Private Investment of a Public Project? A Multi-Period Analysis

Engelbert J. Dockner*)
Department of Finance
University of Vienna
Brünner Straße 72
1210 Vienna, Austria

Kazuo Nishimura
Institute of Economic Research
Kyoto University
Yoshidahonmachi Sakyoku
Kyoto 606, Japan

Abstract

We formulate a dynamic model of voluntary investment in a public project. We distinguish two alternative scenarios. One in which private agents have full knowledge about the accumulated aggregated contributions and one in which each agent only has knowledge about his personal contributions. For both scenarios we investigate whether or not the public project will be efficiently funded by the agents. It turns out that in the case where agents do not know the aggregated cumulative contributions, there are no private contributions and the public good will not be provided. In the case if agents know the cumulated aggregated contributions there exists an equilibrium in which agents will efficiently contribute to the public project.

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*) Corresponding author.

1 Introduction

The private provision of public goods is a fact of market economies. Consumers voluntarily contribute to public radio stations, public parks, etc. These empirical observation is in sharp contrast to many theoretical models that find that private investment in public goods generally results in under-funding. The reason behind this result is a simple free rider problem. If agents have to contribute to something that is not privately consumed each one free rides on the contributions of the others and hence the product is either not sufficiently produced or not produced at all (see Schelling (1960) or Cornes and Sandler (1983), (1996)).

While many theoretical models that deal with private provisions of public goods are essentially static, time is an essential ingredient of many voluntary contribution schemes. This has led economists to look into the problem of dynamic voluntary provisions of public goods. Fershtman and Nitzan (1991) and Admati and Perry (1991) were among the first to discuss the free rider problem in a dynamic game framework. Fershtman and Nitzan (1991) use a linear quadratic differential game in which agents invest in a public stock of capital. Utility of every agent depends on the current level of the stock of capital and investment results in quadratic investment costs. In this framework Fershtman and Nitzan derive an open-loop and a linear Markov perfect equilibrium of the infinite horizon game. They demonstrate that dynamic voluntary contributions to a public project worsens the free rider problem (see Dockner et al. (2000) for a discussion of this result).

Admati and Perry (1991) use an alternate move game to study dynamics and efficiency of voluntary contributions to a public project. They show that if players are impatient and their disutility from the contributions is a convex function then the dynamic contribution game has a unique subgame perfect equilibrium. With costs not to high the project is completed in equilibrium, otherwise the agents will not contribute.

Dockner et al. (1996) study a dynamic public bads model. In this model two countries emit pollutants that accumulate a stock of pollution. Assuming convex costs of pollution and linear utility derived from the production of goods, they are able to characterize Markov perfect equilibria that exhibit a dynamic version of the free rider problem.

To some up, many dynamic models of voluntary contributions of a public good reported here result in a free rider problem. The incentive to free ride depends, however, on the ability of agents to condition their current contribution on the aggregated accumulated amount already spent for the public project.

In a recent paper, Marx and Matthews (2000) investigate dynamic voluntary contributions to a public project by means of a simple capital accumulation game where agents have perfect information about the economy, but imperfect information about the individual contributions. They are able to characterize Nash equilibria for this game and show that contrary to Fershtman and Nitzan (1991) and Admati and Perry (1991) contributions over time enhance efficiency. In particular if players have identical preferences, the number of time periods is large and there is low discounting the public good is provided efficiently. Efficiency enhancements in case of dynamic contributions have also been derived in the paper by Wirl (1996). He uses the linear quadratic differential game introduced by Fershtman and Nitzan and derives nonlinear Markov strategies which Pareto dominate the linear one.

In a game of joint exploitation of a renewable resource Dutta and Sundaram (1993) analyze the tragedy of the commons, i.e. overexploitation of the resource relative to the first best solution when two or more players have access to the same pool of fish. Allowing players to choose Markovian strategies they show that strategic interactions in fact can lead to under-exploitation. If we relate this result to the case of the provision of public goods, private (strategic) interactions of agents could result in an "overproduction". The intuition that Dutta and Sundaram provide for this result is the following. Their model is one with structural dynamics, i.e. there is a state variable, the asset stock, that changes over time with the actions taken by the players. Obviously, this state variable is a sufficiently accurate statistic for the history of the game. If players use these state dependent (Markovian) strategies they act cooperatively even without direct conditioning on the history of actions.

A different version of the free rider problem is also studied by Ihori and Itaya (2001). They introduce a differential game in which different interest groups compete for the restructuring of tax burdens. It is shown that the strategic interactions of interest groups in the process of fiscal reconstruction result in a severe free rider problem.

In this paper we take up the issue of dynamic investment in public projects and introduce a nonlinear version of the model by Marx and Matthews (2000). Contrary to Marx and Matthews, we assume that each agent has a continuous nonlinear utility function, where utility is derived from the stock of the public good accumulated through the contributions of the individuals. Moreover, we assume that the public stock of capital depreciates at a constant rate. In this setting we first characterize the efficient dynamic provision of the public good, i.e. we derive the optimal solution for the case that the public good is produced collectively by all agents. Next, we derive Nash equilibria for the case of multi-period private investments. In this case we distinguish two alternative scenarios. One in which each agent only knows his contributions and does not know the accumulated aggregated amount, and the other one in which agents can condition their current actions on the aggregated accumulated volume. We are able to characterize Nash equilibria for both scenarios and get the following results. In case each agent only knows his contributions there exists a unique Nash equilibrium with no contributions by private agents. In the case agents can condition their actions on the current level of the public stock of capital, there exist many Nash equilibria one of which has the property that there will be an efficient private provision of the public project. Hence, members of society do not need a social planner for the project to be completed but need only care for their private investment strategies. This result is derived without any use of trigger strategies. Each agent only uses a nonlinear Markov strategy which generates the result.

Our paper is organized as follows. In the next section we present the model. In Section 3 we discuss results for the cases when the good is provided collectively and privately. We also contrast the outcome of the dynamic game with that of the corresponding static game. Finally, Section 4 summarizes the results and concludes the paper.

2 A dynamic model of private investment of a public project

Consider a society that consists of N members which we refer to as players or agents. Each agent voluntarily contributes over time to a public project. The investment of agent i at time t in the public project is given by z_t^i. Aggregated investment in period t is denoted by $Z_t = \sum_{i=1}^N z_t^i$. Aggregated investment builds up a public stock of capital which is denoted by X_t at time t. We assume that the public capital stock is used by private agents even during the time it is build up. To capture the use of the public capital stock and its deterioration we assume a constant rate of depreciation equal to $\beta < 1$. Under these assumptions the accumulation equation for public capital is given by

$$X_{t+1} = \sum_{i=1}^{N} z_t^i + (1-\beta)X_t = Z_t + (1-\beta)X_t, \tag{1}$$

where X_0 is the given initial condition. We might assume that when the fund drive starts there is already some strictly positive initial capital stock, i.e. $X_0 > 0$.

Each agent derives instantaneous utility from the public stock of capital and agrees that $\bar{X}$ is the size of the capital stock for which saturation occurs and marginal utility is driven down to zero. Hence, instantaneous utility of agent i is given by

$$u^i(X_t) = \begin{cases} v^i(X_t) & \text{for } X_t \leq \bar{X}, \\ \bar{v}^i & \text{for } X_t \geq \bar{X} \end{cases} \tag{2}$$

where $v^i(\bar{X}) = \bar{v}^i$ holds. From the specification (2) it is clear that we are dealing with nonlinear and continuous utility. The paper by Marx and Matthews (2000) uses linear utility and a discontinuous jump at $\bar{X}$. Based on this single period utility function each agent chooses a sequence of private investment so as to maximize the discounted sum of future net utility, i.e.

$$\max \sum_{t=0}^{\infty} \rho^t \left[u^i(X_t) - z_t^i \right] \tag{3}$$

subject to the constraint

$$X_{t+1} = \sum_{i=1}^{N} z_t^i + (1-\beta)X_t,$$

and the initial condition X_0. $0 < \rho < 1$ is the constant discount rate.

Before we continue to look at the efficient provision of the public good let us introduce the following assumption. We specify the state space as being $S \equiv [0, \infty)$.

Assumption 1 *$u^i(X) : S \to [0, \infty)$ is C^2, increasing and strictly concave for $X \geq 0$ with $u^i(0) = 0$, and*

$$0 \leq u^i(X) \leq (r + \beta)X \ \text{ for all } X \geq 0 \ .$$

Moreover,

$$u^i_X(0) > r + (\beta/N),$$

where r is the constant rate of interest, i.e. $r \equiv 1/\rho - 1 \geq 0$.

Assumption 1 guarantees that the problem covers the case where free riding is an issue. Hence, it corresponds to the assumptions made by Marx and Matthews (2000).

To derive the efficient provision of the public good we define the sum of individual utilities,

$$U(X_t) \equiv \sum_{i=1}^{N} u^i(X_t).$$

For the sum of individual utility functions the following properties hold.

Lemma 1 *$U(X_t)$ is C^2 and strictly concave and there exists a unique level of the public stock of capital X^*, $0 < X^* < \bar{X}$, such that $U_X(X^*) = r + \beta$.*

Proof: From Assumption 1 it follows that $U(X)$ is strictly concave. Moreover,

$$U_X(0) > Nr + \beta$$

and since $U_X(X_t) = 0$ for $X_t \geq \bar{X}$ there exists a unique X^* such that $U_X(X^*) = r + \beta$ holds. □

The result of Lemma 1 establishes the existence of a unique golden rule capital stock for the public good. What the role of this golden rule capital stock is, will be explored in the next Section.

3 Social and private provision of the public good

In this section we analyze our dynamic investment problem. Altogether we are dealing with three different scenarios. First, as a point of reference, we derive the optimal solution for the collusive problem. This problem corresponds to the case where the public good is provided collectively. Next, we assume agents act independently and choose their investment strategies on the basis of their individual utility functions. We distinguish two alternative scenarios depending on the strategy and hence on the amount of information available to the agents when they choose their investments. Fist, we assume that agents use open-loop strategies. This scenario covers the case where agents only know their personal contributions and ignore any knowledge about the size of the capital stock. Second, we look at the case where agents use Markov strategies. In this case agents are capable of conditioning their current investments on the level of the aggregated accumulated contributions. We will compare the equilibria for both scenarios and relate them to the outcome when the public good is provided efficiently. Finally, we derive the equilibrium for the corresponding static game which we will also use to compare static with dynamic voluntary contributions.

3.1 Social provision of the public good

As a starting point we investigate whether or not it is efficient to provide the public good socially. For that reason we derive the efficient allocation as the outcome of the collusive problem:

$$\max_{\{Z_t\}} \sum_{t=0}^{\infty} \rho^t \left[U(X_t) - Z_t \right] \tag{4}$$

subject to the state equation (1) and the initial condition X_0.

Before we characterize the first best provision of the public good we introduce the following notation. We define

$$W(X) \equiv U(X) - (r+\beta)X,$$

and notice that since $W(X)$ is strictly concave and because Lemma 1 holds, there exists a unique maximum of $W(X)$ given by X^*. With the definition of $W(X)$ and Assumption 1 we get

$$W(X) \leq (N-1)(r+\beta)X \quad \text{for all } X \geq 0. \tag{5}$$

Using $W(X)$ and substituting state equation (1) into the objective function (4) the collusive problem can be rewritten as

$$\begin{aligned} \max_{\{Z_t\}} \sum_{t=0}^{\infty} \rho^t \left[U(X_t) - Z_t \right] &= \max_{\{X_{t+1}\}} \sum_{t=0}^{\infty} \rho^t \left[U(X_t) - X_{t+1} + (1+\beta)X_t \right] \\ &= U(X_0) + (1-\beta)X_0 + \max_{\{X_{t+1}\}} \left\{ \sum_{t=0}^{\infty} \rho^{t+1} W(X_{t+1}) \right\}. \end{aligned} \tag{6}$$

Writing the collusive problem as in (6) allows us to derive its unique optimal solution.

Theorem 1 *Under Assumption 1 and* $\lim_{X\to\infty} U_X(X) < r+\beta$ *there exists a unique optimum for the problem of socially providing the public good. Optimal dynamic investment levels are give by*

$$Z_t = \max\left\{0, X^* - (1-\beta)X_t\right\} \tag{7}$$

which result in an evolution of the public stock of capital given by

$$X_{t+1} = \max\left\{(1-\beta)X_t, X^*\right\}. \tag{8}$$

Proof: In order to maximize

$$U(X_0) + (1-\beta)X_0 + \sum_{t=0}^{\infty} \rho^{t+1} W(X_{t+1})$$

subject to the state constraint (1) it suffices to maximize $W(X_{t+1})$ subject to $X_{t+1} \in [(1-\beta)X_t, \infty)$. This maximization problem, however, has the solution given by (8) and (7). □

The optimal solution to the collusive problem is characterized by a most rapid approach path, i.e. to choose the investment levels in such a way so as to reach the steady state as quickly as possible. In case the initial capital stock, X_0, is below the steady state level, X^*, it is optimal to make an impulse investment equal to $X^* - (1-\beta)X_0$ and reach the steady state in the next period. In case the initial state is above the steady state level adjusted for depreciation, there is over-funding and it is optimal to set contributions to zero and let the stock depreciate until the steady state level is reached. This observation provides us with the next result.

Theorem 2 *Let $T \geq 1$ be the point in time at which for the efficient (optimal collusive) solution the steady state, X^*, is reached. The maximal value of the discounted social welfare function is given by*

$$U_{\max} = \sum_{t=0}^{T-1} \rho^t U\left((1-\beta)^t X_0\right) - \rho^{T-1}\left[X^* - (1-\beta)^T X_0\right] + \frac{\rho^T}{1-\rho}\left[U(X^*) - \beta X^*\right]. \quad (9)$$

Proof: This follows from the structure of the optimal solution (7) and (8). Substituting this optimal solution into the objective function (4) results in (9). □

If it is the case that $X_0 < \frac{X^*}{1-\beta}$ the steady state is reached in the first period, i.e. $T = 1$. Hence, the maximum value of the discounted social welfare function is equal to

$$U_{\max} = U(X_0) - [X^* - (1-\beta)X_0] + \frac{\rho}{1-\rho}\left[U(X^*) - \beta X^*\right],$$

or after some manipulations

$$U_{\max} = W(X_0) + (1+r)X_0 + \frac{1}{r} W(X^*). \quad (10)$$

This level of maximum social utility is the reference level that needs to be matched in case of private provision of the public good.

3.2 Private provision of the public good when agents only observe their individual investments

After we derived our benchmark we will next analyze if private investment will provide the public good in the same amount as socially desirable. In this subsection we make, however, the assumption that each agent only knows his contributions. Neither the contributions of the others nor the accumulated aggregated amount is known. This means that given an expectation of others contributions each agent chooses a sequence of investments for the public project that depends only on calendar time, i.e. the strategy spaces of the agents are the set of investment sequences $\{z_t^i\}_{t=0}^{\infty}$. Under this assumptions the problem of agent i becomes

$$\max_{\{z_t^i\}} \sum_{t=0}^{\infty} \rho^t \left[u^i(X_t) - z_t^i\right], \tag{11}$$

subject to the constraint

$$X_{t+1} = z_t^i + Z_t^{-i} + (1-\beta)X_t \tag{12}$$

where $Z_t^{-i} = \sum_{j=1,j\neq i}^{N} z_t^j$ is an expectation by agent i on others' contributions in period t. In this optimization problem each agent takes Z_t^{-i} as given and solves for his individually rational investment level.

Theorem 3 *Under Assumption 1 the unique Nash equilibrium in case agents have only information about their own contributions is not to contribute to the public project.*

Proof: If we substitute (12) into the objective function of agent i, (11), the optimization problem of each individual can be rewritten as

$$\max_{\{z_t^i\}} \left\{ u^i(X_0) + (1-\beta)X_0 + \sum_{t=0}^{\infty} \rho^{t+1} \left[u^i(X_{t+1}) - (r+\beta)X_{t+1}\right] + \sum_{t=0}^{\infty} \rho^t Z_t^{-i} \right\} \tag{13}$$

and after substitution of (12) into this last equation this equals

$$\begin{aligned} \max_{\{z_t^i\}} \Bigg\{ & u^i(X_0) + (1-\beta)X_0 + \sum_{t=0}^{\infty} \rho^t Z_t^{-i} \\ & + \sum_{t=0}^{\infty} \rho^{t+1} \left[u^i\left(z_t^i + Z_t^{-i} + (1-\beta)X_t\right) - (r+\beta)\left(z_t^i + Z_t^{-i} + (1-\beta)X_t\right)\right] \Bigg\} \end{aligned}$$

Given Assumption 1 it is obvious that the unique maximum for the objective function of player i is not to contribute, i.e. $z_t^i \equiv 0$. Hence, the unique optimal response of agent i independently of what the other agents contribute is to set z_t^i equal to zero. This implies that the unique dominant strategy is not to contribute. □

The no-contribution equilibrium results in a stock dynamics given by

$$X_t = (1-\beta)^t X_0,$$

and the level of discounted utility of agent i is given by

$$u_{\max}^i \equiv \sum_{t=0}^{\infty} \rho^t \left[u^i(X_t) - z_t^i\right] |_{z_t^i \equiv 0, X_t = (1-\beta)^t X_0} = \sum_{t=0}^{\infty} u^i((1-\beta)^t X_0).$$

Since in case of no contributions the existing capital stock will be run down geometrically, $(1-\beta)^t X_0 \leq X_0$, we get

$$\sum_{t=0}^{\infty} \rho^t u^i((1-\beta)^t X_0) \leq \sum_{t=0}^{\infty} \rho^t u^i(X_0) = \frac{u^i(X_0)}{1-\rho}$$

Off course, in case $X_0 = 0$ the maximum value of discounted utility of each agent is zero.

The equilibrium strategy in case that an agent does neither take the others' contributions into account nor the aggregated amount, is not to contribute. The negative result arises because in a dynamic setting with commitment each player cannot only free ride on the current contributions of the other agents but also on all future contributions.

3.3 Private provision of the public good when agents condition their investment on the current level of the stock

The last section has demonstrated that private investment results in no funding and hence low welfare, when agents only know their investments and do not observe the investments of the others. Here we change this assumption and allow for the case that each agent observes the aggregated accumulated investment in the public good and can use this information to make his investment decision. Hence, we assume each agent chooses investment as a function of the current stock of capital, i.e. $z_t^i = z^i(X_t)$. In this sense agents use Markovian strategies. Under this assumption the investment problem of agent i becomes the following : Choose a decision rule $z^i(X_t)$ such that for given decision rules of the opponents the discounted sum of net utility

$$\sum_{t=0}^{\infty} \rho^t \left[u^i(X_t) - z_t^i\right]$$

is maximized subject to the accumulation constraint (1).

Theorem 4 *Let D be a strictly positive constant. Under Assumption 1, and if agents choose stock dependent decision rules, there exists a Nash equilibrium with strictly positive investment by each player i over time. The equilibrium investment decision rules are given by*

$$z^i(X_t) = \frac{1}{N-1}\left(\sum_{j=1}^{N}\left[(r+\beta)X_t - u^j(X_t)\right] + D - (N-1)\left[(r+\beta)X_t - u^i(X_t)\right]\right). \quad (14)$$

Proof: Let us assume that all players except i choose an investment strategy $z^j(X_t)$ as in (14). We now derive the best response of agent i to these given Markovian strategies of the rivals. In light of (13) the objective function of agent i can be rewritten as

$$u^i(X_0) + (1-\beta)X_0 + \sum_{j=1, j\neq i}^{N} z^j(X_0) + \sum_{t=0}^{\infty} \rho^{t+1}\left[u^i(X_{t+1}) - (r+\beta)X_{t+1} + \sum_{j=1, j\neq i}^{N} z^j(X_{t+1})\right].$$

The best response of agent i needs to satisfy the Euler equation which for general t is given by

$$-(r+\beta)+u_X^i(X_t)+\sum_{j=1,j\neq i}^{N} z_X^j(X_t)=0 \tag{15}$$

Substituting (14) as the strategies of the rival firms into the left hand side of the Euler equation of agent i shows that (15) holds as an identity. This implies that the value of the objective function of player i is a constant, independently of the strategy player i chooses as a best response to the rivals' strategies. Therefore (15) is a best response of player i, which establishes that

$$z^i(X_t)=\frac{1}{N-1}\left(\sum_{j=1}^{N}\left[(r+\beta)X_t-u^j(X_t)\right]+D-(N-1)\left[(r+\beta)X_t-u^i(X_t)\right]\right)$$

is an equilibrium. Since there are no restrictions on the constant D, it can be choosen such that $z^i(X_t)>0$ for all $X\geq 0$. $\square$

With the investment rules (14) the state dynamics are given by

$$X_{t+1}=H(X_t)\equiv(1-\beta)X_t+\frac{N}{N-1}(r+\beta)X_t-\frac{1}{N-1}\sum_{i=1}^{N}u^i(X_t)+\frac{ND}{N-1}.$$

Before we continue to explore the welfare effects of this equilibrium we need to show feasibility, i.e. $H(X_t)\geq 0$ for $X_t\geq 0$. We get

$$\begin{aligned}
H(X_t) &= \frac{1}{N-1}\left[(Nr+\beta+N-1)X_t-\sum_{i=1}^{N}u^i(X_t)+ND\right]\\
&\geq \frac{1}{N-1}\left[(Nr+\beta+(N-1)\beta)X_t-\sum_{i=1}^{N}u^i(X_t)+ND\right]\\
&= \frac{1}{N-1}\sum_{i=1}^{N}\left[(r+\beta)X_t-u^i(X_t)+D\right]\\
&> 0.
\end{aligned}$$

The first result in this subsection is very surprising and needs some further elaboration. We have established that when agents use Markovian strategies and base their investment decisions on the level of cumulated aggregated contributions there is an equilibrium in which everybody makes a strictly positive contribution to the public project. This is in contrast to many existing results in the literature. We even have to stress the fact that Assumption 1 strongly favors free riding of the agents.

Although we have established that there is a Markov perfect equilibrium in which it is optimal for each agent to privately contribute a strictly positive investment in each period, it is not clear how the level of the public capital stock with private contributions compares to the case of the social optimum. Therefore we now consider the steady state level of the capital stock for the case of the social and the private contributions to the public project.

The steady state level of the capital stock in case of the Markovian equilibrium is implicitly given by

$$(Nr + \beta)\hat{X} = U(\hat{X}) - ND,$$

while in the case of the social provision of the public good it is given by

$$U_X(X^*) = r + \beta.$$

To indicate the dependence of the steady state level $\hat{X}$ on D, we make use of the notation $\hat{X}(D)$.

We can now compare the two steady state levels and get the following result.

Theorem 5 *If $U_X < r + \beta$ for all levels of $X > 0$ satisfying $(Nr + \beta)X = U(X)$ (which means $U_X(\hat{X}(0)) < r + \beta$), then there exists a $\hat{D} > 0$, such that $\hat{X}(\hat{D}) = X^*$.*

Proof: Follows immediately from the properties of the utility function $U(X)$. □

This last result has an important economic interpretation. It shows that the Markov equilibrium can support the long run efficient public capital stock as a Nash equilibrium.

In order to understand the economics of the last result, let us assume that the social utility function is given by $U(X) = X^\alpha$ in the relevant range for X. For this specification we get

$$X^* = \left(\frac{\alpha}{r + \beta}\right)^{\frac{1}{1-\alpha}},$$

and $\hat{X}(0)$ is given by

$$\hat{X}(0) = \left(\frac{1}{Nr + \beta}\right)^{\frac{1}{1-\alpha}}.$$

Now for $U_X(\hat{X}(0)) < r + \beta$ and hence $X^* < \hat{X}(0)$,

$$N < \frac{1}{\alpha} + \frac{\beta}{r}\left(\frac{1-\alpha}{\alpha}\right)$$

has to hold. But given N and α this only holds if r is sufficiently close to 0 or ρ sufficiently close to 1. Hence, in this example we can claim that there exists $0 < \hat{\rho} < 1$ such that for any $\rho \in (\hat{\rho}, 1)$ we have $\hat{X}(\hat{D}) = X^*$ for some $\hat{D} > 0$. This means that the agents have to be far sighted for the equilibrium with private investment to result in an efficient long run public capital stock.

We pointed out in the introduction that papers dealing with voluntary provisions of public goods in a dynamic setting either come to the conclusion that the free riding problem becomes more severe or cooperation can be sustained as an equilibrium behavior by relying on agents' threats and retaliation. The paper by Gaitsgory and Nitzan (1994) makes explicit use of trigger strategies to support the first best provision of the public good. Also the efficient equilibria in the paper by Marx and Matthews (2000) make use of what they call a grim strategy porfile. Any deviation from equilibrium play is met by a maximal feasible punishment.

Our main result does not rely on any explicit use of trigger strategies or punishments. When agents use Markovian strategies a Nash equilibrium exists with strictly positive private contributions that result in an efficient steady state level of the capital stock. The intuition behind the result is that the aggregated accumulated contributions X_t are a sufficient statistic for past behavior and possible deviation from a contribution equilibrium. Therefore in case of a discount rate close to zero Markovian strategies generate a long run equilibrium that enhances the private provision of public goods.

Finally, we would also like to compare our result to the static case. The corresponding static game to our dynamic model is given by

$$\max_{z^i} \left\{ u^i(Z) - z^i \right\},$$

with the collusive problem equal to

$$\max_{Z} \left\{ \sum_{i=1}^{N} u^i(Z) - Z \right\}.$$

Under Assumption 1 with $r + \beta < 1$ and $Nr + \beta > 1$ we get that the unique static social optimum is characterized by

$$U_X(\hat{X}) = 1,$$

and that the unique static equilibrium for private contributions results in no provision of the public good.

4 Conclusions

We have introduced a simple dynamic model of private investment in a public project where agents have nonlinear continuous utility and investment costs are linear. In this setting we characterized the unique solution of the collusive problem which we interpret as the case when the public good is provided collectively. Next we investigated whether or not the public good will be provided if agents privately invest. It turned out that in the case when agents have neither information about the current contributions of the rivals nor about the aggregated accumulated amount spent no provision of the public good is the unique equilibrium. In the case agents use Markovian strategies there exists an equilibrium in nonlinear strategies that results in strictly positive investment by all agents. This equilibrium generates a long run public capital stock that is identical to the first best outcome. Hence, we conclude that efficient provision of a public good is a dynamic equilibrium behavior of rational investors.

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