

# Impact of Non-Classical Measurement Error on Measures of Earnings Inequality and Mobility

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## Abstract

Measures of inequality and mobility based on self-reported earnings reflect attributes of both the joint distribution of earnings and the joint distribution of measurement error and earnings. While classical measurement error would increase measures of inequality and mobility there is substantial evidence that measurement error in earnings is mean reverting. In this paper we present the analytical links between mean reversion and other sources of non-classical measurement error on measures of inequality and mobility. The empirical importance of non-classical measurement error are explored using the Survey of Income and Program Participation matched to tax records.

## 1 Introduction

What is the impact of measurement error on measures of inequality and mobility? Measures of inequality and mobility based on self-reported earnings reflect both attributes of the joint distribution of earnings and the joint distribution of measurement error and earnings. Measurement error can, therefore, lead to potentially large bias in estimates of inequality in the marginal distribution of earnings and estimates of mobility across the joint distribution.

While that classical measurement error would lead to upward bias in estimates of inequality and mobility, the evidence reviewed in (Bound, Brown, and Mathiowitz 2001) shows that measurement error is not classical. Measurement error is mean reverting, in the sense that persons with low earnings tend to overstate their earnings and persons with high earnings understate their earnings. This

offsets the inequality increasing effects of classical measurement error, The impact of non-classical measurement error on mobility is less clear since mobility measures are based on the joint distribution of reported earnings in two periods. This introduces the possibility that earnings and lagged earnings suffer from the same form of measurement error and that measurement errors in the two periods are correlated

In this paper we present the analytical links between the statistical properties of measurement error and the properties of standard measures of inequality and mobility. We use the Survey of Income and Program Participation (SIPP) and matched tax records to provide empirical evidence on the importance of the links developed in the analytical section.

This paper is divided into six sections. The following section reviews the theoretical and empirical literature on measurement error in earnings. We then provide an analytical framework which allows for non-classical measurement error in both earnings and lagged earnings. Section 4 describes the data used in section 5 to provide estimates of the quantitative importance of the factors we develop in the analytical section. In section six we draw conclusions based on these findings.

## **2 Review of Literature**

While there is a substantial literature on measurement error in earnings, reviewed in (Bound, Brown, and Mathiowitz 2001), this literature has focused primarily on the implications of measurement error for studies where earnings appear either as a dependent variable or as an independent variable, but not both. As the following section makes clear, a new set of issues arise when studying earnings mobility since mobility measures describe the relationship between earnings in two different periods. As a result, there is potentially measurement error in both variables being studied. This introduces the possibility that the two sources of measurement error may not only be correlated with true earnings and lagged earnings but that the two sources of measurement error may themselves be correlated

While the literature on the impact of measurement error has

largely focused on single source of measurement error, these studies provide the foundation for our study. The first studies to examine the role of measurement error in the PSID were based on a validation study in which a sample of 418 workers in a large manufacturing plant reported their earnings using the same instrument as used in the PSID. (Duncan and Hill 1985) analyzed the difference between the firm's payroll records for these respondents and the earnings they reported on the PSID questionnaire. Since the firm was highly unionized, it is not surprising that respondents had higher mean earnings and lower variance of earnings than a nationally representative sample of workers. More importantly (Duncan and Hill 1985) found that the variance of measurement errors was large even among this group of workers, many of who worked under an explicit union contract. The variance of measurement error was 30 percent as large as the variance of payroll earnings in 1981 and 15 percent as large in 1982. Furthermore, the measurement error in this validation study is mean reverting (Bound and Krueger 1991) find roughly as large measurement error when CPS earnings are compared to Social Security administrative (SSA) records. This and the follow-up study by (Bollinger 1998) also find negative correlation between the measurement error and SSA earnings<sup>1</sup>. Both studies also find positive correlation in measurement error across the two years of matched CPS data. The finding that measurement error is large and mean reverting is corroborated by (?) who use data from a second wave of data collected from the same firm in the PSID validation study by (Duncan and Hill 1985). Since workers had to be continuously working for the same firm, the sample size decreases substantially. But even this sample of workers continuously employed by the same firm over a six year period (1982 to 1986) exhibited the same mean reversion and positive correlation in measurement error six years apart. Furthermore the size of the measurement error is large—the variance of measurement error is 15 to 30 percent as large as the variance of earnings from administrative records plus the variance of measurement error<sup>2</sup>.

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<sup>1</sup>This mean reversion in reported earnings is largely driven by low earnings males who overstate their SSA earnings.

<sup>2</sup>See (Bound, Brown, Duncan, and Rodgers 1994) Table 1 who report the ratio of measurement error to total variation in PSID earnings in 1982 and 1986 to be .151 and .302. (Bound and Krueger 1991) develop the relationship between this measure of the relative importance of

These studies and the wider literature reviewed in (Bound, Brown, and Mathiowitz 2001) largely treat earnings from administrative records as being free of measurement error. Abowd and Stinson (2005) develop an alternative approach in which they allow measurement error in both reported earnings and administrative records.

The two studies most closely related to ours are (Pischke 1995) and Fields(2006) who both provide empirical estimates of the impact of measurement error on specific measures of mobility<sup>3</sup> Following the literature developed in (Abowd and Card 1989) and, (MaCurdy 1982), Pischke examines the impact of measurement in the PSID on estimates of the variance of permanent and transitory earnings. This earnings components model is estimated in order to assess whether transitory earnings shocks are under-reported in the PSID. Fields(2006), focus is on different concepts of mobility and how measures of these different concepts are affected by measurement error. While both studies provide useful information on the net impact of measurement error on specific measures of mobility, neither provides the analytical links between the non-classical structure of measurement error and measures of mobility.

### 3 Analytical Links

In this section we develop the analytical links between the statistical structure of non-classical measurement error and summary measures of inequality and mobility. We develop these analytical links by focusing on the joint distribution of log earnings and lagged log earnings. The variances of the marginal distributions provide summary measures of inequality and the correlation between log earnings and lagged log earnings provides a summary measure of mobility.

We build on (Bound, Brown, and Mathiowitz 2001) who considers the impact of non-classical measurement error in a bivariate regression of  $y$  on  $x$ , where either  $y$  or  $x$ , but not both are measured with error. This restriction is unlikely to be met in our case since  $y$

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measurement error to the reliability ratio which they define as  $\frac{var(y^*)}{var(y)}$ , where  $y^*$  is log earnings in the administrative data set,  $y = y^* + v$  is reported log earnings. and  $v$  is measurement error.. They point out that in the case of classical measurement error  $\frac{var(y^*)}{var(y)} = \frac{var(y^*)}{var(y^*)+var(v)}$  which is one minus the ratio reported by (Bound, Brown, Duncan, and Rodgers 1994).

<sup>3</sup>(Coder and Scoon-Rogers 1996) also examines measurement error in the SIPP but does not examine its effect on mobility.

is log earnings and  $x$  is lagged log earnings, which are typically obtained from reported earnings in two different years from the same longitudinal instrument. .

We will show that allowing for measurement error in both  $y$  and  $x$  adds several new routes by which non-classical measurement error can affect the correlation in earnings . Measurement error in reported values of  $y$  and  $x$  may not only be correlated with the true values of  $y$  and  $x$  but may also be correlated with each other.

### 3.1 Generalized Measurement Error Model

Following the standard notation consider the following bivariate relationship between  $y^*$  and  $x^*$

$$y^* = \beta x^* + \varepsilon \tag{1}$$

where  $y^*$  and  $x^*$  are true log earnings and lagged log earnings. Both  $y^*$  and  $x^*$  are, however, subject to measurement error leading to the observed values of  $y$  and  $x$  :

$$y = y^* + v \tag{2a}$$

$$x = x^* + \mu \tag{2b}$$

Within this framework  $\sigma_{y^*}^2$  and  $\sigma_{x^*}^2$  provide summary measures of inequality in earnings and lagged earnings. Likewise the correlation between  $y^*$  and  $x^*$ ,

$$\rho = \beta \frac{\sigma(x^*)}{\sigma(y^*)} \tag{3}$$

provide a summary measure of mobility<sup>4</sup>.

#### 3.1.1 Impact on Measures of Inequality

The impact of measurement error on inequality in earnings is given by

$$\sigma_y^2 - \sigma_{y^*}^2 = 2cov(y^*, v) + var(v) \tag{4}$$

where  $\sigma_y^2$  is the variance of measured log earnings and  $\sigma_{y^*}^2$  is the variance of true log earnings. 4 simply shows that the difference between the variance of measured earnings and the variance of of

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<sup>4</sup>See Fields (2006) for alternative measures of mobility.

actual earnings depends both on the variance of measurement error and the covariance of measurement error and true earnings. While larger variance of measurement error will unambiguously lead to an upward bias in inequality, this will be offset by mean reversion in measurement error ( $cov(y^*, v) < 0$ ). If  $\frac{cov(y^*, v)}{var(v)} < -0.5$  then measured inequality will understate the degree of inequality.

The impact of measurement error on the trend in inequality can be expressed in terms of changes in measured inequality,  $\Delta = \sigma_y^2 - \sigma_x^2$ , and true changes in inequality,  $\Delta^* = \sigma_{y^*}^2 - \sigma_{x^*}^2$ ,

$$\Delta - \Delta^* = 2 [cov(y^*, v) - cov(x^*, \mu)] + [var(v) - var(\mu)] \quad (5)$$

which states that increases in inequality may be overstated if measurement error is increasing ( $var(v) - var(\mu) > 0$ ) or if mean reversion in measured earnings is declining ( $cov(y^*, v) - cov(x^*, \mu) > 0$ )

### 3.1.2 Impact on Measures of Mobility

Since  $\rho$  depends on  $\beta$  as well as the variances in the two marginal distributions, we focus on the impact of non-classical measurement error on  $\beta_{yx}$ , the linear projection of  $y$  on  $x$ . The impact of measurement error on  $\beta_{yx}$  can be derived by substituting equation 2a into 1 :

$$y = \beta x + v - \beta \mu + \varepsilon \quad (6)$$

This implies that :

$$\beta_{yx} = \frac{cov(x, \beta x + v - \beta \mu + \varepsilon)}{var(x)} \quad (7a)$$

$$\begin{aligned} &= \beta + \frac{cov(xv)}{var(x)} - \beta \frac{cov(x\mu)}{var(x)} + \frac{cov(x, \varepsilon)}{var(x)} \\ &= \beta(1 - \beta_{\mu x}) + \beta_{vx} + \beta_{\varepsilon\mu} \frac{var(\mu)}{var(x)} \end{aligned} \quad (7b)$$

where  $\beta_{\mu x} = \frac{cov(\mu x)}{var(x)}$  and other coefficients are defined similarly<sup>5</sup>.

It is useful to rewrite 7a in terms of the underlying parameters that capture the non-classical nature of measurement error,  $\beta_{vy^*}$ ,  $\beta_{\mu x^*}$ ,  $\beta_{v\mu}$ ,  $\beta_{v\varepsilon}$  and  $\beta_{\varepsilon\mu}$ . The first two parameters,  $\beta_{vy^*}$  and  $\beta_{\mu x^*}$ , allow

<sup>5</sup>Like (Bound and Krueger 1991) we do not distinguish between sample and population regression coefficients since any bias from measurement error does not depend on  $n$ .

measurement error in  $y$  and in  $x$  to be correlated with the true values  $y^*$  and  $x^*$ <sup>6</sup>. The third parameter,  $\beta_{v\mu}$  allows measurement errors in  $y$  and in  $x$  to be correlated with each other. Measurement error in  $y^*$  and  $x^*$  may also be correlated with  $\varepsilon$ , as captured by  $\beta_{v\varepsilon}$  and  $\beta_{\varepsilon\mu}$ . In the Appendix we show that equation 7a can be written in terms of these underlying parameters.<sup>7</sup>:

$$\beta_{yx} = \beta \left( 1 + \left\{ (\beta_{vy^*} - \beta_{\mu x^*}) \frac{\text{var}(x^*)}{\text{var}(x)} \right\} \right) + \left\{ (\beta_{vy^*} - \beta_{v\varepsilon}) \frac{\text{var}(\varepsilon)}{\beta \text{var}(x)} \right\} + \left\{ [\beta_{v\mu} + \beta_{\varepsilon\mu} - \beta] \frac{\text{var}(\mu)}{\text{var}(x)} \right\} \quad (8)$$

This expression shows that the non-classical nature of measurement error in earnings,  $y^*$ , and lagged earnings,  $x^*$ , affects  $\beta_{yx}$  through three different channels, indicated by the braces.

The first term in braces captures the effect of mean reversion, where  $v$  and  $\mu$  are allowed to depend on  $y^*$  and  $x^*$  respectively ( $\beta_{vy^*} \neq 0$  and/or  $\beta_{\mu x^*} \neq 0$ ). Consider the case where measurement error in both earnings and lagged earnings are mean reverting. The term in braces shows that the net effect of mean reversion depends on offsetting effects of mean reversion in earnings and lagged earnings<sup>8</sup>. If there is equal mean reversion in reported earnings and lagged earnings then  $\beta_{vy^*} = \beta_{\mu x^*}$  and the term in braces is zero. As a result, mean reversion does not lead to bias estimates of  $\beta$ . If earnings and lagged earnings are obtained from the same survey instrument, administered in two different years, it is likely that mean reversion is similar in both years

The second term in braces in equation 8 offers a somewhat more subtle source of bias in  $\beta_{yx}$ . Following the previous literature we have discussed the non-classical relationship between  $y^*$  and  $v$  without distinguishing between two sources of variation in  $y^*$ . Since  $y^* = \beta x^* + \varepsilon$ ,  $y^*$  can vary either because of variation in  $\beta x^*$  or because of variation in  $\varepsilon$ . The second term in braces allows mean reversion in  $y$  to differ depending on whether the variation in  $y^*$  is a result

<sup>6</sup>Since  $x = x^* + \mu$ , there is a built in correlation between  $x$  and  $\mu$ . Therefore,  $\beta_{\mu x} \neq 0$  even if  $\beta_{\mu x^*} = 0$ . A similar argument applies to  $\beta_{vy}$ . (See (Bound and Krueger 1991))

<sup>7</sup>We also show that the results in (Bound, Brown, and Mathiowitz 2001) are special cases of this general expression.

<sup>8</sup>Since  $\beta_{vy^*}$  and  $\beta_{\mu x^*}$  are both negative  $\beta_{vy^*} - \beta_{\mu x^*} > 0$  if  $|\beta_{\mu x^*}| > |\beta_{vy^*}|$

of variation in  $x^*$  or because of variation in  $\varepsilon$ . Since  $\varepsilon$  may reflect factors such as unanticipated shocks to  $y^*$ , this may affect reporting error differently than changes in  $x^*$ . If  $\beta_{v\varepsilon} = \beta_{vy^*}$  then the second term in braces is zero and, as a result, this factor does not contribute to bias estimates of  $\beta$ .

Finally, the last term in braces shows that the standard attenuation bias, captured by  $-\beta \frac{\text{var}(\mu)}{\text{var}(x)}$ , may be partially offset by positive covariance between  $v$  and  $\mu$  or between  $\varepsilon$  and  $\mu$ . The former would happen if measurement error in  $y$  and  $x$  were positively correlated ( $\beta_{v\mu} > 0$ ). In terms of our application to earnings mobility, people who overstate their earnings in one period may tend to overstate their earnings in the other periods. Positive correlation between transitory shocks to earnings,  $\varepsilon$ , and measurement error in lagged earnings ( $\beta_{\varepsilon\mu} > 0$ ) also offsets attenuation bias. If  $\beta_{v\mu} + \beta_{\varepsilon\mu} - \beta = 0$ , then attenuation bias is fully offset by these two factors.

In summary, we have shown the analytical links between different forms of non-classical measurement error and  $\beta_{yx}$ . While the direction of the bias introduced by relaxing the classical assumptions depends on the sign and magnitudes of  $\beta_{vy^*}$ ,  $\beta_{\mu x^*}$ ,  $\beta_{v\mu}$ ,  $\beta_{v\varepsilon}$  and  $\beta_{\varepsilon\mu}$  we have shown the conditions under which these biases are offsetting.

### 3.1.3 Classical measurement error

It is useful to contrast these general results with the special case where  $\mu$  and  $v$  are classical random measurement error. Under the assumptions of classical measurement error  $\beta_{vy^*} = \beta_{\mu x^*} = \beta_{v\mu} = \beta_{v\varepsilon} = \beta_{\varepsilon\mu} = 0$ . Assuming that measurement error is not mean reverting ( $\beta_{vy^*} = \beta_{\mu x^*} = 0$ ) implies that measured inequality unambiguously leads to an overstatement of inequality (equation 4 shows that  $\sigma_y^2 - \sigma_{y^*}^2 > 0$ .) Measurement error affects the trend in inequality only if the variance of measurement error changes over time. Since mean reversion is assumed away it cannot affect the trend in inequality.

Classical measurement error also leads to an overestimate of mobility as measured by the correlation in earnings. Equation 8 simplifies to

$$\beta_{yx} = \beta \left( 1 - \frac{\text{var}(\mu)}{\text{var}(x)} \right) \quad (10)$$

which reduces to the standard result

$$\beta_{yx} = \beta \frac{\text{var}(x^*)}{\text{var}(x)} < \beta \quad (11)$$

since  $\text{var}(x) > \text{var}(x^*)$ . Similarly<sup>9</sup>

$$\rho_{yx} = \rho_{y^*x^*} \frac{\sigma(x^*)\sigma(y^*)}{\sigma(x)\sigma(y)} < \rho_{y^*x^*} \quad (15)$$

Classical measurement error, therefore, increases mobility as well as inequality.<sup>10</sup>

In summary we have shown that non-classical measurement error can lead to potentially offsetting effects on measures of inequality and mobility. In the following sections we provide estimates of the quantitative size of each of these effects and their net impact on estimates of inequality and mobility

## 4 Data

This study uses data from the 1984, 1990, 1993 and 1996 panels of the Survey of Income and Program Participation (SIPP) which are matched to the Detail Earnings Record (DER) at the Social Security Administration<sup>11</sup>. SIPP respondents were interviewed in waves four months apart, and at each wave they were asked to report their earnings for the previous four months, with detailed information on

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$$\rho_{yx} = \beta \frac{\text{var}(x^*)}{\text{var}(x)} \frac{\text{sd}(x)}{\text{sd}(y)} = \beta \frac{\text{var}(x^*)}{\text{sd}(x)\text{sd}(y)} \quad (12)$$

$$= \left( \beta \frac{\text{sd}(x^*)}{\text{sd}(y^*)} \right) \frac{\text{var}(x^*)}{\text{sd}(x)\text{sd}(y)} \frac{\text{sd}(y^*)}{\text{sd}(x^*)} \quad (13)$$

$$= \rho_{y^*x^*} \frac{\text{sd}(x^*)\text{sd}(y^*)}{\text{sd}(x)\text{sd}(y)} < \rho_{y^*x^*} \quad (14)$$

<sup>10</sup>The increase in inequality and the reduction in  $\rho_{yx}$  are directly related since  $\rho_{yx} = \beta_{yx} \frac{\text{sd}(x)}{\text{sd}(y)} = \beta \frac{\text{var}(x^*)}{\text{var}(x)} \frac{\text{sd}(x)}{\text{sd}(y)} = \frac{\beta \text{var}(x^*)}{\text{sd}(x)\text{sd}(y)}$ . Therefore,  $\text{sd}(x)\text{sd}(y) > \text{sd}(x^*)\text{sd}(y^*)$  so the increase in inequality caused by classical measurement error leads directly to the conclusion that  $\rho_{yx} < \rho$ .

<sup>11</sup>The obvious alternative to the SIPP is the PSID validation study which has been used extensively to obtain estimates of measurement error. The primary limitation of this alternative data set is that it includes earnings data from workers in a single manufacturing firm in Detroit. Workers had to stay in the firm in order to be included in the longitudinal data set. Therefore, this data set necessarily excludes all between firm changes in earnings which have been shown to be an important source of earnings mobility.

up to two jobs. We restrict our sample to respondents with valid SIPP earnings in all 12 months of the calendar year (including zero earnings)<sup>12</sup>.

We compare these measures of annual earnings from the SIPP, with the counterpart constructed from the Detailed Earnings Records files that contain earnings information from W-2 forms for all jobs held by the respondent<sup>13</sup>. The DER does not suffer from the standard limitations of FICA tax records which are top-coded at the FICA max and exclude jobs in sectors not covered by the FICA tax, such as state and local government workers.

The data in the DER are matched to SIPP respondents on the basis of their self-reported Social Security numbers. Respondents who fail to give their Social Security numbers or give invalid Social Security numbers cannot be matched and are, therefore, dropped. The match rates for our analysis sample range from 77 percent for the two most recent panels to 86 percent for the earlier panels

While this matching procedure allows us to compare reported earnings in SIPP with earnings in fairly inclusive administrative records, SIPP earnings may differ from DER earnings for several reasons. First, SIPP respondents are only asked to report earnings on up to two jobs in any month. If the respondent held more than two jobs, either simultaneously or in succession, then the earnings in the additional jobs are missed in the SIPP but not in the DER.

In order to construct an earnings variable that is as close as possible to the SIPP earnings variable we exclude self-employment earnings and deferred earnings. The DER also differs from the SIPP since SIPP earnings are top-coded at \$150,000 per year and replaced by the mean earnings of persons classified by demographic characteristics<sup>14</sup>. We, therefore, impose a similar procedure to the DER. In each year, values above \$150,000 are replaced by the mean of earnings of persons with earnings above this threshold disaggregated by gender, ethnicity, race, match status—a total of 24 categories in

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<sup>12</sup>Partial years in each panel are dropped since we require full year records to be comparable to the DER. The following are the full years covered by each panel

1984 panel:1984-85

1990 panel:1990-91

1993 panel:1994-95

1999 panel:1996-99

<sup>13</sup>DER records are available starting in 1978.

<sup>14</sup>SIPP top coding is also based on monthly earnings and earnings within each wave which is not available in the DER.

each year.

Our analysis sample includes males, 25 to 62, not attending school with positive yearly earnings. We also impose the sample restriction that respondents must have valid earnings in  $t$  and  $t - 1$  in order to insure that variances and covariances come from the same sample. This restriction insures that the key identity in equation 8, holds exactly. Relaxing the balanced panel restriction does not alter our conclusions.

We primarily use the 1996 SIPP panel to estimate the impact of measurement error on the level of inequality and mobility. However, we also use the other panels to determine the impact of measurement error on changes over time in measures of inequality and mobility.

## 5 Results

### 5.1 Summary Statistics

Table 1 presents descriptive statistics for the 1996 panel, which is the basis for much of our analysis. Since roughly thirty percent of the earnings in this sample are imputed by SIPP, and since imputations introduce an additional source of measurement error that can be avoided by dropping imputed earnings, we show all our results for the full sample and the sub-sample of non-imputed earnings. Our full sample includes 3,742 observations of positive yearly earnings, of which 2,587 yearly observations include no months with imputed earnings.<sup>15</sup>

The demographic and employment characteristics of the full sample and the sub-sample are shown in columns 1 and 2. The average age in both columns is just above 41 and both columns show similar levels of education. Only race and ethnicity are substantially different across columns. Blacks are under-represented and Hispanics are over-represented in the sample that includes only non-imputed earnings. Employment characteristics are, however, similar with both the full sample and the subsample being largely composed of full-time, year-round workers

Table 2 presents means and variances of log earnings in the DER

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<sup>15</sup>A yearly earnings observation is classified as non-imputed if none of the monthly observations are imputed.

and SIPP, as well as the mean and variance of measurement error. These summary statistics indicate that measurement error is large in the SIPP. Mean log earnings are understated by .15 for the full sample and by .12 for the sample of non-imputed earnings. One possible reason for this large mean discrepancy is that the SIPP includes only information on two jobs.

Not only does SIPP provide an underestimate of mean earnings, the variance in measurement error is also large. The signal to noise ratio, as measured by  $\frac{\text{var}(DER)}{\text{var}(error)}$  is roughly 2.0 when cases with imputed earnings are included. This signal to noise ratio increases only to 2.6 when imputed earnings are excluded. For comparison with other studies we also show what is known in the literature as the reliability ratio for classical measurement error,  $\frac{\text{var}(DER)}{\text{var}(DER)+\text{var}(error)}$ . The reliability ratio, is .67 for all cases and .73 for cases with non-imputed earnings. This is consistent with the value of .7 reported for the PSID in table 1 of (Bound, Brown, Duncan, and Rodgers 1994) and for values around .8 found in the CPS by (Bound and Krueger 1991) table 6.

If measurement error were classical then the large variance in measurement error would lead to substantially greater measures of inequality in the SIPP than in the DER. However, Table 2 shows just the opposite. The variance of log earnings in DER earnings of .68 is somewhat larger than the variance in SIPP earnings for all respondents (.56) or respondents with non-imputed earnings (.53). As we have shown, this is the result of measurement error being mean reverting. This mean reversion more than offsets the increase in the variance of reported earnings from the noise in the data. In terms of our previous notation the variance in SIPP understates the degree of inequality because  $|\sigma_{y^*v}| > \sigma_v^2$

## 5.2 Structure of Measurement Error

Table 2 implies that measurement error in the SIPP is mean reverting, which is consistent with studies of other data sets. As we showed earlier there are several other dimensions of non-classical measurement error which have potentially large impacts on measures of mobility. Table 3 shows the more general structure of measurement error in the SIPP by displaying estimates of the key parameters in

equation 8 which shows the relationship between  $\beta_{yx}$  and  $\beta$  and these parameters of the error distribution ( $\beta_{vy^*}$ ,  $\beta_{\mu x^*}$ ,  $\beta_{v\mu}$ ,  $\beta_{v\varepsilon}$  and  $\beta_{\varepsilon\mu}$ ).

The first row of Table 3 verifies that measurement error is mean reverting. Our point estimate of -.339 for  $\beta_{vy^*}$  is significant at conventional levels. It indicates that measurement error in log earnings ( $v$ ) is strongly negatively correlated with log earnings ( $y^*$ ). A one percent increase in log earnings is accompanied offsetting under-reporting of earnings. This results in reported earnings increasing by roughly one third less than the true increase in earnings. Similarly our point estimate of -.292 for  $\beta_{\mu x^*}$  indicates that measurement error in lagged log earnings ( $\mu$ ) is also strongly negatively correlated with lagged log earnings ( $x^*$ ).

While the point estimates are not significantly different from each other,  $\beta_{vy^*}$  is larger in absolute value than  $\beta_{\mu x^*}$  which implies that the point estimate for the first term in brackets in equation 8 ,  $\left\{ (\beta_{vy^*} - \beta_{\mu x^*}) \frac{var(x^*)}{var(x)} \right\}$ , is negative. The negative difference in point estimates implies that mean reversion leads to a negative bias in  $\beta_{yx}$  and  $\rho_{yx}$ . Since the magnitude of the bias also depends on the  $\frac{var(x^*)}{var(x)}$  we assess the overall magnitude of this and other sources of bias in the following section

The second term in brackets in equation 8 ,  $\left\{ (\beta_{vy^*} - \beta_{v\varepsilon}) \frac{var(\varepsilon)}{\beta var(x)} \right\}$ , shows that the bias depends not only on the relationship between measurement error and earnings,  $\beta_{vy^*}$ , but also on the degree to which earnings reflects deviations from the predicted value based on prior earnings,  $\varepsilon$  The estimated value of -.550 for  $\beta_{v\varepsilon}$  in row 3 of table 3 is substantially larger in absolute value than the value of -.339 for  $\beta_{vy^*}$  shown in row 1. This indicates that mean reversion is substantially greater when earnings deviate from their predicted values based on lagged earnings,  $\beta x^*$ . A one percent increase in transitory earnings,  $\varepsilon$ , is accompanied by an offsetting .550 reduction in measurement error, which is substantially larger than the .339 reduction in measurement error from a one percent change in  $y^*$ , which includes the effects of  $\beta x^*$  as well as  $\varepsilon$ . This implies that respondents tend to understate their earnings even more when their earnings reflect deviations from expected earnings. This under-reporting of deviations from expected earnings increases the correlation in reported earnings across time This results in an upward bias in  $\beta_{xy}$  as an estimate

of  $\beta$  which includes the mobility increasing effects of deviations from expected earnings.

Finally, the sign of the third term in brackets in equation 8 ,  $\left\{ [\beta_{v\mu} + \beta_{\varepsilon\mu} - \beta] \frac{\text{var}(\mu)}{\text{var}(x)} \right\}$ , depends on whether two additional sources of non-classical measurement,  $\beta_{v\mu}$  and  $\beta_{\varepsilon\mu}$ , are sufficiently strong to offset the standard source of attenuation bias, given by  $-\beta \frac{\text{var}(\mu)}{\text{var}(x)}$ . If  $\beta_{v\mu}$  and  $\beta_{\varepsilon\mu}$  are larger than  $\beta$  then they can fully offset the attenuation bias of classical measurement error

The first of these potentially offsetting effects,  $\beta_{v\mu}$ , allows measurement error in earnings and lagged earnings to be correlated. If measurement errors are positively correlated then this will offset the negative impact of attenuation bias. Our estimate of .540 for  $\beta_{v\mu}$  indicates that a one percent increase in measurement error in lagged earnings is associated with a 54.0 percent increase in measurement error in earnings. This positive correlation in measurement error results in positive correlation in reported earnings, which offsets the attenuation bias of classical measurement error.

The second potentially offsetting effect of non-classical measurement error,  $\beta_{\varepsilon\mu}$ , allows measurement error in lagged earnings,  $\mu$ , to be correlated with deviations from expected earnings,  $\varepsilon$ . This allows responses to questions in period  $t$  about earnings in  $t - 1$  to be affected by deviation from expected earnings in the interview period. For example, respondents experiencing transitorily high earnings in period  $t$  may tend to extrapolate these high earnings back to period  $t - 1$  and thus to overstate prior earnings.. The positive and significant point estimate of .198 for  $\beta_{\varepsilon\mu}$  in the bottom row of Table 3 is consistent with this behavior. Since estimates of both  $\beta_{v\mu}$  and  $\beta_{\varepsilon\mu}$  are positive these sources of non-classical measurement error offset the attenuation bias of classical measurement error.

In summary, non-classical measurement error in the SIPP introduces a set of factors that tend to offset the attenuation bias of classical measurement error. The only source of non-classical measurement error that reinforces classical attenuation bias is mean reversion. And since it is only the difference in mean reversion of measurement error in earnings and lagged earnings that matters, its impact is likely to be small.

In the next section we present the resulting estimates of  $\beta_{yx}$  and

$\rho_{yx}$  for the SIPP and DER to see the net impact of non-classical measurement error. We then disaggregate this net impact into the component parts based on the individual non-classical factors described in this section.

### 5.3 Impact on $\beta_{yx}$ and $\rho_{yx}$

Table 4 shows the net impact of the non-classical nature of measurement error on estimates of  $\beta_{yx}$ , the elasticity of log earnings with respect to lagged earnings, and on  $\rho_{yx}$ , the correlation between log earnings and lagged log earnings. While there are many other measures of mobility we start with these two simple measures which allow us to isolate the impact of the individual components of non-classical measurement described in the previous sections <sup>16</sup>.

Row 1 of Table 4 shows estimates of the elasticity and correlation from the DER. The estimated elasticity of .868 indicates that a one percent increase in lagged earnings in one period is associated with a .868 percent increase in earnings in the following period. This is slightly higher than the .845 elasticity for the full SIPP sample and nearly identical to the .867 elasticity for the sample of non-imputed earnings. Likewise the correlations are similar in the two data sets. The DER yields a correlation of .834 while the SIPP yields estimates of .831 and .86, depending on whether or not imputed earnings are included.

While the SIPP and DER estimates are remarkably similar, this does not mean that measurement error is not important. In fact table 2 showed that the variance of measurement error is large, which would lead to large attenuation bias if measurement error were classical. However, as the previous section has shown, non-classical measurement error is important and may be sufficiently large to offset the effects of classical measurement error.

In order to access the importance of the individual sources of measurement error described in the previous sections, we use equations 4 and 8 to calculate  $var(y)$ ,  $\beta_{yx}$  and  $\rho_{yx}$  under a set of counterfactual situations. The top panel of table 5 replicates the estimated values of  $var(y)$ ,  $\beta_{yx}$  and  $\rho_{yx}$  from the DER and SIPP shown in Tables 2

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<sup>16</sup>For description of the impact of measurement error on a more extensive set of measures see Fields(2006).

and 4. These are shown for comparison with the counterfactual values in the bottom panel. The four rows in the bottom panel show the values under the following counterfactual assumptions.

Row 1 calculates  $var(y)$ ,  $\beta_{yx}$  and  $\rho_{yx}$  under the counterfactual assumption that all measurement error in SIPP is classical by setting  $\beta_{vy^*}$ ,  $\beta_{\mu x^*}$ ,  $\beta_{\varepsilon\mu}$  and  $\beta_{v\mu}$  in equations 4 and 8 all equal to zero. Given the large measurement error shown in table 2 this would naturally lead to a high value of  $var(y)$  since the random measurement error would add to the variance of log earnings. Similarly random measurement error would lead to spurious mobility as earnings would become less correlated with lagged earnings. This would lead to low values of  $\beta_{yx}$  and  $\rho_{yx}$ . Row 1 of the bottom panel shows that if all measurement error were classical then the variance of log earnings in the SIPP would be roughly 1.5 times larger than the variance of log earnings in the DER (1.027 versus .684). Similarly classical random measurement error would lead to an overestimate of mobility as reflected in lower correlation in earnings across time. If measurement error were classical, both  $\beta_{yx}$  and  $\rho_{yx}$  would be half as large in the SIPP as in the DER.  $\beta_{yx}$  would be .433 in the SIPP, compared to .868 in the DER. Similarly,  $\rho_{yx}$  would be .408 and .834 in these two data sets

While classical measurement would lead to substantially higher  $var(y)$  in the SIPP than the DER Table 2 showed that SIPP actually has a lower variance. Similarly classical measurement error would lead to lower values of  $\beta_{yx}$  and  $\rho_{yx}$  in the SIPP than in the DER, while Table 4 indicates very similar values. As we showed this is the result of the three bracketed terms in equation 8 offsetting the effects of classical measurement error. We identify the relative importance of each of these bracketed terms by introducing them sequentially.

Row 2 of the bottom panel relaxes the constraints on the third bracketed term by allowing  $[\beta_{v\mu} + \beta_{\varepsilon\mu}]$  to take on the positive value based on the values of  $\beta_{v\mu}$  and  $\beta_{\varepsilon\mu}$  shown in Table 3. This allows measurement errors in earnings and lagged earnings to be positively correlated ( $\beta_{v\mu} = .540$ ) and allows measurement error in lagged earnings to be positively correlated with deviations from expected earnings ( $\beta_{\varepsilon\mu} = .198$ ). These sources of non-classical measurement error have a large impact on  $\beta_{yx}$  and  $\rho_{yx}$ . Under this counterfactual, the SIPP estimate of the elasticity increases to .813, which brings it

much closer to the DER value of .868. Similarly the SIPP value for  $\rho_{yx}$  would be .766, which makes up much of the gap with the DER value of .834. Thus, allowing  $\beta_{v\mu}$  and  $\beta_{\varepsilon\mu}$  to be non-zero largely offsets the attenuation bias of classical measurement error. This gap closing effect of non-classical measurement error is primarily the result of allowing for correlation in measurement error in earnings and in lagged earnings since  $\beta_{v\mu}$  is more than twice as large as  $\beta_{\varepsilon\mu}$

Row 3 of the bottom panel relaxes the classical assumption that the second term in brackets in equation is zero by letting  $(\beta_{vy^*} - \beta_{v\varepsilon})$  take its value of .211 in Table 3<sup>17</sup>. Since this term is positive, its effect is to further raise  $\beta_{yx}$  from .813 to .892 and to raise  $\rho_{yx}$  from .766 to .840. Both counterfactual values of  $\beta_{yx}$  and  $\rho_{yx}$  are somewhat above their SIPP and DER values shown in the top panel.

Finally Row 4 of the bottom panel allows for mean reversion by allowing  $(\beta_{vy^*} - \beta_{\mu x^*})$  to take its value of -.047 in Table 3<sup>18</sup>. Since  $(\beta_{vy^*} - \beta_{\mu x^*})$  is negative and since this term determines the sign of the first term in brackets in equation 8, relaxing the classical assumption of no-mean reversion leads to lower values of  $\beta_{yx}$  and  $\rho_{yx}$ . But the effect is small. Allowing for mean reversion lowers the elasticity from .892 to .845 and lowers the correlation from .840 to .831.

Since the impact of mean reversion on the variance of the marginal distribution,  $var(y)$ , depends on  $\beta_{vy^*}$ , rather than on the difference between  $\beta_{vy^*}$  and  $\beta_{\mu x^*}$ , the impact is substantially larger. Allowing for mean version cuts the value of  $var(y)$  nearly in half, from 1.027 to .563.

In summary, classical measurement error would have led to substantially higher values of  $var(y)$  and to lower values  $\beta_{yx}$  and  $\rho_{yx}$  in the SIPP than the DER. The fact that the SIPP shows lower values of  $var(y)$  than the DER reflects the strong mean reversion in measurement error which more than offsets the inequality increasing effect of the large variance of measurement error. The fact that the SIPP and DER give very similar values for the correlation in earnings is largely the effect of another source of non-classical measurement error. While classical measurement error assumes that

<sup>17</sup>  $(\beta_{vy^*} - \beta_{v\varepsilon}) = .550 - .339 = .211$

<sup>18</sup> Since equation 8 holds exactly and since all the relevant classical assumptions have been relaxed, we arrive back at the SIPP values in the top panel.

errors in reported earnings and lagged earnings are independent we find strong correlation in these two sources of measurement error. These correlated errors largely offset the attenuation bias of classical measurement error. As a result estimates of the correlation in earnings are very similar in these two data sets.

## **6 Sections to be completed**

### **6.1 Shorrocks measure of mobility**

The preceding has focused on the correlation in earnings as a measure of mobility. This measure has been used since it allowed us to draw direct links between the structure of the measurement error and mobility. We are currently exploring whether similar links can be developed for the mobility concept developed in (Shorrocks 1978)

### **6.2 Impact of measurement error on trends**

We have shown that the structure of measurement error in the SIPP leads to underestimates of the mean and variance of log earnings but has little impact on the correlation in between earnings and lagged earnings. These biases in levels do not necessarily imply biases in trends since trends in the mean, variance and correlation of earnings are affected by changes in the structure of measurement error not the levels. We will, therefore, also present evidence on the effect of measurement error on trends in the mean, variance and correlation of earnings

### **6.3 Non-linearities**

The analytical and empirical work so far has focused on the linear relationships between log earnings, lagged log earnings and various aspects of measurement error. Since these linear relationships may be misleading we will also present plots that do not impose linearity.

## **7 Conclusion**

We have presented a general framework that can be used to analyze the impact of non-classical measurement on measures of inequality

and mobility. While classical measurement error leads to upward bias in both inequality and mobility, non-classical measurement error introduces a set of potentially important offsetting factors. For example, mean reverting measurement error can lead to downward bias in estimates of inequality which offsets the increased variance from classical measurement error. Likewise, correlated measurement error in earnings and lagged earnings can fully offset the attenuation bias in estimates of the correlation in earnings.

Our empirical application shows that there is substantial measurement error in SIPP earnings. This measurement error is, however, far from classical. Consistent with prior studies of the PSID, we find substantial mean reversion in measurement error of earnings in the SIPP. Measurement error is, however, not only correlated with earnings but measurement errors in earnings and lagged earnings are themselves correlated. The net impact of non-classical measurement error is that inequality is underestimated in SIPP. The correlation in earnings across time is, however, similar in the SIPP and administrative records. This is the result of large but offsetting bias in measure of earnings and lagged earnings

## A Appendix

In this appendix we show how equation 8 is derived from equation 7a. We then show that results in Bound are special cases of equation 8.

### A.1 Derivation of equation 8

To derive equation 8 start with equation 7a

$$\beta_{yx} = \beta(1 - \beta_{\mu x}) + \beta_{vx} + \beta_{\varepsilon\mu} \frac{\text{var}(\mu)}{\text{var}(x)} \quad (16a)$$

$\beta_{\mu x^*}$  is introduced by recognizing that  $\beta_{\mu x} = \frac{\text{cov}(\mu, x^* + \mu)}{\text{var}(x)} = \beta_{\mu x^*} \frac{\text{var}(x^*)}{\text{var}(x)} + \frac{\text{var}(\mu)}{\text{var}(x)}$ . Therefore,

$$\beta_{xy} = \beta(1 - \beta_{\mu x^*} \frac{\text{var}(x^*)}{\text{var}(x)} - \frac{\text{var}(\mu)}{\text{var}(x)}) + \beta_{vx} + \beta_{\varepsilon\mu} \frac{\text{var}(\mu)}{\text{var}(x)} \quad (17)$$

$\beta_{vy^*}$  is introduced by recognizing that  $\beta_{vx}$  can be written in terms of  $\beta_{vy^*}$  and  $\beta_{v\mu}$ :

$$\beta_{vx} = \frac{\text{cov}(vx)}{\text{var}(x)} = \frac{\text{cov}(v, x^* + \mu)}{\text{var}(x)} \quad (18)$$

$$\begin{aligned} &= \frac{\text{cov}\left(v, \frac{y^*}{\beta} - \frac{\varepsilon}{\beta} + \mu\right)}{\text{var}(x)} \\ &= \frac{\text{cov}(v, y^*) - \text{cov}(v\varepsilon) + \beta\text{cov}(v, \mu)}{\beta\text{var}(x)} \end{aligned}$$

$$\beta_{vx} = \frac{\beta_{vy^*} \text{var}(y^*)}{\beta \text{var}(x)} - \frac{\beta_{v\varepsilon} \text{var}(\varepsilon)}{\beta \text{var}(x)} + \beta_{v\mu} \frac{\text{var}(\mu)}{\text{var}(x)} \quad (19)$$

Substituting 18 into 17:

$$\beta_{yx} = \beta\left(1 - \beta_{\mu x^*} \frac{\text{var}(x^*)}{\text{var}(x)} - \frac{\text{var}(\mu)}{\text{var}(x)}\right) + \frac{\beta_{vy^*} \text{var}(y^*)}{\beta \text{var}(x)} \quad (20)$$

$$- \frac{\beta_{v\varepsilon} \text{var}(\varepsilon)}{\beta \text{var}(x)} + \beta_{v\mu} \frac{\text{var}(\mu)}{\text{var}(x)} + \beta_{\varepsilon\mu} \frac{\text{var}(\mu)}{\text{var}(x)} \quad (21)$$

$$= \beta\left(1 - \beta_{\mu x^*} \frac{\text{var}(x^*)}{\text{var}(x)}\right) + \frac{\beta_{vy^*} \text{var}(y^*)}{\beta \text{var}(x)} - \frac{\beta_{v\varepsilon} \text{var}(\varepsilon)}{\beta \text{var}(x)} \quad (22)$$

$$+ [\beta_{v\mu} + \beta_{\varepsilon\mu} - \beta] \frac{\text{var}(\mu)}{\text{var}(x)} \quad (23)$$

$$= \beta\left(1 - \beta_{\mu x^*} \frac{\text{var}(x^*)}{\text{var}(x)}\right) + \frac{\beta_{vy^*} \text{var}(y^*)}{\beta \text{var}(x)} - \frac{\beta_{v\varepsilon} \text{var}(\varepsilon)}{\beta \text{var}(x)} \quad (24)$$

$$+ [\beta_{v\mu} + \beta_{\varepsilon\mu} - \beta] \frac{\text{var}(\mu)}{\text{var}(x)} \quad (25)$$

This expression can be simplified by recognizing that  $\text{var}(y^*) =$

$\beta^2 \text{var}(x^*) + \text{var}(\varepsilon)$ . Substituting this into 20 yields:

$$\beta_{yx} = \beta \left(1 - \beta_{\mu x^*} \frac{\text{var}(x^*)}{\text{var}(x)}\right) + \frac{\beta_{vy^*} \beta^2 \text{var}(x^*) + \text{var}(\varepsilon)}{\beta \text{var}(x)} \quad (26)$$

$$- \frac{\beta_{v\varepsilon} \text{var}(\varepsilon)}{\beta \text{var}(x)} + [\beta_{v\mu} + \beta_{\varepsilon\mu} - \beta] \frac{\text{var}(\mu)}{\text{var}(x)} \quad (27)$$

$$= \beta \left(1 - \beta_{\mu x^*} \frac{\text{var}(x^*)}{\text{var}(x)}\right) + \beta \beta_{vy^*} \frac{\text{var}(x^*)}{\text{var}(x)} + \frac{\beta_{vy^*} \text{var}(\varepsilon)}{\beta \text{var}(x)} \quad (28)$$

$$- \frac{\beta_{v\varepsilon} \text{var}(\varepsilon)}{\beta \text{var}(x)} + [\beta_{v\mu} + \beta_{\varepsilon\mu} - \beta] \frac{\text{var}(\mu)}{\text{var}(x)} \quad (29)$$

$$= \beta \left(1 + \left\{ (\beta_{vy^*} - \beta_{\mu x^*}) \frac{\text{var}(x^*)}{\text{var}(x)} \right\}\right) \quad (30)$$

$$+ \left\{ (\beta_{vy^*} - \beta_{v\varepsilon}) \frac{\text{var}(\varepsilon)}{\beta \text{var}(x)} \right\} + \left\{ [\beta_{v\mu} + \beta_{\varepsilon\mu} - \beta] \frac{\text{var}(\mu)}{\text{var}(x)} \right\}$$

which is equation 8. This gives  $\beta_{xy}$  in terms of  $\beta_{vy^*}, \beta_{\mu x^*}, \beta_{v\mu}, \beta_{v\varepsilon}$  and  $\beta_{\varepsilon\mu}$  the five parameters that capture the non-classical nature of the measurement error.

## A.2 Comparison with Bound

(Bound, Brown, and Mathiowitz 2001) considers measurement error in either  $y$  or  $x$ , but not both. Since there is only one source of measurement error  $\beta_{v\mu} = 0$ . (Bound, Brown, and Mathiowitz 2001) further assumes that  $\beta_{\varepsilon\mu} = \beta_{v\varepsilon} = 0$  Equation 16a, therefore simplifies to

$$\beta_{yx} = \beta(1 - \beta_{\mu x}) + \beta_{vx} \quad (32a)$$

Likewise equation 18 simplifies to

$$\beta_{vx} = \frac{\beta_{vy^*} \text{var}(y^*)}{\beta \text{var}(x)} \quad (33)$$

and equation 8 reduces to

$$\beta_{yx} = \beta \left(1 + (\beta_{vy^*} - \beta_{\mu x^*}) \frac{\text{var}(x^*)}{\text{var}(x)}\right) + \beta_{vy^*} \frac{\text{var}(\varepsilon)}{\beta \text{var}(x)} - \beta \frac{\text{var}(\mu)}{\text{var}(x)} \quad (34)$$

If there is only measurement error in  $x$  and this measurement error is allowed to depend on  $x^*$  but is assumed to be independent

of  $y^*$  (and independent of  $\varepsilon$ ) then equation 34 simplifies to

$$\beta_{yx} = \beta(1 - \beta_{\mu x^*}) \frac{\text{var}(x^*)}{\text{var}(x)} - \beta \frac{\text{var}(\mu)}{\text{var}(x)} \quad (35)$$

$$= \beta(1 - \beta_{\mu x}) \quad (36)$$

which matches (Bound, Brown, and Mathiowitz 2001) p3713.

While (Bound, Brown, and Mathiowitz 2001) does not present an explicit expression for the impact of non-classical measurement error in  $y$  they conclude that mean reverting measurement error ( $\beta_{vy^*} < 0$ ) leads to downward bias in estimates of  $\beta$ . This conclusion is consistent with 34. Let  $\beta_{\mu x^*} = 0$ . so

$$\beta_{yx} = \beta(1 + \beta_{vy^*} \frac{\text{var}(x^*)}{\text{var}(x)}) + \beta_{vy^*} \frac{\text{var}(\varepsilon)}{\beta \text{var}(x)} - \beta \frac{\text{var}(\mu)}{\text{var}(x)} \quad (37)$$

$$= \beta(1 + \beta_{vy^*} \left( \frac{\text{var}(x^*)}{\text{var}(x)} + \frac{\text{var}(\varepsilon)}{\beta \text{var}(x)} \right)) - \beta \frac{\text{var}(\mu)}{\text{var}(x)} \quad (38)$$

Since  $x = x^*$  then  $\text{var}(x^*) = \text{var}(x)$  and  $\text{var}(\mu) = 0$ , so

$$\beta_{yx} = \beta \left[ 1 + \beta_{vy^*} \left( 1 + \frac{\text{var}(\varepsilon)}{\beta \text{var}(x)} \right) \right] \quad (39)$$

which is consistent with the conclusion in (Bound, Brown, and Mathiowitz 2001) that mean reverting measurement error ( $\beta_{vy^*} < 0$ ) leads to downward bias.

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**Table 1: Descriptive Statistics**

	<b>All</b>	<b>Non-imputed</b>
<b>black</b>	0.086	0.060
<b>hispanic</b>	0.087	0.094
<b>age</b>	41.6	41.3
<b>education</b>		
<b>&lt;12</b>	8.4	8.7
<b>12-15</b>	61.1	60.1
<b>16+</b>	30.5	31.1
<b>hours</b>	2215	2231
<b>week</b>	50.6	50.8
<b>imputed earnings</b>	0.309	0.000
<b>obs</b>	3742	2587

Note:

(1) 1996 SIPP Panel

**Table 2: Mean and Variance of  
Log Earnings and Measurement Error**

	<b>Mean</b>	<b>Variance</b>	<b>obs</b>
<b><u>Log earnings</u></b>			
<b>DER</b>	10.53	0.68	
<b>SIPP</b>			
<b>All</b>	10.38	0.56	3742
<b>Non-imputed</b>	10.42	0.53	2587
<b><u>Measurement Error</u></b>			
<b>All</b>	-0.15	0.34	3742
<b>Non-imputed</b>	-0.12	0.26	2587
<b><u>Signal to noise ratio</u></b>			
<b>All</b>		1.99	
<b>Non-imputed</b>		2.64	
<b><u>Reliability ratio</u></b>			
<b>All</b>		0.67	
<b>Non-imputed</b>		0.73	

Note:

(1) SIPP 1996 Panel

(3) Signal to noise ratio= $\text{var}(\text{DER})/\text{var}(\text{error})$

(2) Reliability ratio= $\text{var}(\text{DER})/[\text{var}(\text{DER})+\text{var}(\text{error})]$

**Table 3: Structure of measurement error**

	<b>All</b>	<b>Non-imputed</b>
$\beta_{vy^*}$	-0.339	-0.284
	( 0.010 )	( 0.011 )
	[ 0.480 ]	[ 0.465 ]
$\beta_{\mu x^*}$	-0.292	-0.243
	( 0.010 )	( 0.012 )
	[ 0.439 ]	[ 0.156 ]
$\beta_{ve}$	-0.550	-0.533
	( 0.019 )	( 0.021 )
	[ 0.428 ]	[ 0.477 ]
$\beta_{v\mu}$	0.540	0.482
	( 0.016 )	( 0.019 )
	[ 0.487 ]	[ 0.460 ]
$\beta_{\mu\mu}$	0.198	0.21
	( 0.014 )	( 0.019 )
	[ 0.230 ]	[ 0.224 ]

Note:

- (1) 1996 SIPP Panel
- (2) Standard error in parentheses
- (3) Correlation in brackets



**Table 4: Elasticity and Correlation**

	$\beta$	<b>corr</b>
<b>DER</b>	0.868 ( 0.009 )	0.834
<b>SIPP</b>		
<b>All</b>	0.845 ( 0.009 )	0.831
<b>Non- imputed</b>	0.867 ( 0.011 )	0.865

Note:

(1) SIPP 1996 Panel

(2) Standard error in parentheses

**Table 5: Impact of Non-classical Measurement Error**

	<u>var (y)</u>	<u><math>\beta_{yx}</math></u>	<u>corr<sub>yx</sub></u>
<b>DER</b>	0.684	0.868	0.834
<b>SIPP</b>	0.563	0.845	0.831
<b><u>Counterfactual</u></b>			
<b>Classical</b>	1.027	0.433	0.408
<b>Non-classical</b>			
<b><math>(\beta_{v\mu} + \beta_{\varepsilon\mu}) \neq 0</math></b>	1.027	0.813	0.766
<b><math>(\beta_{vy*} - \beta_{v\varepsilon}) \neq 0</math></b>	1.027	0.892	0.840
<b><math>(\beta_{vy*} - \beta_{\mu x*}) \neq 0</math></b>	0.563	0.845	0.831

Note:

(1) 1996 SIPP Panel