N)$. On the other hand, if a merger does not involve either firm $\iota(1|N)$ or firm $\iota(2|N)$, or both, it cannot be AS-decreasing since $dAS(\cdot)/dc_{\iota(k|N)} = 0$ for $k > 2$. Hence, one of the two mergers, say M_1, must involve firm $\iota(1|N)$, while merger M_2 involves firm $\iota(2|N)$. Lemma 8, Property 5, then implies that M_2 is CS-decreasing in isolation. Hence, prior to both mergers, $p_{\iota(1|N)} = c_{\iota(2|N)}$ (since, otherwise, if $p_{\iota(1|N)} < c_{\iota(2|N)}$, M_2 could not be CS-decreasing) and $\overline{c}_{M_2} > c_{\iota(2|N)}$, i.e., M_2 is cost-increasing. Since M_1 is AS-decreasing in isolation and does not involve firm $\iota(2|N)$, it must be cost-increasing (Lemma 8,Property 4). But Lemma 8, Property 1, implies that a cost-increasing merger must remain AS-nonincreasing.

Finally, note that if merger M_1 is cost-reducing and AS-decreasing in isolation, it must involve firms $\iota(1|N)$ and $\iota(2|N)$ (Lemma 8, Property 4). But, as shown above, this implies that merger M_2, in isolation, is AS-nondecreasing. ■

We now turn to the interaction between mergers that, in isolation, affect consumer surplus in opposite directions.

Proposition 8 *Suppose that merger M_1 is AS-increasing (and therefore cost-reducing) in isolation, while merger M_2 is AS-decreasing in isolation but AS-increasing once merger M_1 has taken place. Then:*

(i) *Merger M_2 is a cost-reducing merger.*

(ii) *Merger M_1 is AS-increasing conditional on merger M_2 taking place.*

(iii) *The joint profits of the firms involved in merger M_1 are weakly higher if both mergers take place than if neither merger takes place.*

Proof. Part (i) follows immediately from Lemma 8, Property 1.

The proof of part (ii) is along the same lines as that of part (i) of Proposition 2.

To see part (iii), note that since M_2 is a cost-reducing merger that is AS-decreasing in isolation, it must involve firms $\iota(1|N)$ and $\iota(2|N)$ (see Lemma 8, Property 4). Hence, if no merger takes place, the joint profit of the firms in M_1 is zero. ■

We now turn to the optimal merger approval policy for an antitrust authority concerned with maximizing (discounted) aggregate surplus when multiple disjoint mergers may be proposed over time. The extensive form of the game is described in Section 3, but we replace the myopic CS-based merger policy considered there by the following myopic AS-based merger policy:

Definition 4 *A **myopic AS-based merger policy** approves at each stage a proposed merger that is cost-reducing and AS-nondecreasing given the market structure at the start of that stage, if any such mergers exist.*

It can readily be verified that, under this myopic AS-based merger policy, all feasible mergers being proposed in each period after any history is a weakly dominant subgame perfect equilibrium. In this equilibrium, the set of approved mergers in every period t is the AS-maximizing set given the set of feasible mergers $\mathfrak{F}_t$. Further, the outcome maximizes discounted aggregate surplus, given the actual realized sequence of feasible mergers.

5 Conclusion

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