Mis-match, Re-match, and Investment

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Very Preliminary
This Version: January 4, 2009

Abstract

In markets where the pattern of matching matters for total surplus, restrictions on side payments that can be made to partners (non-transferable utility) will typically induce inefficient assignments. This provides a possible justification for "associational redistribution": a social planner who could enforce a match that differs from the market outcome may raise aggregate social surplus. Policy remedies to overcome this static inefficiency are complicated by dynamic incentive effects when individuals' productive types result from investments made before they match. In contrast to transferable utility models, which always have an efficient equilibrium, investments will typically be distorted; this occurs despite symmetric information about agents' characteristics. Moreover, if investment itself takes place in a matching environment (e.g. schools), the effects can be exacerbated. We study schooling and labor market policies that have empirical counterparts, assessing the differential effects of early-stage and later-stage policies on education choice, inequality, and exclusion.

1 Introduction

Many economic decisions are undertaken in settings where private and social payoffs depend not only on own characteristics but also those one cooperates

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with. This holds in particular for environments such as teams, firms, or schools where an individual's productivity is affected by those of his peers. Moreover, an agent's productive attributes may not be exogenous but subject instead to choices made before sorting occurs. Hence, allocations and policies have to be evaluated in terms of both the static efficiency of output generation and the dynamic incentives they set for agents' investment choices.

In competitive environments with perfect information, perfectly transferable utility, and no widespread externalities, policy has a limited role to play: a (near-)efficient allocation is always in the equilibrium set (Cole et al., 2001, Felli and Roberts, 2002).

There may be reason for concern, however, when there is matching market failure, for instance due to asymmetric information about other agents' attributes, search frictions or widespread externalities. Market failures such as these may generate inefficient levels of output and investment or undesirable levels of inequality. The latter in particular has been cited as a justification for policy intervention that directly interferes with sorting outcome, that is *associational redistribution* (see Coate and Loury, 1993, Durlauf, 1996a, who seems to have coined the term).

This paper emphasizes another source of inefficient matching, namely limited transferability between matched partners. Limited transferability has many causes: incentive problems with worker effort or commitment problems in distributing profit shares are just two. Often a significant benefit from a work environment takes the form of reputation or enhanced future earnings, and these cannot be divided arbitrarily. Other reasons not to expect utility to be fully transferable are legal constraints on output sharing, risk aversion, or "behavioral" considerations.

At least since Becker (1973), it is known that when payoffs are not fully transferable, the equilibrium match may fail to maximize social surplus. Thus nontransferability is a possible reason why associational redistribution might be desirable. Often, peer effects will matter not only in the labor market but earlier on, when investments not undertaken in solitude but in social environments and peers' attributes matter. Thus policy intervention both at early and late stages might be justified, and this raises the question of how to optimally time such interventions.

The setup we employ to analyze various forms of associational redistribution is as follows. Agents are characterized by a binary type reflecting whether they are privileged in terms of access to education or not. Agents choose an environment for investing in education which can be integrated (heterogenous) or segregated (homogeneous). When investing in education they face a fixed cost that depends on the investment environment.

Education investment determines the probability of a high education outcome. On the labor market agents match into firms whose output depends on members' education outcomes. The production technology is such that heterogeneity in firms is more productive, and would be the outcome under unrestricted side payments. We model nontransferable utility in the simplest possible way: output must be shared equally within firms. The result is segregation by educational achievement in the labor market. Thus, the equilibrium outcome is inefficient from the point of view of aggregate surplus.[1]

We then go on to evaluate several associational redistribution policies that have empirical counterparts. When sorting in education is inefficient on the labor market an immediate remedy is an *achievement based* policy that re-matches agents based on educational attainment.

But when evaluating such an achievement based policy in relation to the laissez-faire outcome in a dynamic model with investments, a trade-off emerges. On the one hand, an achievement based policy augments output through re-sorting, but on the other it depresses investments by rewarding a low education outcome with a chance of obtaining a good match. The adverse incentive effect may be partially mitigated by a re-matching policy that conditions not on results of choices but on exogenous information correlated with education outcomes, such as agents' personal backgrounds.

Such a *background based* policy is most effective and dominates an achievement based policy when the desired re-matching of educational outcomes can be replicated by a match in terms of backgrounds. This is most likely to occur when underprivileged agents are abundant. On the other hand, in economies where the underprivileged are minorities, both achievement based

[1]Unless accompanied by compensation, that is monetary transfers, associational redistribution typically does not yield Pareto improvements. In a nontransferable utility framework the use of side payments is severely limited. Nevertheless we evaluate allocations in terms of aggregate surplus. This can be defended on the grounds of taking an ex ante perspective before agents' types have realized, a view that e.g. potential parents may take when voting on educational policy.

and background based policies could be desirable.

When agents invest in social environments where peers' attributes matter for own choices in affecting payoff also associational redistribution at the investment stage may potentially improve aggregate surplus. In our model, such a *school integration* policy reduces segregation of schools with respect to background. While this policy serves to extend access to education it does not further interfere with a laissez-faire labor market allocation. Hence, school integration is beneficial if it is cost efficient at the schooling stage.

One can also ask how policies introduced in the labor market *and* the investment stage interact. When there is a labor market policy in place, school integration may affect investment incentives beyond reducing exclusion. If an achievement based policy is in place on the labor market school integration further erodes investment incentives since the chance of obtaining a good match when not investing increases. Indeed, if laissez-faire is better than an achievement based policy when schools segregate, this is also the case when schools integrate. School integration when a background based policy is used on the labor market reduces the informational content of students' backgrounds for labor market re-matching. This effect is so strong that when schools integrate allocations under laissez-faire and a background based policy on the labor market coincide. In short, school integration is most likely a substitute rather than a complement to labor market re-matching policy.

Finally, we consider a policy that re-matches the labor market conditioning on agents' choices of investment environments. Such a *club based* policy is sometimes used in regulating university access by assigning quotas to high-schools, or neighborhoods. This policy turns out to be remarkably successful in trading off output and incentive provision effects both on the labor market and at the investment stage. A club based policy rewards agents who integrate at the investment stage by letting them choose their match on the labor market, thus maintaining high investment incentives and ensuring some integration at school. Additionally, the policy induces some integration also on the labor market, since to satisfy incentive compatibility in a sorting equilibrium at the investment stage positive measures of the two segregated and the integrated types environments are needed.

The literature on school and neighborhood choice (see among others Bénabou, 1993, 1996, Epple and Romano, 1998) typically finds too much segregation in types. This may be due to market power (see e.g. Board, 2008) or widespread externalities (see also Durlauf, 1996b, Fernández and Rogerson, 2001). Supposing that attributes are fixed aggregate surplus may be raised by an adequate policy of bribing some individuals to migrate, an argument also put forward by de Bartolome (1990). Fernández and Galí (1999) compare matching market allocations of school choice with those generated by tournaments and find that the latter may dominate in terms of aggregate surplus when capital markets are imperfect leading to nontransferabilities. They do not consider investments before the match.

Peters and Siow (2002) present a model where agents invest in attributes before matching on a marriage market. They use a strictly nontransferable utility setup, similar to our labor market framework, and find that allocations are constrained Pareto optimal (with the production technology they study, aggregate surplus is also maximized). They do not discuss policy nor do they allow for peer effects in the investment environment. Gall et al. (2006) analyze the impact of timing of investment on allocative efficiency. Finally, a number of recent studies considers investments before matching in the presence of asymmetric information (see e.g. Bidner, 2008, Hopkins, 2005, Hoppe et al., 2008). These studies tend to focus on the amount of wasteful signalling and do not consider policies of associational redistribution.

The paper proceeds as follows. Section 2 lays out the labor market and discusses effectiveness of policies of associational redistribution in terms of sorting, incentives, and exclusion. Section 3 presents the schooling stage and a policy of school integration. Section 4 considers effectiveness of policies at both schooling stage and labor market and introduces club based policies. Section 5 concludes, and the appendix contains the more tedious calculations.

2 A Labor Market

A labor market is populated by a continuum of agents I with unit measure. Agents are characterized by their educational attainment a which is either high h or low ℓ. Denote the measure of h agents by $q \in [0, 1]$. On the labor market agents match into firms of size two and jointly produce output. Profit y in a firm depends on agents' education outcomes. Assume that

$$y(\ell, \ell) < y(\ell, h) = y(h, \ell) < y(h, h).$$

For most of the paper profits in firms have decreasing differences, that is

$$2y(h,\ell) > y(h,h) + y(\ell,\ell).$$

This is best motivated by a technology that combines two different tasks, one human capital intensive and one less so, say engineering and design versus actual manufacturing. Kremer and Maskin (1996) propose a production function that has this feature. Another possible reason for decreasing differences is diversification between two tasks, or contractual frictions, for instance if cost of capital or information rents are decreasing in the scope of the project. Denote by $w(a,a')$ the wage of an agent with educational attainment a when matching with an agent whose educational attainment is a'. Wages are positive and sum up to the firm's profit, $w(a,a') + w(a',a) = y(a,a')$. Agents derive utility from wage income.

To provide a benchmark solve now for the competitive labor market equilibrium, that is a stable match of agents into firms. With decreasing differences there exist wages $w(h,\ell) \geq 0$ and $w(\ell,h) \geq 0$ with $w(h,\ell) + w(h,\ell) = y(h,\ell)$ such that

$$w(h,\ell) \geq y(h,h)/2 \text{ and } w(\ell,h) > w(\ell,\ell)/2.$$

This implies that given wages $w(.)$ there is no distribution of profits in segregated firms such that agents in integrated firms were better off forming a segregated firm. Hence, in labor market equilibrium measure $\min\{q, 1-q\}$ of integrated firms emerge, the remainder segregates. Market wages are determined by scarcity, that is $w(h,\ell) = y(h,h)/2$ if $q > 1/2$, $w(\ell,h) = y(\ell,\ell)/2$ if $q < 1/2$, and $w(h,\ell) \in [y(h,h)/2, y(h,\ell) - y(\ell,\ell)/2]$ if $q = 1/2$.

2.1 Nontransferable Utility

The example above tacitly assumed that utility was perfectly transferable on the labor market. This means agents can contract on the distribution of profits within a firm without affecting productive efficiency, that is the size of the profits. There are a number of plausible reasons of why this assumption may be violated in applications. Lack of access to or imperfections on the credit market, limited liability and moral hazard within the firm are one reason not to expect perfectly transferable utility. Others are incomplete contracts and renegotiation, risk aversion, legal constraints and regulation, or behavioral concerns.

To facilitate exposition assume an extreme case of non-transferabilities, that is strictly nontransferable utility. This means only a single vector of payoffs to agents is feasible in any firm.[2] To keep notation bearable assume that profits are shared equally in firms, that is

$$y(h,h) = 2w_{hh}, y(h,\ell) = y(\ell,h) = 2w_{h\ell} = 2w_{\ell h}, y(\ell,\ell) = 2w_{\ell\ell} = 0.$$

Abbreviate $W = w_{hh}$ and $w = w_{h\ell}$ and assume that

$$W < 1.$$

This ensures that wages are typically bounded above by 1 which permits interpretation of investments induced by expected wages as probabilities.

When utility is nontransferable the equilibrium labor market allocation looks quite different. Despite decreasing differences of firm profits integration is no longer possible in equilibrium. Suppose that a positive measure of (h,ℓ) firms form and h agents obtain wage w. Then any two h agents have a profitable deviation by starting a (h,h) firm earning W each, a contradiction to stability. Hence, under strictly nontransferable utility only homogeneous firms emerge.

As long as there are positive measures of high and low types (i.e. $0 < q < 1$), and there is a diversity benefit in production ($2w > W$), aggregate surplus is strictly lower when utility is nontransferable. For instance, if $q < 1/2$, surplus is $2qw$ if utility is transferable, while it is qW if not; if $q > 1/2$, surplus is $(1-q)2w + (2q-1)W$ if utility is transferable, and qW if not.

Nontransferability of utility may therefore distort the matching pattern and reduce aggregate surplus (see Legros and Newman, 2007). Indeed this seems to provide a powerful justification for associational redistribution on the labor market when transferability of utility is severely impeded. Consider a policy of associational redistribution that assigns h agents to ℓ agents whenever possible. Any remaining agents match into homogeneous firms

[2]While this may be motivated for instance by Nash bargaining in renegotiations within the firm, all results in the paper are robust to allowing for some transferability by letting wages vary around equal sharing by some amount small enough.

using uniform rationing. Call this an *achievement based* policy. This policy replicates the matching pattern under transferable utility and achieves an increase in aggregate surplus for any exogenously given distribution of educational attainments, as measured by q in our example.

Significant parts of active labor market policy are interpretable as achievement based policies, e.g. employment subsidies, in particular those payable to employers. By targeting the long term unemployed or unemployed youth such policy effectively re-sorts the labor market conditioning on educational achievements or rather lack thereof. Most industrialized countries use some variety of wage subsidy or workfare programs: in the US this was the Targeted Jobs Tax Credit (TJTC) and later on the Work Opportunity Tax Credit (WOTC) , Germany introduced integration subsidies as part of the Hartz policy reform, the UK introduced wage subsidies as part of the New Deal for Young People, and France uses payroll tax subsidies for minimum wage labor contracts and wage subsidies for the unemployed young.

2.2 Education Investments

Educational attainments, and agents' attributes on markets in general, often result from individual choice, however. In this case q is not exogenous, and welfare consequences of a policy forcing rematches on the labor market are more subtle, since agents' incentives for education investment depend on the labor market wages which in turn may depend on whether utility is transferable or not. That is, distortions on the labor market due to nontransferable utility may affect education formation through both price and pattern.

Suppose therefore that a measure $s \in [0, 1]$ of agents have the opportunity to invest in education. s is best understood as the fraction of the population with access to schooling. Let s be given exogenously for the moment. When investing agents exert effort $e \in [0, 1]$ to acquire education. Specifically, spending effort e yields a high education outcome h with probability e and a low education outcome ℓ with probability $(1 - e)$. Exerting effort e comes at a utility cost, $e^2/2$. The measure of high achievers, i.e. agents with education h, q, is now endogenous and given by $q = se$.

Let $w(h)$ and $w(\ell)$ be the expected wages of a high and a low achiever. Then, at the margin, investment incentives are given by $e = w(h) - w(\ell)$.

Under strictly nontransferable utility the labor market segregates as derived above, and therefore $w(h) = W, w(\ell) = 0$ implying that $e = W$.

When utility is perfectly transferable investment incentives depend on whether q is anticipated to be greater or lower than $1/2$. If $q > 1/2$, then being a high achiever offers a chance of being matched with a low or a high achiever (since high achievers are in excess supply). As a pair of high achievers obtain $w(h) = W$ each, this is also what they get when matched with a low achiever,[3] who gets the residual $w(\ell) = 2w - W$. Therefore investments are $e = 2(W - w)$. If $q < 1/2$, low achievers are in excess supply, get a wage $w(\ell) = 0$, and high achiever match only with low achievers obtaining wage $w(h) = 2w$. Investments in this case are $e = \min\{2w; 1\}$, strictly greater than in the case $q > 1/2$. In equilibrium the anticipated q must coincide with its realization se, for instance, if $q > 1/2$, $e^{TU} = 2(W - w)$ and therefore we need that $s2(W - w) > 1/2$. We have the following result (all proofs missing from the text are in the appendix).

Lemma 1 *Let s be the measure of agents who have access to investment. Suppose that there is transferable utility.*

(i) If $W - w > \frac{1}{4s}$, $e^{TU} = 2(W - w)$ and $q > 1/2$,

(ii) If $W - w < \frac{1}{4s} < w$, $e^{TU} = \frac{1}{2s}$ and $q = 1/2$,

(iii) If $w < \frac{1}{4s}$, $e^{TU} = 2w$ and $q < 1/2$.

By Lemma 1 the social return from education, and therefore e^{TU}, decreases with q. Under nontransferable utility the private return from education is independent of q. Hence, given W, the difference in education investment under nontransferable (LF) and transferable utility (TU) increases in q. By the lemma, a high q regime is favored by larger s. It turns out that this comparative static is equivalent to the condition in the following corollary.

Corollary 1 *Comparing investment levels when utility is perfectly transferable and strictly nontransferable yields*

$$e^{LF} > e^{TU} \Leftrightarrow W > \frac{1}{2s}.$$

[3]There is indeed equal treatment on the labor market under transferability: if a high achiever gets strictly more than another high achiever, the latter can match with the partner of the former and contract for a wage that is slightly lower.

Since for a given s the measure of high achievers is proportional to the investment, the comparative statics result for the measure of high achievers q holds both under transferable and nontransferable utility.

2.3 Achievement Based Policy

Since mismatches due to nontransferable utility distort investment incentives, the case for associational redistribution seems even more compelling when the measure of high achievers is endogenous. This intuition is incomplete, however, because *given nontransferabilities on the labor market* enforcing the "correct" sorting may in fact worsen investment incentives.

For instance, recall that under transferable utility the labor market wage adjusts as to provide the long market side with its autarky payoff (i.e. W for high and 0 for low achievers). For instance, if low achievers are in excess supply they get a wage of zero and high achievers get a wage of $2w$. If utility is strictly nontransferable, however, low achievers must obtain a payoff greater than zero under an achievement based policy (i.e. when forcing integration), since they obtain w with a positive probability (induced by uniform rationing), while high achievers get w with certainty. Hence, investment incentives are weaker than under laissez-faire as the return is lower in the good state (high achiever) and higher in the bad state (low achiever).

Indeed, in any equilibrium under forced integration low achievers must be in excess supply. Suppose the contrary, $q > 1/2$. Then high achievers match into integrated firms with probability $\frac{1-q}{q}$ and get w, and into segregated firms with probability $\frac{2q-1}{q}$ obtaining W. Low achievers get w for sure. Investment is therefore $e = \frac{2q-1}{q}(W - w)$, and $es > 1/2$ by our assumption. It is not possible to satisfy both conditions simultaneously for any value of $s \in [0, 1]$, a fact established in the proof of the following proposition.

Proposition 1 *Under an achievement based policy the measure of educated agents is less than* $1/2$ *for any* $s \in [0, 1]$*. Investment in education is*

$$e^A = \frac{1 - 2q}{1 - q} w,$$

with $q = \frac{1}{2} - \left[\sqrt{s^2w^2 + \frac{1}{4}} - sw\right]$.

Clearly, $e^A < w < W = e^{LF}$. This means forcing integration on the labor market worsens investment incentives. But since firms produce more output under integration than under segregation, there is also a positive re-sorting effect on aggregate surplus. Whether a labor market policy based on achievement improves upon the laissez-faire allocation thus depends on whether the gain in output is large enough to offset investment distortions. Aggregate surplus under laissez-faire is $S^{LF} = sW^2/2$. An achievement based policy induces total surplus of

$$S^A = se^A \left(2w - \frac{e^A}{2}\right).$$

Therefore the achievement based policy improves on laissez-faire when (recall that $W = e^{LF}$),

$$\underbrace{e^{LF}(2w - W)}_{\text{output gain given LF effort}} > \underbrace{(e^{LF} - e^A)2w}_{\text{output loss given rematch}} \underbrace{-\frac{1}{2}((e^{LF})^2 - (e^A)^2)}_{\text{savings in costs}}. \quad (1)$$

The LHS captures the surplus added by integration under the achievement based policy keeping investment at its laissez-faire level. The RHS measures the effects on investment: a lower output given the rematch and a savings in cost due to lower incentives.

The LHS decreases with W. In the RHS, e^A is independent of W but the term $e^{LF}(2w - \frac{e^{LF}}{2}) = W(2w - \frac{W}{2})$ increases in W. Therefore, there exists a unique value of W for which condition (1) holds with equality. Let $W_0(w, s)$ be this cutoff value.

Corollary 2 *Total surplus under an achievement based policy is higher than under laissez-faire if and only if* $W \leq W_0(w, s)$.

In the appendix we show that $W_0(w, s) \geq 0$ increases in w and decreases in s, and that $W > \sqrt{3}w$ is a sufficient condition for $S^{LF} > S^A$.

Figure 1 depicts the cutoff $W_0(w, s)$ as a function of w that separates the areas LF and A when $s = 1$.

2.4 Background Based Policy

Although an achievement based policy may increase aggregate surplus compared to laissez-faire, this is always accompanied by a downward distortion

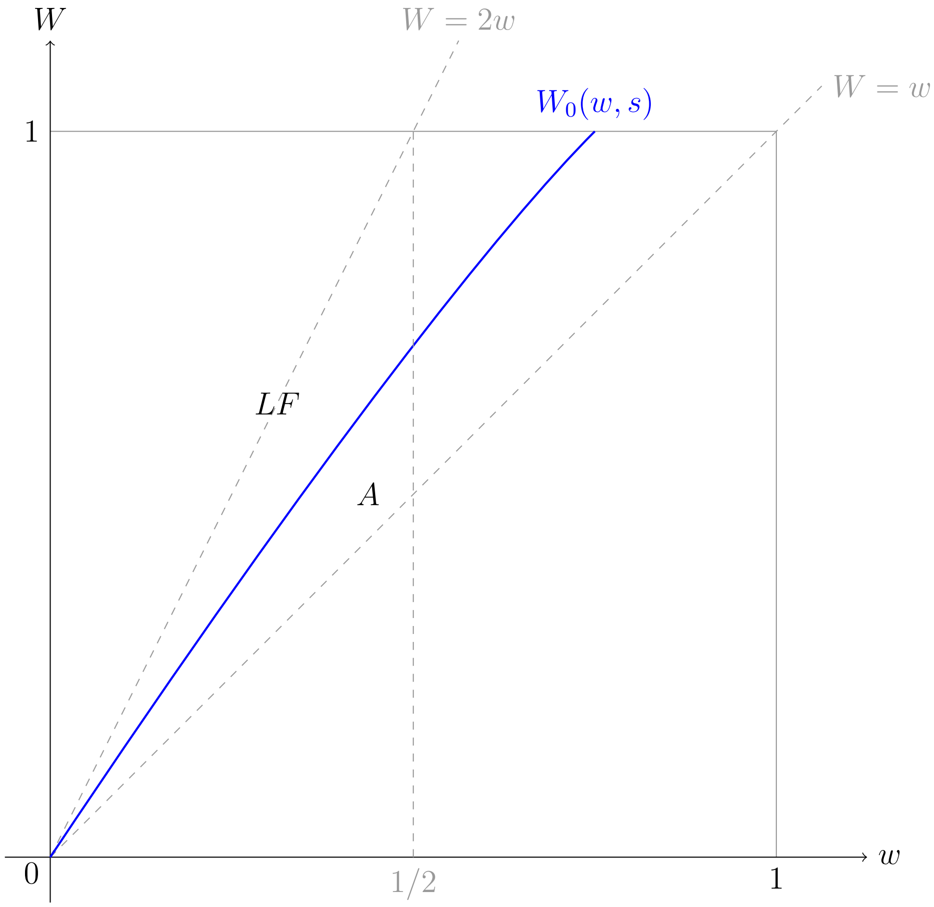


Figure 1: Laissez-faire versus Achievement Based Policies

of investment incentives. Specifically, for agents with access to education the wedge in payoffs between high and low achievement is less under an achievement based policy. A natural way to ameliorate investment distortions is therefore to decrease the low achievement payoff for agents with access to education by giving priority in integrated firms to agents without access to education (who become low achievers).

Suppose that agents can be identified by whether or not they have access to education. This may be the case when access depends on observable information such as socio-economic characteristics of agents' neighborhoods or parents. These characteristics can be thought of as determining whether agents are *privileged* in terms of access, or *underprivileged*. Hence, an agent i's *background*, or type, is $b_i \in \{U, P\}$. Consistent with the analysis above the measure of agents with background P is s, while the remainder $1-s$ has background U. A background based policy integrates as much as possible U and P agents, but otherwise lets the agents form firms as they like.

Consider first the case $s \leq 1/2$. Each U agent is matched with a P agent with probability $\frac{s}{1-s}$ and with a U agent with probability $\frac{1-2s}{1-s}$; agents of type P are matched with an agent of type U with probability one. There are therefore measure s of (U, P) matches and $\frac{1}{2} - s$ of (U, U) matches.

As under an achievement based policy a privileged agent with high achievement is matched into an integrated firm (h, ℓ) obtaining wage w with certainty. A privileged low achiever, however, has now probability zero of matching into a (h, ℓ) firm, since integration is in terms of background and underprivileged agents become low achievers. This reduces expected payoff of a privileged low achiever compared to an achievement based policy. Hence, when $s < 1/2$ a background based policy induces a redistribution towards underprivileged agents (having a higher chance to obtain wage w) and stronger incentives for privileged agents. Since the measure of firms (h, ℓ) is $se^B > se^A$, total output is greater under a background based policy.

This implies that when $s < 1/2$ a background based policy dominates an achievement based policy. Therefore it will yield a higher social surplus than under laissez-faire also in the neighborhood of the curve $W_0(w, s)$. The exact comparison with the laissez-faire allocation follows the same lines as above: laissez-faire induces better incentives but more inefficient matches, and it can be shown that there is a cutoff value $W_2(w, s) > W_0(w, s)$ such that laissez-faire and background base policies yield the same total surplus. When $s < 1/2$ this cutoff is independent of s and equal to $\sqrt{3}w$.

When $s > 1/2$, a privileged agent optimally invests

$$e^B = \underbrace{\frac{1-s}{s}w}_{\text{match with a } U \text{ agent}} + \underbrace{\frac{2s-1}{s}W}_{\text{match with a high achiever } P \text{ agent}} . \tag{2}$$

As $e^A < w$ we have $e^B > e^A$, and a background based policy induces redistribution towards the underprivileged and stronger incentives for the privileged as in the case above. Yet this does not imply that total surplus is higher under a background based policy. Since $s > 1/2$ some P agents must form (P, P) matches. A background based policy does not prescribe matches in terms of achievement so that these agents segregate as under laissez-faire.

Hence, there is a positive measure of firms (h, h) and (ℓ, ℓ), which is inefficient from an output point of view. That is, for $s > 1/2$ a background based policy induces better incentives and a worse matching than an achievement based policy. An argument similar to the one in Corollary 2 yields a cutoff $W_1(w, s)$ such that a policy based on background is preferable to one based on achievement if $W > W_1$ and the reverse is true if $W < W_0$. Since the amount of mismatch, i.e. the output inefficiency, increases in s so does $W_1(w, s)$ which also increases in w.

The comparison with laissez-faire in case $s > 1/2$ follows the same logic as in the previous section; a background based policy provides worse incentives but a more efficient matching than laissez-faire. The cutoff $W_2(w, s)$ such that both policies are surplus equivalent also increases in w and in s.

This discussion as well as some additional properties of the cutoff values are summarized in the following proposition, and illustrated in Figure 2.

Proposition 2 *There are functions $W_1(w, s)$, $W_2(w, s)$ with the properties*

- $2w \geq W_2(w, s) > W_0(w, s) > W_1(w, s) \geq w$,
- $W_1(w, s), W_2(w, s)$ *are increasing in w and in s,*
- $W_1(w, s) = w$ *for $s < 1/2$, and*
- $W_2(w, s) = \sqrt{3}w$ *for $s < 1/2$,* $\lim_{s \to 1} W_2(w, s) = 2w$,

such that the surplus maximizing policy is

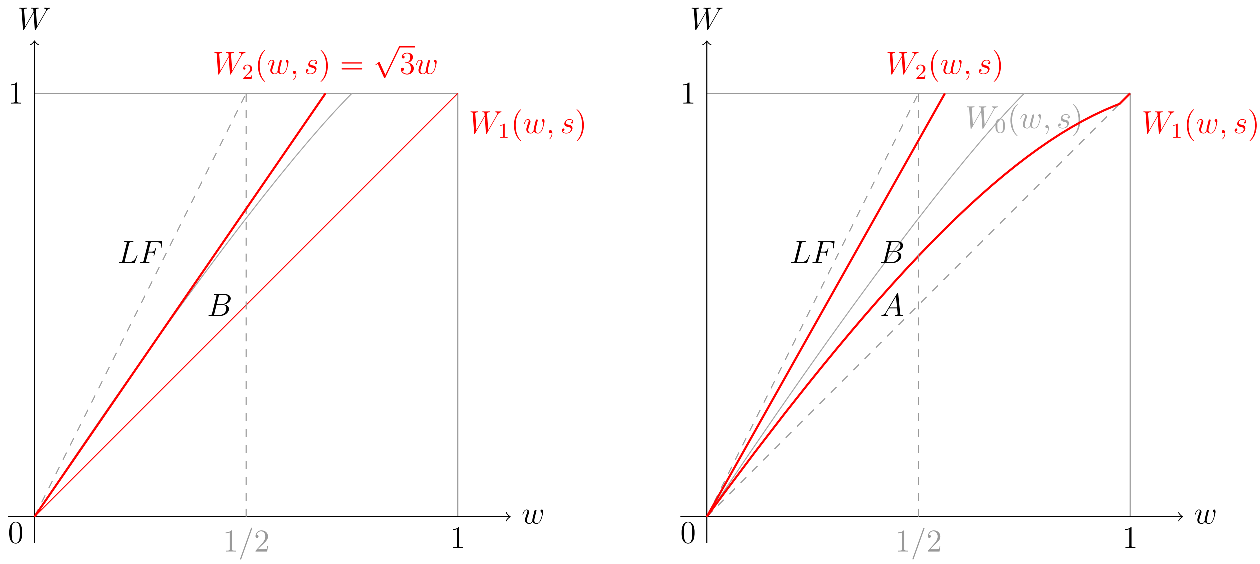
(i) Laissez-faire when $W \geq W_2(w, s)$,

(ii) Background based when $W \in (W_2(w, s), W_1(w, s))$, and

(iii) Achievement based when $W < W_1(w, s)$.

2.5 Access to Education

The preceding analysis assumed that the measure of agents who choose positive investment in education s is exogenous. Lack of access to education may be understood as a high fixed cost agents faced when acquiring education. In



(a) $s < 1/2$

(b) $s > 1/2$

Figure 2: Laissez-faire, Achievement, and Background Based Policies

this case individual returns to education determine the extent of exclusion, since higher returns from investment on the labor market may induce agents to participate in education. In this context downward distortion of investment incentives due to associational redistribution on the labor market may amplify exclusion from education.

For instance, suppose a measure π of agents have background P and face no access cost to education and the remaining measure $1 - \pi$ of agents have background U and incur cost $F > 0$ when investing $e > 0$. In the previous section by $s = \pi$ held always, but now possibly $s > \pi$ if the individual return to education outweighs the access cost F for the U agents.

Under laissez-faire individual payoff from strictly positive investment is $u^{LF} = W^2/2$ independently of s. Therefore, under transferable utility either $F > W^2/2$ and $s = \pi$, or $F \leq W^2/2$ and $s = 1$.

Under an achievement based policy individual payoff from a strictly positive investment depends on s through e^A and is $u^A(s) = (e^A(s))^2/2$.

Under a background based policy an agent's payoff is increasing in the investment of the agents with whom they are supposed to match. Suppose all agents invest. Supposing a symmetric equilibrium U agents solve

$$\max_{e} eW - \frac{e^2}{2} - F.$$

$e = W$ if $W^2/2 > F$ and 0 otherwise. P agents also optimally invest $e = W$ since they find a U agent with high achievement U with certainty if all agents invest. Hence, when $F \leq W^2/2$ there exists an equilibrium allocation under a background based policy such that $s = 1$ and $e^B(1) = W$ giving payoff $u^B(1) = (e^B(1))^2/2$. Note that there may exist other equilibria as well.[4]

Using these results and the solutions for investment and market wages from above reveals that for any value of $s \geq \pi$

$$u^A(s) < u^{TU}(s) \text{ and } u^A(s) < u^{LF} \text{ and } u^B(1) = u^{LF}.$$

Since U agents invest only if $u^j(s) \geq F$, we have the following result.

Proposition 3 *Suppose that U agents face a fixed cost F when choosing $e > 0$. Then the measure of agents who choose $e > 0$, i.e. participate in education acquisition, is greater under laissez-faire than under an achievement or background based policy.*

In case of transferable utility, if case (iii) of Lemma 1 applies at $s = 1$ then $u^{TU} > u^{LF}$, since $e^{LF} = W < e^{TU} = \min\{2w; 1\}$. Hence, for $F \in (W^2/2, (e^{TU})^2/2)$, access is $\pi < 1$ under laissez-faire while it is $s = 1$ under transferable utility. Suppose that we are now in case (i) of Lemma 1. Then, $u^{TU}(1) = 2w - W + 2(W - w)^2$ which is less than $W^2/2$ whenever $3W > 2w + 2$. Since $W < 1$ and $W < 2w$ this is not possible.

That is, participation in education under laissez-faire when utility is non-transferable is never greater than under transferable utility. Proposition 3 states that if U agents are excluded from education under laissez-faire, associational redistribution cannot help to reduce exclusion, even when higher participation would be socially beneficial. This raises the question of whether a social planner may want to facilitate access to education by targeting the

[4] To see this suppose $s = \pi$. Denote a U agent's optimal strictly positive investment by e_u. We show in the Appendix that $e_u < W$ independently of π. Hence, for $e_u \leq F \leq W^2/2$ another equilibrium exists under a background based policy such that $s = \pi$ and $e = e^B$ if $b = P$ and $e = 0$ if $b = U$.

cost of access to education directly, and how such a policy interacts with labor market policies.

To analyze these issues, we turn to a model of schooling where the cost of access depends on the way agents match in school, and where associational redistribution at the school level is the policy instrument used to reduce the access cost.

3 The Schooling Stage

Most education investments are taken not in solitude but rather in a social environment where the behavior of an agents' peers influences own behavior. This may be by way of social norms and role models, learning spill-overs in class, or pure cost externalities. For the purpose of modeling we focus on the last and assume that agents are heterogenous in cost of acquiring education, which depends on an agent's match at school. We focus on heterogeneity in cost of access to education rather than marginal cost of acquiring education. Whereas marginal cost of education may reflect individual ability, access cost captures an agent's socioeconomic background.

We generalize in a simple way the access problem of the previous section. Let $g(.)$ denote the fixed cost an agent incurs when investing in education and suppose it depends on that agent's schooling environment, or *club* (b, b'). Assume that

$$0 = g(P, P) < g(U, P) = g(P, U) = f < g(U, U) = F.$$

Suppose for the rest of this section that the labor market operates under laissez-faire. That is, the payoff for an agent in environment (b, b') is $\max\{W^2/2 - g(b, b'); 0\}$. This does not depend on the measure of agents with strictly positive investment s. This allows an analysis of allocation problems at the schooling stage independently of the labor market.

When utility is perfectly transferable, a U agent can compensate a P agent in a (U, P) environment for the increase in access cost to f. Integrated (U, P) environments are stable if the joint payoff exceeds the sum of segregation payoffs:

$$\max\{W^2 - 2f; 0\} \geq W^2/2.$$

That is, in the benchmark case of perfectly transferable utility there is measure $\min\{\pi; 1/2\}$ of (U, P) schooling environments, and $s = \min\{2\pi; 1\}$ if $4f < W^2$. If $4f > W^2$ schools segregate, i.e. the measure of (U, P) schooling environments is 0, and $s = \pi$.

Under strictly nontransferable utility payoffs from the labor market outcome induce higher utility for P agents when matching into a (P, P) than into a (U, P) environment whenever $f > 0$. Since utility is strictly nontransferable N cannot compensate P agents for this. Hence, an allocation with integrated (U, P) environments cannot be stable since any two P agents matched into (U, P) environments have a profitable deviation. That is, under laissez-faire schools segregate and surplus is $S^{LF} = \pi W^2/2$ as above.

A good example for the laissez-faire outcome is the labor market for physicians in the U.S. The well studied national residency matching program can be interpreted as a labor market that assigns students to residencies at hospitals whose qualities determine lifetime income. The matching program uses a version of the Gale-Shapley algorithm to approach a stable allocation not allowing for side payments making utility nontransferable. Education investments are undertaken in medical schools. In case one is interested in providing minimum health standards in all university hospitals, decreasing differences in the matching surplus provide an adequate formulation.

3.1 School Integration

That is, similar to the labor market the laissez-faire allocation at the schooling stage may fail to internalize positive externalities within schooling environment when utility is (sufficiently) nontransferable. This points to a beneficial role for associational redistribution at the school stage, in particular if such policy can condition on information on backgrounds that is not subject to individual choice. For instance, a policy of school integration that forces agents to invest in integrated (U, P) environments should raise aggregate surplus if bringing in U agents is cost efficient. A *school integration* policy matches U to P agents whenever possible using uniform rationing to assign the remaining agents to homogeneous school environments.

A prime example of a policy pursuing school integration is, of course, busing. More contemporaneously, there exists considerable international variation in policies determining the degree of integration of schools in terms of pupils' background. One indicator of this is the age at which pupils are first sorted into a particular ability stratum, a policy called tracking. This age ranges from 10 in Austria and Germany to 16 and above in the U.S. or most of Scandinavia (see Table 5.20, OECD, 2004).

When the labor market operates under laissez-faire investments are $e^{LF} = W$ by agents in (P,P) environments and $e = 0$ by agents in (U,U) environments. That is, $s = \pi$. Under school integration there are measure 2π of agents in (U,P) schools, and measure $1-2\pi$ in (U,U) schools if $\pi \leq 1/2$; otherwise measure $1-\pi$ of agents are matched into (U,P) schools, and measure $2\pi - 1$ in (P,P) schools. Agents in (U,P) schools invest e^{LF} if $W^2 > 2f$ and $e = 0$ if $W^2 < 2f$.[5] hence, aggregate surplus under school integration is

$$\begin{aligned} S^{SI} &= \min\{2\pi;1\}(W^2/2 - f) + \max\{2\pi - 1;0\}W^2/2 \text{ if } W^2 \geq 2f \text{ and} \\ S^{SI} &= \max\{2\pi - 1;0\}W^2/2 \text{ if } W^2 < 2f. \end{aligned}$$

Therefore $S^{LF} > S^{SI} \Leftrightarrow W^2 < 4f$. This does not depend on the assumption of decreasing differences ($W < 2w$). Hence, school integration may restore the benchmark allocation under fully transferable utility when $4f < W^2$.

To give a specific example, Meghir and Palme (2005) analyze effects of a schooling reform in Sweden that was implemented around 1950. The reform increased compulsory schooling by three years, abolished tracking after grade 6, and imposed a nationally unified curriculum. That is, the policy aimed at decreasing school segregation in backgrounds. It turns out that the policy change increased education acquisition (beyond the new compulsory level for highly able pupils) and labor income for individuals whose fathers had low education, while it did not significantly change education acquisition and lowered wage income for individuals whose father had high education.[6]

[5] If $\pi < 1/2$ and $w^2 < 2f$ there may be other equilibria where nobody invests.

[6] Segregation at school may not only apply to sorting of students. Teachers may share a preference for safe schools and motivated students, possibly to an extent that cannot be compensated by public salaries (see Hanushek et al., 2004).

4 Combining Early and Later Stage Policies

A question potentially important for policy concerns is whether effectiveness of school integration depends on the labor market policy in place. Hence, we are interested in whether associational redistribution on the labor market and at the school stage may act as complements or substitutes, that is whether they reinforce or cancel each other. Two major concerns may arise when evaluating the impact of simultaneous earlier and later stage policies. On the one hand, school integration raises the access cost of P agents which may lead to discouragement due the investment distortion under an achievement based policy. On the other hand, integrating schools dilutes the informativeness of background for educational outcome thus reducing the effectiveness of a background based policy.

In the following we limit our attention to cases that satisfy some parametrical assumptions on access cost.

Assumption 1 (Access Cost) *Let* $f < (W - w)^2 < W^2/2 < F < 2w^2$.

This assumption ensures that f agents always find it profitable to invest when matching into integrated firms, and that high fixed cost F agents find it optimal to invest when paid the full social benefit of turning a (ℓ, ℓ) firm into a (h, ℓ) firm.

4.1 Fully Transferable Utility Benchmark

We start by deriving the fully transferable utility benchmark allocation. The market wage for h agents is $w(h) \in [W, 2w]$ depending on the scarcity of h versus ℓ agents. Investment is $e^{TU} = 2(w(h) - w)$ giving payoff $2(w(h) - w)^2 + 2w - w(h) - g(b, b')$. Since utility is transferable, U agents may compensate P agents for lowering their access cost. Integrated (U, P) environments are stable if the joint payoff exceeds the sum of segregation payoffs:

$$\begin{aligned} (w(h) - w)^2 &> f \text{ if } F > 2(w(h) - w)^2 + 2w - w(h) \text{ and} \\ F &> 2f \text{ if } F < 2(w(h) - w)^2 + 2w - w(h). \end{aligned}$$

That is, given Assumption 1 there is integration both on the labor market and at the schooling stage when utility is perfectly transferable. For investments two interesting cases arise as stated in the following proposition.

Proposition 4 *Suppose Assumption 1 holds. When utility is fully transferable both schools and the labor market integrate. Moreover,*

(i) if $\min\{2\pi; 1\}2(W - w) > 1/2$, $s = \min\{2\pi; 1\}$, *investments are* $e = 2(W - w)$ *and* $q > 1/2$,

(ii) otherwise $q = \min\{2w; 1/2\}$ *and*

- *if* $2w > 1/2$ *and* $\pi \geq 1/2$, $s = 1$ *and investments are* $e = 1/2$,
- *if* $2w > 1/2$ *and* $\pi < 1/2$, $s = 2\pi + \max\{0; 1/2 - 2\pi\sqrt{2F}\}$ *and investments are* $e = \min\{\frac{1}{4\pi}; \sqrt{2F}\}$.
- *if* $2w \leq 1/2$, $s = 1$ *and investments are* $e = 2w$.

Proof: In Appendix.

In case (i) social returns to education are high enough for $q > 1/2$ when all agents in (U, P) and (P, P) schools invest, while (U, U) agents do not. In case (ii) social returns are high enough to induce all agents, even those in (U, U) schools, to invest when $q < 1/2$, but not when $q > 1/2$. Hence, $q = 2w$ if all agents invest but $w < 1/4$. Otherwise $q = 1/2$ and the market price may adjust to make (U, U) agents indifferent between investing or not.

That is, if the measure of P agents and the value added in (h, h) firms, $W - w$, are sufficiently great, high access cost U agents under-invest in education and low access cost P agents over-invest under laissez-faire compared to the benchmark allocation. Otherwise all agents under-invest. This means the market friction at work in this model, namely nontransferable utility, varies in its effect with the characteristics of the economy. Abundance of low access cost P agents in conjunction with a technology that does not value unskilled labor input best describes an industrialized country, whereas the reverse seems true in developing economies. Maintaining this interpretation, our results indicate that non-transferable utility exacerbates inequality of opportunity in industrialized countries by discouraging high access cost agents, while the discouragement effect is universal for developing economies.

4.2 Achievement Based Policy and School Integration

Suppose now that an achievement based policy is used on the labor market in conjunction with integration at school. Recall that the surplus under a achievement based policy net of access cost is given by

$$S^A = se^A\left(2w - \frac{e^A}{2}\right),$$

where $s = \pi$ when schools segregate. S^A increases in the measure s of agents investing $e > 0$. Aggregate surplus under school integration is $S^{SI} = \min\{2\pi; 1\}W^2/2 - \min\{\pi; 1-\pi\}2f$. That is, $S^{SI} > S^A$ if

$$\frac{W^2}{\min\{2\pi; 1\}} - \min\left\{\frac{1-\pi}{\pi}; 1\right\}2f > e^A\left(2w - \frac{e^A}{2}\right). \tag{3}$$

Since this condition is monotone in W,

$$S^{SI} > S^A \Leftrightarrow W > W_3(w, \pi, f),$$

where $W_3(w, \pi, f)$ increases in w and f, and decreases in π for $\pi \leq 1/2$, see appendix for details.

Consider now a policy of achievement based associational redistribution and school integration. When choosing education investment an agent solves

$$\max_e e\frac{1-2q}{1-q}w - \frac{e^2}{2} + \frac{q}{1-q}w - g(b, b').$$

This yields a necessary condition

$$e^{AI} = \frac{1-2q}{1-q}w. \tag{4}$$

e^{AI} depends on s as $q = se^{AI}$. Note that $e^{AI}(s) = e^A(s)$, but $s^A = \pi$ whereas s^{AI} is endogenous. An agent in a (U, P) environment invests if

$$\left(\frac{1-2q}{1-q}\right)^2\frac{w^2}{2} > f.$$

Investment of agents in (U, P) environments depends on q and determines s, which in turn affects q. Two cases may arise: either f is small enough to induce investment by agents in (U, P) environments, or investment by agents in (U, P) environments is discouraged. The following proposition states this.

Proposition 5 *Under school integration and an achievement based policy on the labor market,* *(i) in case* $\sqrt{2f} < e^A(\pi)$, $e^{AI} < e^A$, $q^{AI} > q^A$, $s^{AI} = \min\{2\pi; 1\} > s^A$, *and* $S^{AI} > S^A$ *if* $\pi < 1/2$,

(ii) in case $\sqrt{2f} > e^A(\max\{2\pi - 1; 0\})$, $e^{AI} > e^A$, $q^{AI} < q^A$, $s^{AI} = \max\{2\pi - 1; 0\} < s^A$, *and* $S^{AI} < S^A$.

Proof: In Appendix.

For intermediate cases s adjusts such that $2f = e^A(s)$. That is, there may arise the case, e.g. when $\pi \leq 1/2$ and $w^2/2 < f < (W - w)^2$, that school integration induces zero investments, given an achievement based policy on the labor market, since incentives to invest are depressed.

Finally, suppose integration is enforced at school and compare the allocation under an achievement based labor market policy to the laissez-faire labor market outcome. Using the notation $e^A(s)$ to indicate that investment depends on the measure of agents investing, $S^{SI} > S^{AI}$ if

$$W^2 \;>\; 2e^A(s)\left(2w - \frac{e^A(s)}{2}\right) \text{ with } s = \min\{2\pi; 1\}. \tag{5}$$

Hence, $S^{LF} > S^A$ implies $S^{SI} > S^{AI}$, since under both policies integration induces the same fixed cost. As $e^A(s)$ decreases in s because incentives weaken with higher availability of h agents, per capita surplus net of fixed cost under school integration and achievement based policies is less than with an achievement based policy only. Since the condition is monotone in W,

$$S^{SI} > S^{AI} \Leftrightarrow W > W_4(w, \pi),$$

where $W_4(w, \pi)$ increases in w, decreases in π for $\pi \leq 1/2$, and $W_4(w, \pi) = W_4(w, 1/2)$ for $\pi > 1/2$, see appendix for details. For $\pi = 1$ trivially $W_4(w, 1) = W_1(w, 1) = W_0(w, 1)$. For $\pi < 1$, $W_4(w, \pi) < W_0(w, \pi)$ as pointed out above.

That is, effectiveness of a policy at the schooling stage depends on the policy used on the labor market. For instance, since $W_4 < W_0$, if an achievement based policy on the labor market is preferable to laissez-faire when the schooling stage operates under laissez-faires, this may reverse when introducing school integration. The reverse does not hold, i.e. if laissez-faire on the labor market is preferable to an achievement based policy, introducing school integration does not change this. Hence, school integration and achievement based policies act as substitutes.

4.3 Background Based Policy and School Integration

Turn now to a combination of a background based policy on the labor market and integration at school. Under school integration measure $2\min\{\pi; 1-\pi\}$ of agents are matched into (U, P) environments with access cost f. On the labor market all possible matches between U and P agents are enforced, but given this constraint agents may segregate. Suppose both U and P agents in (U, P) environments invest. The measure of h agents with a U background is $\min\{\pi; 1-\pi\}e$, since agents in (U, U) environments with access cost F do not invest. The measure of h agents with background P is πe. The measure of P agents required to match with U agents is $\min\{\pi; 1-\pi\}$. Hence, if all agents invest, both U and P agents with education h encounter an agent with h and the required background for sure. Therefore U and P agents solve

$$\max_e eW - \frac{e^2}{2} - g(b, b').$$

That is, $e^{BI} = W$ and the labor market payoff is $W^2/2 - f > 0$. Hence, agents facing access cost f or smaller find it indeed profitable to invest whereas (U, U) do not, which is consistent with our assumption.[7] That is, under a background based policy combined with school integration

$$e^{BI} = W = e^{LF} = e^{SI}, s^{BI} = \min\{2\pi; 1\} = s^{SI}, S^{BI} = S^{SI},$$

where the last statement is implied by the preceding two. That is, given there is integration at school an allocation under a background based labor market policy coincides with the one under a laissez-faire labor market. This is because background based policies use information on educational outcome provided by personal background to re-match the labor market. The predictive power of personal background depends positively on the degree of segregation at school. There is some empirical evidence that the degree of tracking influences the dependence of students' educational attainments on parental background.[8]

[7] Note that again there may be multiple equilibria when $w^2 < f$.

[8] See Schütz et al. (2008), Brunello and Checchi (2007) or Ammermüller (2005), for instance, who find that dependence of students' outcomes on their socioeconomic backgrounds depends positively on earlier start of tracking, and number of tracks or private schools. (Waldinger, 2007) raises the issue of causality, however.

Combining school integration with a background based policy on the labor market raises aggregate surplus if $S^{SI} > S^{B}$, that is if

$$
\begin{aligned}
&2\frac{W^2}{2} - 2f > e^B\left(2w - \frac{e^B}{2}\right) \text{ if } \pi \leq \frac{1}{2}, \text{ and} \\
&\frac{W^2}{2} - 2(1-\pi)f > e^B\left((1-\pi)2w + (2\pi-1)W - \pi\frac{e^B}{2}\right) \text{ if } \pi > \frac{1}{2}. \qquad (6)
\end{aligned}
$$

Since $e^B = w$ if $\pi \leq 1/2$, in that case $S^{SI} > S^B$ if and only if $2W^2 > 3w^2 - 4f$. Under Assumption 1 a sufficient condition for this is $W > \sqrt{3}w$. Since also for $\pi > 1/2$ the condition is monotone in W we have

$$S^{SI} > S^B \Leftrightarrow W > W_5(w, \pi, f),$$

where $W_5(w, \pi, f)$ increases in w and f, for $\pi > 1/2$ increases in π, and $W_4(w, \pi) = W_4(w, 1/2)$ for $\pi \leq 1/2$, for details see the appendix. Two interesting cases may arise, as $S^{SI} > S^B$ does not imply $S^{SI} > S^{LF}$ when $S^{LF} > S^B$. Correspondingly, $S^{SI} > S^{LF}$ does not imply $S^{SI} > S^B$ if $S^{LF} < S^B$. Hence, whether school integration is beneficial or not may depend on the labor market policy in place, since school integration affects the functioning of a background based policy.

4.4 Club Based Policies

Due to legal or informational constraints sometimes it may be infeasible to learn agents' types, that is whether $b = P$ or $b = U$. Information on agents' investment environments may be attainable, however. For instance, this could be the socio-economic characteristics of neighborhoods individuals live in, or the performance rank of the school attended. A policy of associational redistribution on the labor market that conditions on the school environment or club (b, b') is called a *club based* policy. Club based associational redistribution can be thought of as measures of placing students from disadvantaged neighborhoods or schools in firms, e.g. school-to-work-policies like the School-to-work Opportunities Act in 1994 in the U.S. which used mentoring or internships. Other examples of club based policies are those that assign places at prestigious schools ensuring high labor market continuation payoffs using a quota based on students' schooling environments.

Define a club based policy formally. Denote the measure of (U, U) environments by s_u, and the one of (P, P) by s_p. Then the remaining measure $1 - s_u - s_p$ of agents match into (U, P) environments.

Definition 1 (Club Based Policy) *A matching $m(.) : I \mapsto I$ is a club based policy on the labor market if*

(i) for $s_p \leq s_u$, $m(i) = j$ and $(b_i, b_{\mu(i)}) = (P, P) \Rightarrow (b_j, b_{\mu(j)}) = (U, U)$,

(ii) for $s_p \geq s_u$ $m(i) = j$ and $(b_i, b_{\mu(i)}) = (U, U) \Rightarrow (b_j, b_{\mu(j)}) = (P, P)$,

(iii) uniform rationing is used to assign the abundant market side.

A club based policy assigns agents from (P, P) environments to agents from (U, U) environments whenever possible while neither conditioning on background nor on educational achievement. Given they comply with this rule agents are free to segregate in educational attainment, in particular if they come from (U, P) environments.

Suppose first $s_u = 1 - \pi$ and $s_p = \pi$, that is all environments are segregated. Then P agents match to U agents with probability $(1 - \pi)/\pi$ if $\pi > 1/2$ and with certainty if $\pi \leq 1/2$. Since $F > W^2/2$ agents in (U, U) environments do not invest and P agents solve

$$\max_e ew - \frac{e^2}{2} \text{ if } \pi \leq 1/2 \text{ and}$$
$$\max_e e\left(\frac{s_u}{s_p}w + \left(1 - \frac{s_u}{s_p}\right)W\right) - \frac{e^2}{2} \text{ if } \pi > 1/2.$$

Interior solutions satisfy $e^C = W - (s_u/s_p)(W - w)$ if $\pi > 1/2$, and $e^C = w$ otherwise. Hence, $e^C = e^B$ if $s_p = \pi$ and $s_u = 1 - \pi$, and a club based policy coincides with a background based policy when schools segregate.

Here we limit our attention to the case $\pi > 1/2$.[9] To check whether segregation at school is an equilibrium suppose a pair of agents match into a (U, P) environment. On the labor market these agents are not subject to regulation, that is they segregate in education outcome (the measure of h agents is positive as $\pi > 1/2$). Hence, strictly positive investments solve

$$\max_e eW - \frac{W^2}{2} - f.$$

[9] Otherwise some integration remains an equilibrium outcome, although multiplicity of equilibria becomes an issue, see the appendix.

$e = W$ generates payoff $W^2/2 - f > 0$, so (U, P) agents invest. Agents segregate into schools if payoffs are higher in segregated than in integrated clubs for P or U agents, that is if

$$(e^{CB})^2 > W^2 - 2f \text{ or } 2e^{CB}w > W^2 - 2f,$$

since (U, U) have expected payoff $e^C w$ due to uniform rationing. The first condition implies the second as $2w > e^C$. Hence, schools segregate and background and club based policies coincide if and only if

$$2f > \frac{1-\pi}{\pi} 2w(W - w) - W(2w - W).$$

Suppose this is not the case. Then agents in segregated clubs obtain higher payoff by forming (U, P) clubs. Therefore $s_u + s_p < 1$. Let $s_u = 0$ and suppose $s_p \geq 1/2$ for the moment, then a (U, P) agent is matched to a (P, P) agent with certainty, and, assuming (P, P) agents invest, solves

$$\max_e eW - c(e, U, P).$$

As $W^2 > 2f$, (U, P) and (P, P) agents invest. When deviating to segregated schools U agents obtain ew, P agents obtain $W^2/2$. Since $W^2/2 > W^2/2 - f$ all P agents in (U, P) clubs can profitably deviate to a (U, U) school. An analogous argument applies to the case $s_p < 1/2$ although there may be multiple equilibria unless $w^2/2 > f$. Hence, $s_u > 0$ and $s_p > 0$.

Since (U, P) agents may segregate on the labor market they invest $e = W$. Incentive compatibility for school sorting binds for P agents, so that s_p and investment of (P, P) agents makes them indifferent between (U, P) and (P, P) schools, see Appendix for details. This implies the following proposition.

Proposition 6 *If $2f < \frac{1-\pi}{\pi} 2w(W - w) - W(2w - W)$, a club based policy induces*
(i) integration at school, $s_p + s_u < 1$ and $s_u > 0$, and on the labor market as the measures of (ℓ, ℓ), (ℓ, h), and (h, h) firms all are positive,
(ii) investments $e = 0$ in (U, U), $e = W$ in (U, P) and $e = (W^2 - 2f)/(2w) > e^B$ in (P, P) environments.
Otherwise club based and background based policies coincide.

The next proposition evaluates welfare under a club based policy.

Proposition 7 *Let $\pi > 1/2$. A club based policy dominates achievement based, background based, school integration policies and any combination thereof in terms of aggregate surplus if*

$$W^2 - 2f > 2w\left(W - \frac{1-\pi}{\pi}(W-w)\right).$$

Proof: In Appendix.

That is, if access cost in integrated school is sufficiently small to induce integration, a club based policy successfully trades off investment incentive effects and output effects from re-matching. Negative effects of integration on the labor market are curbed by conditioning integration on club membership rather than on achievement. Negative effects of school integration due to reducing quality of screening in backgrounds or clubs are limited as in any sorting equilibrium incentive compatibility requires positive measures of (U, U), (U, P), and (P, P) environments. Finally, a club based policy captures the benefits from re-matching at least partially both at school and on the labor market, since the measures of all environment types are positive.

Note that Proposition 7 also applies when firm profits have increasing differences, i.e. $2w < W$. Recall that then segregation on the labor market maximizes output all else equal.[10] Now Proposition 7 implies that when fixed costs in integrated schools are low enough to admit school integration under a club based policy, this ensures that a club based policy dominates laissez-faire independently of the properties of the match surplus.

An illuminating example of club based policies is admission of high school graduates to public universities in Texas. In late 1996, Texas state universities abolished affirmative action based on race in response to the Fifth Circuit Court decision in *Hopwood vs. Texas.* In 1997 the Texas Top 10 Percent law was instituted with the stated aim to preserve minority attendance rates. This scheme guarantees automatic admission to Texas state universities for students who graduate among the best ten percent of their class. Since Texan high schools were highly segregated this was expected to counteract any adverse effect of abolishing affirmative action to campus

[10]Then the labor market segregates in education also in the fully transferable utility benchmark allocation. Wages in the benchmark allocation coincide with equilibrium wages under strictly nontransferable utility. Hence, for this point (sufficiently) strictly nontransferable utility is needed at the schooling stage, but not on the labor market.

diversity, tacitly assuming that composition was not affected by the policy change. Kain et al. (2005) report that

> Hopwood had a devastating effect on minority enrollment in Texas selective public universities, reducing the African-American and Hispanic share of entering classes by 37 percent and 21 percent between 1996 and 1998.

That is, after about two decades of affirmative action in Texas its removal triggered a sudden reversal to segregation. This may indicate that affirmative action policies were ineffective in changing beliefs, or that segregation in higher education was not entirely belief-based. Kain et al. (2005) further conclude that the Texas Top 10 percent law was not effective in preserving campus diversity since the top slots were disproportionally taken by non-minority students. Long (2004) confirms both observations in a broader study covering US-wide substitution of affirmative action by high school quotas.

Parents appear to have reacted to incentives, as Cullen et al. (2006) report some evidence of strategic re-sorting by good students into worse peer-groups in Texas. This appears to be consistent with our model where a club based policy may induce re-sorting into schools. If diversity at school is desirable from a social planner's point of view, the Texas Top 10 Percent Law seems a fine case of unintended, yet beneficial consequences.

5 Conclusion

We presented a framework to analyze policies of associational redistribution on the labor market and at school. The framework imposes strictly nontransferable utility serving to focus on the interaction of matching patterns and investment incentives. It remains silent, however, about another source of inefficiencies when utility is transferable, but not perfectly so. Then competition may require inefficient sharing of surplus (see e.g. Legros and Newman, 2008) which in turn affects investment incentives. While beyond the scope of the present paper, more research on this topic appears to be desirable.

In the present approach policies aim at replicating the fully transferable utility matching outcome, that is integration, as a benchmark. In a more complex derivation of nontransferable utility, nontransferabilities may affect the optimal matching, however. See Gall et al. (2008) for an example when information rents decrease in the scope of the project, so that the optimal matching involves integration when there is asymmetric information generating nontransferabilities, but segregation under perfect information.

Labor market policies need to trade off output efficient sorting and provision of adequate incentives for pre-match investments. Conditioning labor market re-matching on observable information not subject to individual choice, such as background, appears beneficial when it is linked to education outcome. Early stage intervention, i.e. at school, does not distort incentives and provides benefits when integrating schools is cost efficient. In that sense early stage policies are more effective than later stage policies, supporting the conclusion Heckman (2008) draws from analyzing policies aimed at promoting cognitive development, albeit from a different angle.

Earlier and later stage policy are interdependent, however. School integration may limit the informational content provided by individual background and reduce the effectiveness of screening, rendering background based policies obsolete. When an achievement based policy is used on the labor market school integration may discourage investment due to low returns to education. Moreover, optimal policies may depend on characteristics of the economy. For instance, if privileged agents are scarce, a background based policy dominates an achievement based policy. This does not hold for economies where the privileged abound. Abundance of under-privileged agents best describes developing countries, suggesting that the use of achievement based policies should be restricted to developed economies.

Finally, we identify a labor market policy that looks promising in terms of trading off incentive provision and efficient sorting both at the early and the later stage: club based associational redistribution re-matches the labor market conditioning on individual school choices. It yields some integration both on the labor market and at school while inducing higher investments than other policies. This result is particular encouraging since there is no reason to expect this policy to be optimal. A highly interesting direction for future research is to explore optimal mechanisms of associational redistribution in sequential assignment markets when utility is nontransferable.

A Mathematical Appendix

A.1 Proof of Lemma 1

To maximize expected utility $u = ew(h) + (1-e)w(\ell) - \frac{e^2}{2}$, a necessary condition for optimal investment is therefore $e^{TU} = 2(w(h) - w)$.

We have established in the text that if $q > 1/2$, agents with education h are abundant and obtain wage $w(h) = W$, agents with ℓ obtain wage $w(\ell) = 2w - W$. Hence, $e^{TU} = 2(W - w)$ and the realized q is $s2(W - w)$ which is indeed greater than $1/2$ only if $W - w > \frac{1}{4s}$.

If $q < 1/2$, h agents are scarce, so that $w(h) = 2w$ and $w(\ell) = 0$. Because $e^{TU} = \min\{2w; 1\}$, the realized $q = se^{TU}$ is less than $1/2$ only if $w < \frac{1}{4s}$.

Finally, if $q = 1/2$ a continuum of wages is consistent with a stable allocation, $w(h) \in [W, 2w]$ and $w(\ell) = 2w - w(h)$. Agents choose education investment $e^{TU} = 2(w(h) - w)$. However the realized $q = s2(w(h) - w)$ is equal to $1/2$ only if $w(h) = w + \frac{1}{4s}$. Therefore $e^{TU} = \frac{1}{2s}$ in this case.

Suppose first $q = s2(w(h)-w) > 1/2$. This is only consistent with $s2(W-w) > 1/2$. $q < 1/2$ is only consistent with $s2w < 1/2$. For intermediate cases, that is $s(W - w) < 1/4 < sw$, $q = 1/2$. Therefore $e = 2w(h) - 2w = 1/2s$, that is $w(h) = w + s/4$.

A.2 Proof of Corollary 1

When $W - w > \frac{1}{4s}$, the assumption that $2w > W$ implies that $W > \frac{1}{2s}$. In this case, $e^{LF} - e^{TU} = 2w - W > 0$.

When $w < \frac{1}{4s}$, since $2w > W$, the condition implies that $W < \frac{1}{4s}$. In this case, $e^{LF} - e^{TU} = W - 2w < 0$.

In the intermediate case, where $W - w < \frac{1}{4s} < w$, $e^{TU} = \frac{1}{2s}$ and therefore $e^{LF} - e^{TU}$ is positive only if $W > \frac{1}{2s}$.

A.3 Proof of Proposition 1

Given a policy of associational redistribution based on educational achievement which assigns h agents to ℓ agents whenever possible, an agent chooses effort e to solve

$$\max_e \quad e\left(\frac{1-q}{q}w + \frac{2q-1}{q}W\right) + (1-e)w - \frac{e^2}{2} \text{ if } q > 1/2,$$
$$\max_e \quad ew + (1-e)\frac{q}{1-q}w - \frac{e^2}{2} \text{ if } q \leq 1/2. \tag{7}$$

Supposing $q > 1/2$, a necessary condition for investment is

$$e = \frac{2q-1}{q}(W-w).$$

In equilibrium $se = q$ must hold. Since e above increases in q and se increases in s, it is sufficient to verify that $q > 1/2$ can occur when $s = 1$. $e = q$ implies

$$q^2 - 2(W-w)q + (W-w) = 0$$

but the discriminant is $(W-w)^2 - (W-w) = (W-w)(W-w-1)$ which is negative since $W - w < 1$. Therefore in any equilibrium $q \leq 1/2$ and

$$e^A = \frac{1-2q}{1-q}w < w. \tag{8}$$

Replacing e^A by q/s and solving for q yields the expression in the proposition (the other solution is greater than 1). Clearly, the solution is less than 1/2.

Since $e^A < w$ both

$$e^A < e^{LF} \text{ and } e^A < e^{TU}.$$

This is because an achievement based policy provides ℓ agents with a chance $q/(1-q)$ to obtain wage w thus lowering gains from investing compared to both the laissez-faire and the transferable utility outcome. That is, enforcing integration based on educational achievement offers some insurance against the low education outcome distorting education investments downwards. Nevertheless, better sorting through re-matching on the labor market may potentially compensate for adverse incentive effects.

A.4 Proof of Proposition 2

Note that matching is constrained in terms of background types but not in terms of educational achievements, the labor market segregates in education outcomes *given* the policy constraints. For instance, if $s > 1/2$, within the (P, P) matches, agents form either (h, h) or (ℓ, ℓ) firms. Hence, under a background based policy P agents choose effort e to solve

$$\begin{cases} \max_e ew - \frac{e^2}{2} & \text{if } s \leq 1/2 \\ \max_e e\left(\frac{1-s}{s}w + \frac{2s-1}{s}W\right) - \frac{e^2}{2} & \text{if } s > 1/2. \end{cases}$$

Interior solutions satisfy $e^B = w$ if $s \leq 1/2$, and $e^B = W - (1-s)(W - w)/s$ if $s > 1/2$. That is, $e^{LF} > e^B \geq w > e^A$ for $s \in (0, 1)$. Hence, a background based policy mitigates incentive distortions compared to an achievement based policy.

A.5 Proof of Corollary 2

Under an achievement based policy investments satisfy (8). With $q = se^A$

$$e^A = w + \frac{1}{2s} - \sqrt{w^2 + \frac{1}{4s^2}}.$$

Simple calculations show that the derivative of e^A with respect to s is negative, the derivative of $q^A = se^A$ with respect to s is positive, and e^A increases in w.

The condition $S^A > S^{LF}$ holds if e^A solves the quadratic equation ${e^A}^2 - 4we^A + W^2 < 0$. Solving yields

$$e^A > 2w - \sqrt{4w^2 - W^2}. \tag{9}$$

Since $e^A < w$, a necessary condition for $S^A > S^{LF}$ is that $W^2 > 3w^2$. Finally, using (1), $W_0(w, s)$ solves

$$W(2w - W) = (W - e^A)2w - \frac{1}{2}(W^2 - {e^A}^2) \tag{10}$$

By Proposition 1, differentiating q with respect to w, and using $q = se^A$, e^A is an increasing function of w. Therefore the RHS of (10) decreases in w, and increases in W since $W < 1$. The LHS increases in w and decreases in W since $w < W$. Hence as w increases, W must increase to restore equality. Hence, $W_0(w, s)$ increases in w. It increases in s because e^A increases in s, hence the RHS of (10) decreases in s and W must increase to restore equality.

A.6 Proof of Proposition 3

A.6.1 Derivation of the Cutoff $W_1(w, s)$

For $s < 1/2$ we established in the text that a background based dominates an achievement based policy. While both policies induce exactly the same matching pattern – each P is matched with a U and there is the same measure of (h, ℓ) firms for a given e – since $e^B > e^A$, there are more integrated firms, hence, as $2w > W$, surplus is higher. Therefore, $W_1(w, s) = w$ as claimed.

If $s > 1/2$ screening by background loses its effectiveness as a measure $2s - 1$ of P agents (inefficiently) segregate in educational outcome; this does not occur under an achievement based policy. Total surplus under a background based policy is then

$$S^B = e^B \left((2\pi - 1)W + (1 - \pi)2w - \pi \frac{e^B}{2} \right). \tag{11}$$

$S^B > S^A$ if and only if

$$s(e^B - e^A) \left(2w - \frac{1}{2}(e^B + e^A) \right) > (2s - 1)e^B(2w - W). \tag{12}$$

The LHS captures the gain through better incentive provision under a background based policy, while the RHS gives the benefit from re-sorting under an achievement based policy. Since (12) strictly relaxes as W increases,

$$S^B > S^A \Leftrightarrow W > W_1(w, s).$$

The cutoff $W_1(w, s)$ increases in s as efficiency gains from re-sorting under a background based policy increase relative to an achievement based policy as s decreases (which decreases the effectiveness of screening by background). $W_1(w, 1) = W_0(w, 1)$, as for $s = 1$ a background based policy implies the laissez-faire outcome. Therefore $W_0(w, s) > W_1(w, s)$ for $s < 1$ and the difference decreases in s.

Finally, we show that $W \geq \sqrt{3}w$ implies $S^B > S^A$.

Note first that e^B increases in s while e^A decreases in s. Therefore both output and incentive effect in condition (12) move in the same direction. Simple calculations reveal that the derivative of W_1 with respect to s is positive for $s \in [1/2, 1]$ and $W_1(w, s)$ may not be monotone in w. For a sufficient condition, solving the quadratic expression $S^B > S^A$ for e^A yields

$$e^A < 2w - \sqrt{4w^2 - \frac{2}{s}S^B}. \tag{13}$$

$2S^B > 3sw^2$ gives a sufficient condition:

$$\begin{aligned} 2((2s-1)W + (1-s)w)((2s-1)W/2 + (1-s)3w/2) &> 3s^2w^2 \\ \Leftrightarrow (2s-1)W^2 + 4(1-s)Ww &> 3w^2. \end{aligned}$$

Solving this quadratic expression in W yields the condition

$$W > \left(\frac{\sqrt{3(2s-1) + 4(1-s)^2}}{2s-1} - 2\frac{1-s}{2s-1} \right) w.$$

Since $(\sqrt{3(2s-1)+4(1-s)^2} - 2(1-s))/(2s-1) < \sqrt{3}$ a sufficient condition for $S^B > S^A$ is $W \geq \sqrt{3}w$.

A.6.2 Derivation of the Cutoff $W_2(w, s)$

We compare now the background based policy to laissez-faire.

When $s \leq 1/2$, under laissez-faire there are $se^{LF}/2$ firms of type (h, h) contributing to total output and the surplus is $S^{LF} = sW^2/2$. By contrast, with a background based policy there are se^B firms (h, l) contributing to total output and surplus is $S^B = 3w^2/2$. Therefore, when $s < 1/2$, $S^{LF} > S^B$ if and only if $W > \sqrt{3}w$; hence $W_2(w, s) = \sqrt{3}w$ as claimed.

Consider now the case $s > 1/2$. In this case, $S^{LF} > S^{BB}$ if and only if

$$\begin{aligned} (e^{LF} - e^B) \left[(1-s)2w + (2s-1)W - \frac{s}{2}\left(e^{LF} + e^B\right) \right] \\ > (1-s)e^{LF}(2w - W). \end{aligned} \tag{14}$$

Manipulating condition (14) and solving for W yields

$$W > \frac{4s - 2 + \sqrt{1 - 4s + 7s^2}}{3s - 1} w := W_2(w, s).$$

Clearly, $W_2(w, s)$ increases in w and calculating the derivative of W_2 with respect to s it can be checked that W_2 increases also in s. Note that $W_2(w, s) \to 2w$ as $s \to 1$. Bounds on W_2 for $s \in [1/2, 1]$ are given by

$$\sqrt{3}w = W_2(w, 1/2) \leq W_2(w, s) \leq W_2(w, 1) = 2w.$$

$W_2(w, s) \geq \sqrt{3}$ implies in particular that $W_2^2(w, s) \geq 3w^2$ which implies that $S^{LF} > S^A$, see above, and therefore $W_2(w, s) > W_0(w, s)$.

A.7 Proof of Proposition 4

Start with the labor market allocation and investment choice. Since $2w > W$ there exist wages $w(h)$ for h agents and $2w - w(h)$ for ℓ agents such that integration is a stable labor market outcome. When investing an agent solves

$$\max_e ew(h) + (1-e)w(\ell) - \frac{e^2}{2} - g(b, b'),$$

where market wages are (i) $w(h) = W$ and $w(\ell) = 2w - W$ if $q > 1/2$, (ii) $w(h) \in [W, 2w]$ and $w(\ell) = 2w - w(h)$ if $q = 1/2$, and (iii) $w(h) = 2w$ and $w(\ell) = 0$ if $q < 1/2$. Corresponding optimal interior investments are

$$e = \begin{cases} e_0 = 2(W-w) & \text{if } q > 1/2 \\ [2(W-w), \min\{2w; 1\}] & \text{if } q = 1/2 \\ e_1 = \min\{2w; 1\} & \text{if } q < 1/2 \end{cases}$$

Strictly positive investment is profitable if $e^2/2 > g(b, b')$.

Denote by ρ the measure of (U, U) agents. ρ is endogenous and depends on school choice. $q > 1/2$ implies $e = 2(W - w)$, so that (U, U) agents do not invest since $F > W^2/2$. This is only consistent if $(1-\rho)e > 1/2$, that is $(1-\rho)2(W-w) > 1/2$.

$q < 1/2$ implies $e = \min\{2w; 1\}$, so that (U, U) agents invest since $F < 2w^2$. This is only consistent if $\min\{2w; 1\} < 1/2$, that is $w < 1/4$.

If $1/4 \geq w \geq W - 1/(4(1-\rho))$, $q = 1/2$ must hold. To have $q = 1/2$ either $(1-\rho)2(w(h)-w) = 1/2$ if $1/(4(1-\rho)) \in [W-w, \sqrt{F}]$, or $2(w(h)-w)^2 = F$ and measure $1/2 - (1-\rho)\sqrt{F}$ of (U, U) agents invest $e = \sqrt{F}$ which makes them indifferent between $e > 0$ and $e = 0$.

Turn now to the school stage when the measure ρ is determined. If $q > 1/2$ payoffs at the school stage are given by

$$2w - W \text{ if } g(b, b') = F,$$
$$2w - W + 2(W-w)^2 - g(b, b') \text{ if } g(b, b') < F,$$

Hence, a U agent values a P agent at $2(W-w)^2 - f$, and a P agent values a U agent at $-f$. Hence, schools integrate, that is $1 - \rho = \min\{2\pi; 1\}$, if

$$(W-w)^2 > f.$$

Otherwise schools segregate and $1 - \rho = \pi$. On the other hand, if $q = 1/2$, payoffs at school are given by $2w - w(h) + 2(w(h)-w)^2 - g(b, b')$ if $g(b, b') < F$

and $2w - w(h)$ if $g(b, b') = F$, or by $F - g(b, b')$ for all agents. Since $2f < 2(W - w)^2 \leq 2(w(h) - w)^2 \leq F$ under our assumptions schools integrate and $1 - \rho = \min\{2\pi; 1\}$. In case $q < 1/2$ all agents invest as noted above.

Using this on $(1 - \rho)2(W - w) \geq 1/2$ gives condition (i) in the statement. Otherwise, if $2\min\{2\pi; 1\}\sqrt{2F} > 1$ we have $2e = 1/\min\{2\pi; 1\}$. If $2\min\{2\pi; 1\})\sqrt{F} \leq 1$ $e = \sqrt{2F}$ and some (U, U) agents invest, verifying the assertions in case (ii).

A.8 Appendix to Section 4: Combining Policies

SI **versus** A**:**

The cutoff value W_3 is given by

$$W_3 = \begin{cases} \sqrt{2f + e^A(2w - e^A/2)} & \text{if } \pi \leq 1/2 \\ 2\sqrt{(1-\pi)f + q^A(w - e^A/4)} & \text{if } \pi > 1/2 \end{cases}$$

In case $\pi \leq 1/2$ clearly W_3 decreases in π since $e^A(s) < w$ decreases in $s = \pi$. If $\pi > 1/2$, W_3 increases in π if $2f < \frac{\partial S^A}{\pi}$ and decreases otherwise. In both cases W_3 trivially increases in f and w, the latter since e^A increases in w.

AI **versus** SI**:**

Since the LHS of condition (5) strictly increase in W while the RHS does not depend on W, a cutoff for $S^{SI} > S^{AI}$ is given by

$$W_4 := \sqrt{2e^A(\min\{2\pi; 1\})(2w - e^A(\min\{2\pi; 1\})/2)}.$$

Indeed $W > W_4$ implies $S^{SI} > S^{AI}$ and $W < W_4$ implies $S^{SI} < S^{AI}$. W_4 increases in $e^A(s)$. Since e^A increases in w, so does W_4. As e^A decreases in s, W_4 must decrease in π for $\pi < 1/2$ and be constant for $\pi \geq 1/2$.

SI **versus** B**:**

In case $\pi > 1/2$ condition (6) for $S^{SI} > S^B$ can be rewritten to yield

$$W > \left(1 - \frac{1}{4\pi - 1}\right)w + \frac{\sqrt{(1 + \pi(4\pi - 1))w^2 + 4f\pi(4\pi - 1)}}{4\pi - 1} := W_5.$$

This expression clearly increases in w and f, and, since $\pi > 1/2$, also in π.

A.9 Proof of Proposition 5

By (4) given s an agent with fixed cost f invests if

$$e^{AI} > \sqrt{2f} \Leftrightarrow e^A(s) > \sqrt{2f}.$$

Since $e^A(s)$ strictly decreases in s, $e^A(\max\{2\pi-1;0\}) < \sqrt{2f}$ implies that (U,P) agents do not invest if $s = \max\{2\pi;0\}$ which is consistent therewith. If $e^A(\min\{2\pi;1\}) > \sqrt{2f}$ (U,P) agents invest at $s = \min\{2\pi;1\}$. For intermediate f, s is defined by $e^A(s) = \sqrt{2f}$ making (U,P) agents indifferent between investing or not, which is consistent with $\max\{2\pi;0\} < s < \min\{2\pi;1\}$. Aggregate surplus under AI is

$$\max\{2\pi-1;0\}e^{AI}\left(2w - \frac{e^{AI}}{2}\right) \text{ if } e^A(\max\{2\pi-1;0\}) < \sqrt{2f},$$
$$\min\{2\pi;1\}e^{AI}\left(2w - \frac{e^{AI}}{2}\right) - 2\min\{\pi;1-\pi\}f \text{ if } e^A(\min\{2\pi;1\}) > \sqrt{2f}.$$

$S^A > S^{AI}$ if and only if $q^A(2w - e^A(s^A)/2) > q^{AI}(2w - e^A(s^{AI})/2)$. For $e^A(\max\{2\pi-1;0\}) < \sqrt{2f}$, $q^A = \pi e^A(\pi) > q^{AI}$ and $e^A(\pi) < e^A(s^{AI})$, therefore $S^A > S^{AI}$. Let now $e^A(\min\{2\pi;1\}) > \sqrt{2f}$ and suppose $\pi \leq 1/2$ first. Then a sufficient condition for $S^{AI} > S^A$ is

$$\begin{aligned} & 2w\left(2e^A(2\pi) - e^A(\pi)\right) > 2(e^A(2\pi))^2 - \frac{(e^A(\pi))^2}{2} \\ \Leftrightarrow\ & 4\pi w(q^{AI} - q^A) > (q^{AI})^2 - (q^A)^2, \end{aligned}$$

which must be true since $e^A(2\pi) < e^A(\pi) < w$. In case $\pi > 1/2$ a sufficient condition for $S^{AI} > S^A$ is

$$2w\left(q^{AI} - q^A\right) > \left(\frac{3}{2} - \pi - \frac{1}{2\pi}\right)(q^A(1))^2 + \frac{(q^{AI})^2 - (q^A)^2}{2\pi}.$$

This is implied by

$$w\left(1 - \frac{q^A}{q^{AI}}\right) > \left(3 - 2\pi - \frac{1}{\pi}\right)q^{AI}.$$

Since $w \geq e^{AI} = q^{AI}$ and for $1/2 < \pi \leq 1$

$$2\pi + \frac{1}{\pi} - 2 > 1 > \frac{q^A}{q^{AI}},$$

$S^{AI} > S^A$ follows.

A.10 Omitted Details for Proposition 6

From above we know that $s_u > 0$ and $s_p + s_u < 1$. If $\pi > 1/2$ then $s_p = s_u + 2\pi - 1$ and a (U, U) agent is matched to a (P, P) agent with certainty, does not invest, and obtains $e^P w$. A (U, P) agent can match either to a (P, P) or a (U, P) agent and solves

$$\max_e eW - c(e, U, P),$$

supposing that at least (P, P) agents invest. Since $f < W^2/2$ all (U, P) agents invest $e = W$. In this case a (P, P) agent solves

$$\max_e e\left(\frac{s_p - s_u}{s_p}W + \frac{s_u}{s_p}w\right) - c(e, P, P),$$

and therefore

$$e_p = \frac{s_p - s_u}{s_p}(W - w) + w.$$

That is, (U, P) invest more than (P, P) agents. Agents have no incentive to change schools if

$$W^2 - 2f \geq e_p^2 \text{ and } W^2 - 2f \geq 2e_p w,$$

with at most one strict inequality. Since $e_p < 2w$ the second condition must bind, that is $W^2 - 2f = 2e_p w > e_p^2$. This determines measures s_u and s_p since $s_u = s_p + 1 - 2\pi$ by feasibility, so that

$$s_p = (2\pi - 1)2w\frac{W - w}{W^2 - 2f - w^2}.$$

Note that $W^2 - 2f > 2w\left[W - \frac{1-\pi}{\pi}(W - w)\right]$ implies $s_p < \pi$.

That is, $0 < s_u < 1 - \pi$ and $0 < s_p < \pi$, so that measure $s_u e_p > 0$ of (h, ℓ), measure $(1 - s_u - s_p)W + (s_p - s_u)e_p > 0$ of (h, h), and measure $(s_u + s_p)W - s_p e_p > 0$ of (ℓ, ℓ) firms form.

Briefly consider the case $\pi \leq 1/2$. Suppose that $s_p = \pi$ and $s_u = 1 - \pi$ implying payoffs $w^2/2$ for (P, P) and $w^2\pi/(1-\pi)$ for (U, U) agents. Let a pair of agents matches into a (U, P) school. Since (P, P) agents are scarce these agents also match on the labor market implying optimal investments e_u, e_p

satisfy $e_u = w - e_p(2w - W)$ and vice versa. That is, $e = w/(1+2w-W) < w$. Hence, segregation can be supported as an equilibrium outcome if

$$w^2\left(\frac{\frac{3}{2}+2w-W}{(1+2w-W)^2} - \max\left\{\frac{1}{2};\frac{\pi}{1-\pi}\right\}\right) > f.$$

Let now $s_u < 1-\pi$ and $s_p < \pi$. Then (U,P) agents segregate and invest W, (P,P) invest w, and (U,U) agents invest 0 and obtain payoff $w^2 s_p/s_u$, where $s_p/s_u = 1-(1-2\pi)/s_u$. Hence, $s_u < 1-\pi$ can hold in equilibrium if

$$W^2 - w^2 \geq 2f \text{ and } W^2 - 2w^2\left(1-\frac{1-2\pi}{s_u}\right) \geq 2f.$$

Hence, whenever $W^2 - 2f \geq w^2$ there exist $s_u < 1-\pi$ such that $W^2 - 2w^2\left(1-\frac{1-2\pi}{s_u}\right) \geq 2f$, in particular $s_u = 1-2\pi$, that is full school integration, can be supported as an equilibrium outcome.

A.11 Proof of Proposition 7

Assume that

$$W^2 - 2f > 2w\left[W - \frac{1-\pi}{\pi}(W-w)\right]. \tag{15}$$

Then aggregate surplus under a club based policy can be written as

$$\begin{aligned} S^C &= 2s_u e_p w + (s_p - s_u)e_p W + (1-s_p-s_u)(\frac{W^2}{2} - f) - s_p e_p^2 \\ &= s_p\frac{e_p^2}{2} + s_p e_p w + 2(\pi - s_p)(\frac{W^2}{2} - f) = s_p\frac{e_p^2}{2} + (2\pi - s_p)(\frac{W^2}{2} - f) \\ &= \left(W^2 - 2f\right)\left[\frac{(2\pi-1)(W-w)}{2wW} + \pi\right]. \end{aligned}$$

Since $S^{LF} = \pi W^2/2$ the condition $S^C > S^{LF}$ is equivalent to

$$\frac{(2\pi-1)(W-w)+\pi wW}{(2\pi-1)(W-w)+2\pi wW} > \frac{2f}{W^2}.$$

Substituting $\delta := (W-w)/w$ in the above condition its LHS strictly increases in δ. Since $\delta \geq 0$ a sufficient condition for $S^C > S^{LF}$ is $W^2 > 4f$, which is implied by (15).

Under assumption (15) $S^C > S^A$ is implied by

$$2W - 2\frac{1-\pi}{\pi}(W-w) > \frac{3}{2}w.$$

Since $\pi > 1/2$ and $W > w$ this must hold true. Moreover, $S^C > S^A$ implies $S^C > S^{AI}$.

Turn now to a background based policy. Recall that when $\pi > 1/2$ $e^B = W - \frac{1-\pi}{\pi}(W - w)$ and

$$\begin{aligned} S^B &= (1-\pi)e^B 2w + (2\pi - 1)e^B W - \pi \frac{(e^B)^2}{2} \\ &= \frac{W^2}{\pi}\left(\frac{5}{4}(1-\pi)d - \frac{3-4\pi}{2}\right)\left(2\pi - 1 + (1-\pi)\frac{d}{2}\right), \end{aligned}$$

where $d = 2w/W$. Under assumption (15),

$$S^C > W\left(2\pi - 1 + (1-\pi)\frac{d}{2}\right)\frac{(2\pi-1)\left(1-\frac{d}{2}\right) + \pi dW}{\pi}.$$

Hence, $S^C > S^B$ is implied by

$$(2\pi - 1)\left(1 - \frac{d}{2}\right) + \pi dW \geq W\left(\frac{5}{4}(1-\pi)d - \frac{3-4\pi}{2}\right).$$

This can be rearranged to yield

$$(2-d)\pi(1-W) - 1 + \frac{d}{2} \geq \frac{W}{4}\left(5d(1-\pi) - 6\right).$$

This must be true since $d < 2$, $\pi > 1/2$, and $W < 1$ by assumption.

Compare now S^C to $S^{SI} = W^2/2 - 2(1-\pi)f$. $S^C > S^{SI}$ if and only if

$$\frac{1 - \frac{d}{2} + \frac{dW}{2}}{1 - \frac{d}{2} + dW} > \frac{2f}{W^2},$$

where again $d = 2w/W$. The LHS of this condition is strictly decreasing in d. Since at $d = 2$ the condition reduces to $W^2 > 4f$ which holds by assumption the above condition holds for $d \leq 2$.

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