Wage profiles in the Burdett-Mortensen model

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Abstract

The aim of the paper is to show that the Burdett-Mortensen (1998) model is a convenient framework to make predictions about wage growth over experience span over different quantiles of the wage distribution. In a simple representation of the Burdett-Mortensen model closed-form solutions for wage quantiles are derived. The model is then tested on American and German data (CPS-ORG and IABS). In general, the predictions of the model are consistent with the observed data, although still some contradictory results are obtained. This could be due to an implausible shape of the wage distribution derived in a simple Burdett-Mortensen model, which was already addressed in several studies before.

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I. Introduction

The phenomenon of wage dispersion has been attracting much attention of many labor economists. Human capital theory attributed these differences in pay to variation in human capital across workers. But according to Mortensen "Observable worker characteristics that are supposed to account for productivity differences typically explain no more than 30 percent of the variation in compensation across workers…" (Mortensen 2003:1). Although controlling for firm-specific effects can explain about 70% of wage variation[2] the question still remains: Why firms follow different wage policies and why similar workers within one firm are still paid differently? A seminal paper by Burdett and Mortensen (1998) presents a model where identical workers are still paid differently in equilibrium. The model has been modified in several ways to make predictions about the shape of the wage distribution more consistent with observed data (see e.g. Bontemps et al. (2000) and van den Berg and Ridder (1998)); but even in its simplest form the model still sheds much light onto the phenomenon of wage dispersion.

In this paper I present a synthesis of search and human capital theories. The aim of this work is to apply a modified Burdett-Mortensen model to German and US data to explain inter-quantile wage differentials within industries.

The paper is organized as follows: section 2 gives the theoretical background of the Burdett-Mortensen model, section 3 describes the data used for econometric model, section 4 presents the empirical results, and section 5 concludes.

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[2] See Mortensen (2003)

II. Theoretical Model

Consider an economy with M_F firms and M_W workers.[3] Let M_F and M_W be fixed and denote $M = \frac{M_F}{M_W}$. It is assumed that each firm opens but one vacancy. The inflow of job offers to unemployed workers is a stationary Poisson process with the arrival rate λ. Employed workers may search on the job. Workers are homogeneous. Let the arrival rate of job offers to employed workers be also λ. Jobs are also destroyed for exogenous reasons at rate δ. Equilibrium unemployment rate is then $u = \frac{\delta}{\delta + \lambda}$.

The fraction of workers receiving wage w or less is given by:

$$G(w; F) = \frac{\delta F(w)}{\delta + \lambda\left(1 - F(w)\right)}, \qquad (1.1)^4$$

where $F(w)$ is the distribution of wage offers, which is to be endogenously determined. The separation rate of workers in a firm paying a wage w is $\delta + \lambda\left(1 - F(w)\right)$. The first term represents the flow of employed workers into non-employment and the second term is the flow of workers to other firms offering a higher wage than w.

The inflow of workers to a firm paying a wage w is $\frac{\lambda}{M}\left[u + (1-u)G(w;F)\right] = \frac{\delta\lambda}{M\left[\delta + \lambda\left(1 - F(w)\right)\right]}$, where $\lambda u / M$ is the rate of recruiting from non-employment and $\frac{\lambda}{M}(1-u)G(w;F)$ is the flow of workers from other firms paying less than w.

The steady-state labor supply of workers to a firm is then:

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[3] Here I follow a simple representation of the Burdett-Mortensen model given in Manning (2003).

[4] See Manning (2003) for derivations.

$$L(w;F) = \frac{\delta\lambda}{M\left[\delta + \lambda\left(1 - F(w)\right)\right]^2}. \tag{1.2}$$

The profits of a firm solve:

$$\pi(w;F) = \frac{\delta\lambda(p-w)}{M\left[\delta + \lambda\left(1 - F(w)\right)\right]^2}. \tag{1.3}$$

Each firm sets a wage to maximize profits. In steady-state profits of firms should be equal irrespective of the wage set to preserve a non-degenerate wage distribution.[5] Suppose that workers value their leisure at b, which would be the reservation wage for non-employed workers. Then the steady-state level of profits is given by:

$$\pi(w;F) = \frac{\delta\lambda(p-b)}{M\left[\delta + \lambda\right]^2}. \tag{1.4}$$

This gives solution to the equilibrium wage offer distribution:

$$F(w) = \frac{\delta + \lambda}{\lambda}\left[1 - \sqrt{\frac{p-w}{p-b}}\right], \tag{1.5}$$

and the distribution of observed wages:

$$G(w) = \frac{\delta}{\lambda}\left[\sqrt{\frac{p-b}{p-w}} - 1\right]. \tag{1.6}$$

The expected observed wage in the economy is then:

$$E(w) = \frac{\delta}{\delta + \lambda}b + \frac{\lambda}{\delta + \lambda}p. \tag{1.7}$$

The lowest observed wage in the economy is then b and the highest is $p - \left(p - b\right)\left(\frac{\delta}{\delta + \lambda}\right)^2$.

[5] If at a certain wage w' profits could be higher than at any other, all firms would set this wage and the wage distribution would collapse to a one-point distribution.

From (1.7) one could see that expected observed wages increase with productivity. One could also notice that the spread of the wage distribution (the raw difference between the highest and the lowest wage) also goes up with productivity. However, it doesn't make much sense to analyze the raw spread. More interesting from empirical point of view would be to look at quantiles of the wage distribution. And this simple version of the Burdett-Mortensen model provides this information. For example, the median observed wage solves:

$$w_{.5} = \frac{\lambda^2 p + 4\delta\lambda p + 4\delta^2 b}{\left(2\delta + \lambda\right)^2}. \tag{1.8}$$

And the 2nd decile of the observed wage distribution is:

$$w_{.2} = \frac{\lambda^2 p + 10\delta\lambda p + 25\delta^2 b}{\left(2\delta + \lambda\right)^2}, \tag{1.9}$$

or in general form:

$$w^{\theta} = p - \frac{\left(p - b\right)\delta^2}{\left(\theta\lambda + \delta\right)^2}, \tag{1.10}$$

where θ is the respective quantile.

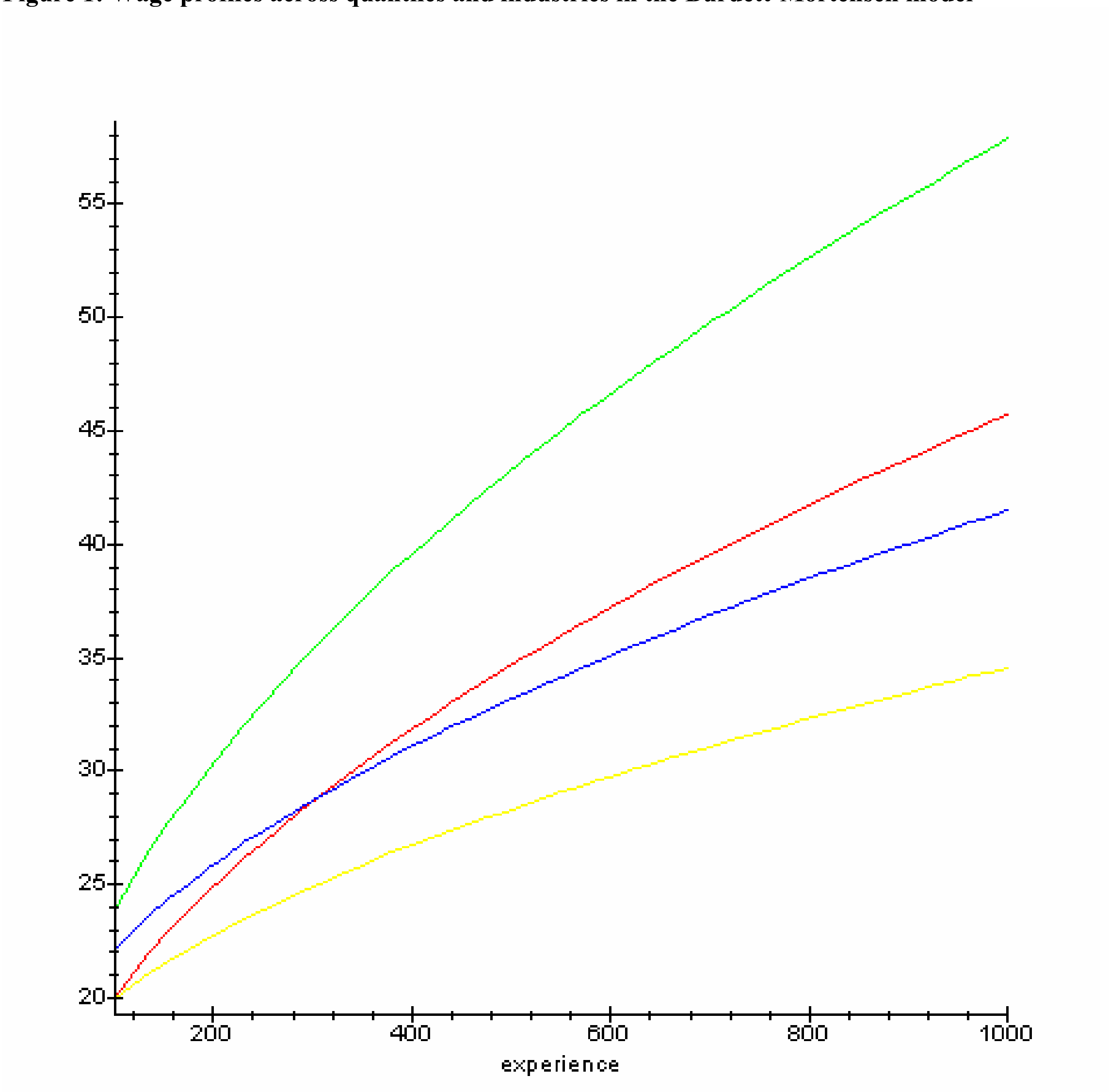
Given that $p \geq b$ (no firm would pay a worker a wage higher than his/her productivity), one could verify that $w_{.5} > w_{.2}$ and the difference between the two is larger the higher is the productivity parameter. It is straightforward to obtain that inter-quantile differences increase with productivity (true for all quantiles).

One could extend the analysis further assuming that the productivity parameter consists of two components: industry-specific productivity parameter and individual-specific. Assume that productivity can be written as a Cobb-Douglas function:

$$p = \mu^{\alpha}\varepsilon^{\beta}, \tag{1.11}$$

where μ is industry-specific component and ε is workers experience. Substituting (1.11) into (1.8) and (1.9) one could see that inter-quantile differences within each industry grow with experience of workers. Assuming that μ is stationary one could verify that differences between the same quantiles between two different industries grow with experience. However, the change in difference between the different quantiles between the two industries with respect to experience is unclear. The next graph illustrates the idea.

Figure 1: Wage profiles across quantiles and industries in the Burdett-Mortensen model



The blue and green curves show the 2nd and 5th wage decile in industry one respectively and the yellow and red curves show the 2nd and 5th wage decile in industry two respectively.[6] One could see that the difference between industry one and two for the same deciles is increasing with experience and inter-quantile differences within each industry are also growing with experience. However, the difference between the second decile in industry one and the 5th decile in industry two is declining up to some point and then grows again.

III. Data and descriptive evidence

The data used for the empirical analysis come from two sources. I use two large micro-data sets, IABS[7] for Germany and the CPS outgoing rotation group (CPS-ORG)[8] for the U.S. For ease of representation I confine my analysis only to years 1990, 1995 and 2000. Moreover, the analysis is restricted to skilled[9] male workers aged below 65 employed full-time in wholesale and retail trade.[10] Appendix one graphs the nonparametric estimates of log wages over potential experience (figures 2 to 7). For Germany for all selected years the differential between the two industries is growing with experience as predicted by the Burdett-Mortensen model. For the U.S. the picture is less clear. It seems that in the United States the inter-industry wage differential remains constant with experience.

IV. Estimation

1. Fractional polynomials

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[6] The parameters are chosen arbitrarily.
[7] See Bender and Haas (2002).
[8] See Webster, 2000, and Gao, 2003, for more thorough descriptions
[9] The issues of harmonization of the two datasets are addressed in Möller and Aldashev (2005).
[10] German data is restricted to West Germany only

The Burdett-Mortensen model outlined in section 2 predicts that inter-quantile differentials within each industry should grow with experience. Moreover, the model predicts the difference between lower quantiles to be higher than between higher quantiles. The problem arises when one wants to specify the function of the effect of experience on wages. Usually it is done in Mincerian way to allow for nonlinearities by introducing a linear and a square term. However, this produces only a limited family of curves. Following Royston and Altman (1994) I applied fractional polynomial of degree two to search for a specification to best suit the data. Figure 8 plots the fractional polynomial estimates vs. Mincerian (linear and a square term) with the lowess smoothing estimates as a benchmark. The graph shows that the fractional polynomial estimates show a better fir especially in the tails.

The polynomial of degree m may be written as:

$$\beta_0 + \beta_1 x + \beta_2 x^2 + \ldots + \beta_m x^m , \tag{1.12}$$

and the fractional polynomial of degree m with m integer and/or $p_1 < \ldots < p_m$ fractional powers:

$$\beta_0 + \beta_1 x^{p_1} + \beta_2 x^{p_2} + \ldots + \beta_m x^{p_m} , \tag{1.13}$$

where for a power p:

$$x^p = \begin{cases} x^p & \text{if } p \neq 0 \\ \ln x & \text{if } p = 0 \end{cases} . \qquad (1.14)^{11}$$

Fractional polynomial may also contain repeated powers. For example if $m = 2$ and $p_1 = p_2 = c$, then the fractional polynomial can be written as:

[11] A Mincerian type regression can be considered as a special case with $m = 2$ and $p = \{1, 2\}$.

$$\beta_0 + \beta_1 x^c + \beta_2 x^c \ln x . \tag{1.15}$$

For a given m (specified a priori) the authors suggest selecting the best power vector $\tilde{p} = (\tilde{p}_1, ..., \tilde{p}_m)$ over the restricted parameter space $\Re$ with the lowest deviance D. Where deviance is defined as $D = -2 \times L$, where L stands for the log-likelihood function. The authors argue that "…experience so far suggests that $\Re = \{-2, -1, -0.5, 0, 0.5, 1, 2, ..., \max(3, m)\}$ …is sufficiently rich to cover many practical cases adequately." (Royston and Altman, 1994: 434) Define $\hat{p}$ as the full MLE of p, which is allowed to vary continuously, i.e. not restricted to $\Re$. The statistic $D(m, p) - D(m, \hat{p})$ has a χ^2-distribution with m degrees of freedom. Hence, the obvious test for any specification against the "best" specification is $D(m, p) - D(m, \tilde{p})$.

2. Strategy

At the first stage I run fractional polynomial regression on cross-sectional data for Germany and the USA (years 1990, 1995, and 2000) with $m = 2$:[12]

$$\ln w_i = \beta_0 + \beta_1 EXP_i^{p_1} + \beta_2 EXP_i^{p_2} + \varepsilon_i . \tag{1.16}$$ [13]

The regression is run for retail and wholesale trade pooled together.[14] Following Royston and Altman (1994) p with lowest deviance is chosen.

The functional relationships between wages and experience are:

$$\ln w_i = \beta_0 + \beta_1 \ln EXP_i + \beta_2 EXP_i^3 + \varepsilon_i , \tag{1.17}$$

for Germany and the USA in 1990,

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[12] Censored observations are excluded. This poses no problem for the US data (0.2-0.3% are censored) but for Germany this is more troublesome (about 8% are censored).

[13] EXP is a transformed variable. EXP=experience+.1 so EXP>0.

[14] Of course one can argue that the functional form itself may vary across industries and the regression has to be run separately for retail and wholesale.

$$\ln w_i = \beta_0 + \beta_1 EXP^{1/2}{}_i + \beta_2 EXP_i^3 + \varepsilon_i\,, \tag{1.18}$$

for the United States in 1995,

$$\ln w_i = \beta_0 + \beta_1 EXP_i^{1/2} + \beta_2 EXP_i^{1/2} \ln EXP_i + \varepsilon_i\,, \tag{1.19}$$

for Germany in 1995,

$$\ln w_i = \beta_0 + \beta_1 EXP_i^{1/2} + \beta_2 EXP_i^2 + \varepsilon_i\,, \tag{1.20}$$

for the US in 2000, and

$$\ln w_i = \beta_0 + \beta_1 EXP_i^{1/2} + \beta_2 EXP_i + \varepsilon_i\,, \tag{1.21}$$

for Germany in 2000.

At the next stage, given the functional form determined at stage one, quantile regressions on differences between 4th and 2nd, 6th and 4th, and 8th and 6th deciles have been run. Figures 10 and 11 present inter-quantile differences for wholesale trade in 1990 for Germany and USA. The results show that for the US the difference between 4th and 2nd is not significantly different from the difference between 8th and 6th whereas the difference between the 6th and 4th decile is significantly lower at least for experience between 7 and 30 years. Theoretically, the difference between 4th and 2nd deciles should be higher than the difference between 6th and 4th; this is confirmed by the results. However, on the other hand, the difference between 6th and 4th should be higher than the difference between 8th and 6th decile. The results show just the opposite. So the theory is only partly confirmed. For Germany the picture is totally different. The difference between 4th and 2nd is not significantly different from the difference between 6th and 4th decile whereas the difference between the 8th and 6th decile is significantly higher at experience below 22 years but lower at experience level above 22 years. This could mean that in wholesale in Germany the lower paid catch up over time with the highest paid group.

In retail in the United States in 1990 the difference between 8th and 6th is not significantly different from the difference between 6th and 4th decile whereas the difference between the 4th and 2nd decile is significantly higher at experience levels from 2 to 30 years. This is in line with theory. For Germany, the difference between 8th and 6th decile is significantly higher than the differences between 6th and 4th and 4th and 2nd deciles (at least for experience between 5 to 40 years), which are not statistically different from each other. This is in discord with the theoretical model. Moreover, the curve showing the differences between 8th and 6th decile is steeper than others. These results indicate that in Germany in retail in 1990 the disparities in the upper tail of the earnings distribution grew faster than disparities in the lower tail.

In 1995 not much has changed for the wholesale trade in the United States. In retail, however, the highest differences are reported between 8th and 6th decile and the lowest between 6th and 4th deciles. This indicates higher dispersion in the upper tail of the distribution. In Germany in wholesale trade the differences between 8th and 6th decile are always greater than differences between 6th and 4th and 4th and 2nd with experience below 30 years. For experience above 30 the inter-quantile differences seem to be similar, so the distribution becomes symmetric. In retail the situation has changed also dramatically since 1990. In 1995 in retail the highest difference is between 4th and 2nd deciles and lowest between 6th and 4th. Moreover, the line showing the difference between 4th and 2nd decile is the steepest. This indicates that wages of the lowest-paid group grow slower than for higher paid groups and the difference in payment between workers in the 2nd decile of the earnings distribution and in 4th decile grows with experience.

In 2000 the picture for the wholesale trade in the USA is not clear. In retail the only unambiguous result is that the difference between 4th and 2nd decile is the highest for experience above 32 years. For Germany in wholesale trade the picture is similar to the year 1995. The highest difference is between 8th and 6th decile up to about 30 years of experience; after this experience level there is no significant difference between the inter-quantile differentials. In retail in Germany the situation has changed since 1995. The highest differential is between 8th and 6th decile for experience above 8 years. Moreover, the difference between 4th and 2nd deciles seems to be constant with experience. After about 20 years of experience the difference between 6th and 4th deciles is significantly higher than the difference between the 4th and 2nd deciles. This is in discord with theory. This suggests that possibly in upper quantiles wages grow faster than predicted by the theory.

V. Conclusion

The aim of the paper was to demonstrate that the model developed by Burdett and Mortensen (1998) is capable not just to establish equilibrium wage dispersion but give various predictions about wage changes over one's experience span. Empirical analysis supports the theoretical view that inter-quantile differences in wages grow with experience[15]. The hypothesis that the difference between lower quantiles should be higher than the difference between upper quantiles is only partly confirmed. However, it must be noted that this might be not very surprising as the equilibrium wage distribution derived in Burdett-Mortensen (1998) has an increasing density, which is not the case in reality.

[15] There were several exceptions. For example, in US retail in 1990 and in German wholesale trade in 1995 – the curves seem to be rather flat. The contradictory case is found in German wholesale trade in 1990 where the difference between 8th and 6th deciles is declining with experience.

Nevertheless, the model is a very convenient tool in analyzing wage profiles over different quantiles of the wage distribution.

References

Bender, S., Haas, A., 2002: “Die IAB-Beschäftigtenstichprobe“, in: Kleinhenz, Gerhard (Hg.): IAB-Kompendium Arbeitsmarkt-und Berufsforschung, Beitr-AB 250, Nürnberg, S. 3 - 12.

Bontemps, C., Robin, J.-M., and van den Berg, G.J., 2000: “Equilibrium Search with Continuous Productivity Dispersion: Theory and Non-Parametric Estimation.” *International Economic Review* 40: 1039-1075.

Burdett, K., and Mortensen, D.T., 1998. “Wage Differentials, Employer Size, and Unemployment.” *International Economic Review* 39: 257-273.

Gao, D., 2003: "Wage Analysis Computations," in Mishel, Lawrence, Jared Bernstein, and Heather Boushey, The State of Working America 2002-2003, Ithaca, New York: Cornell University Press.

Freeman, R.B. and Schettkat, R., 2001: “Skill Compression, Wage Differentials and Employment: Germany vs. the US, *Oxford Economic Papers* 3, 582-603.

Manning, A., 2003: *Monopsony in Motion*. Princeton University Press.

Möller, J., and Aldashev, A., 2005: “Industry Rents, Skills and Experience in Germany and the U.S. – Does the Evidence Support the Insider-Outsider Theory?”, Working paper.

Mortensen, D.T., 2003: *Wage Dispersion: Why Are Similar Workers Paid Differently?* Cambridge, Massachusetts.

Royston, P., and Altman, D.G., 1994: “Regression using Fractional Polynomials of Continuous Covariates: Parsimonious Parametric Modelling.” *Applied Statistics* 43: 429-467.

Royston, P., and Sauerbei, 1999: “Building multivariable prognostic and diagnostic models: Transformation of the predictors by using fractional polynomials.” *Journal of the Royal Statistical Society* A162: 71-94.

Schmitt, J., 2003: “Creating a Consistent Hourly Wage Series from the Current Population Survey’s Outgoing Rotation Group, 1979-2002.” Working paper.

van den Berg, G., J., and Ridder, G., 1998: “An Empirical Equilibrium Search Model of the Labor Market.” *Econometrica* 66(5): 1183-1221.

Webster, D., 2000: “Economic Policy Institute CPS ORG Labor Extracts, 1973-1999, Version 1.31.” Washington, DC: Economic Policy Institute.

Appendix 1

Figure 2: Smoothed log wage over experience span for Germany 1990 (red – retail trade, blue – wholesale trade)

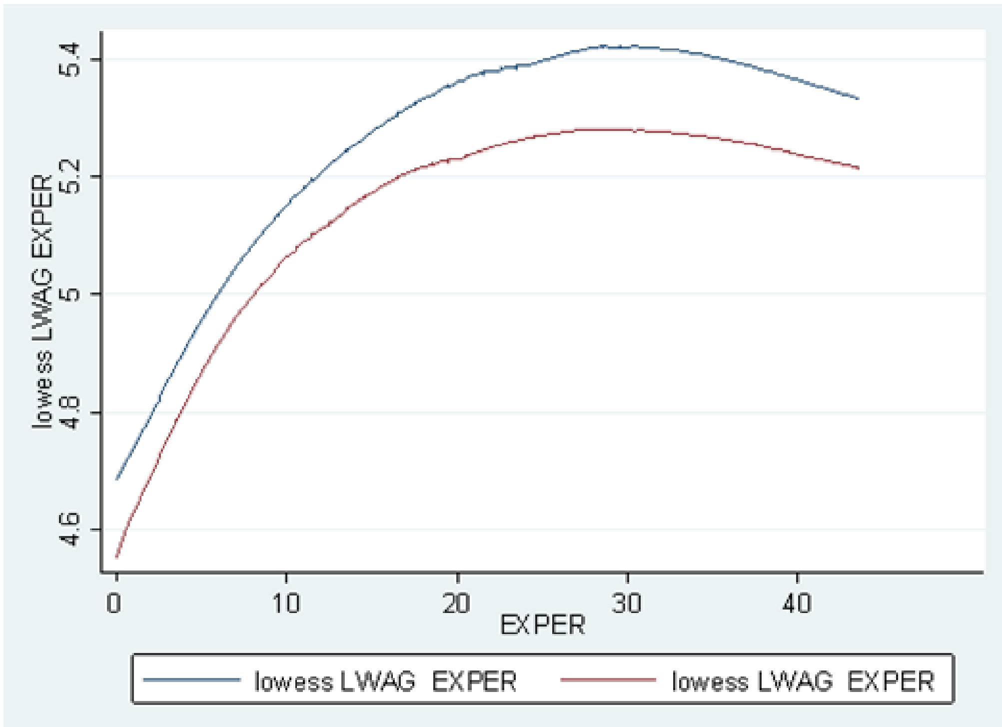


Figure 3: Smoothed log wage over experience span for the United States 1990 (red – retail trade, blue – wholesale trade)

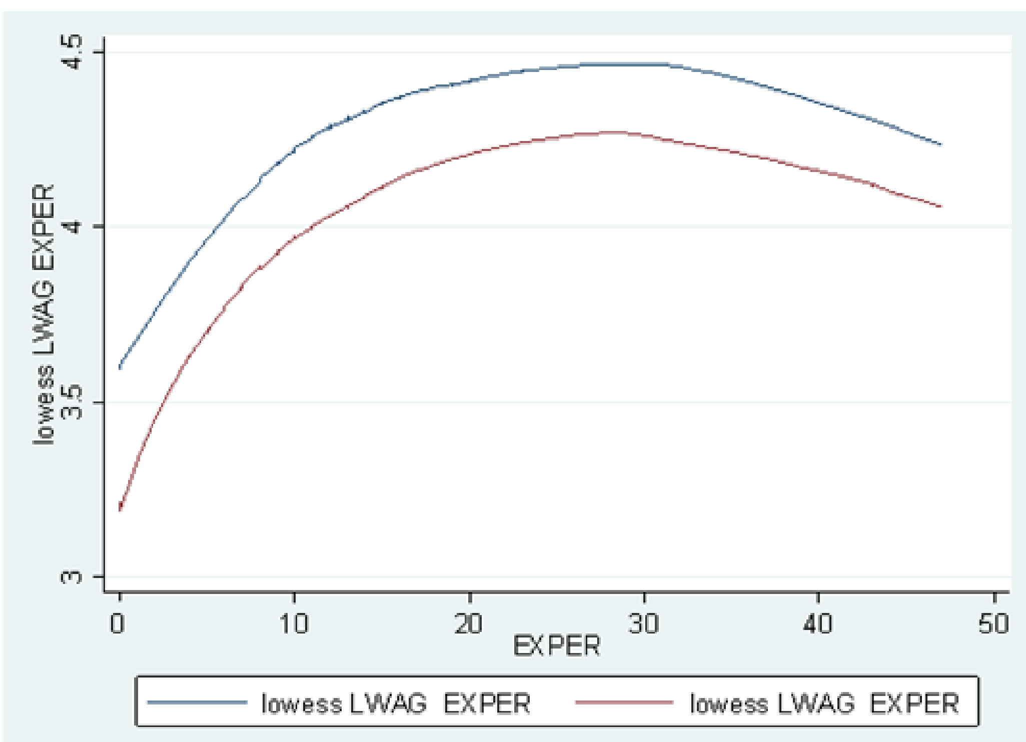


Figure 4: Smoothed log wage over experience span for Germany 1995 (red – retail trade, blue – wholesale trade)

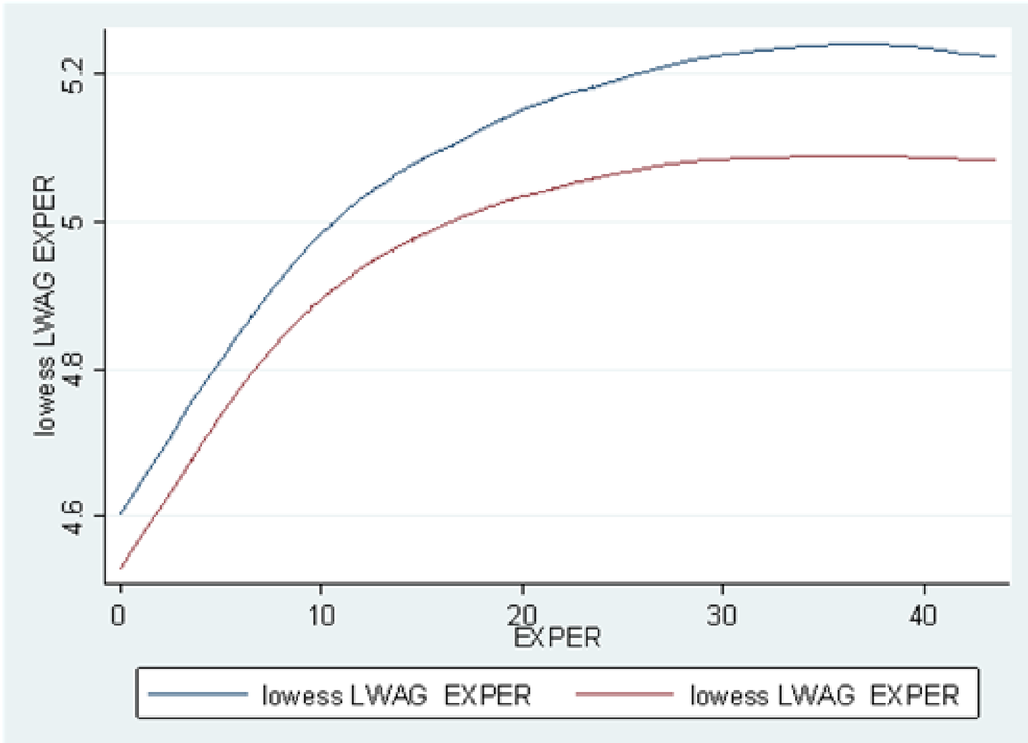


Figure 5: Smoothed log wage over experience span for the United States 1995 (red – retail trade, blue – wholesale trade)

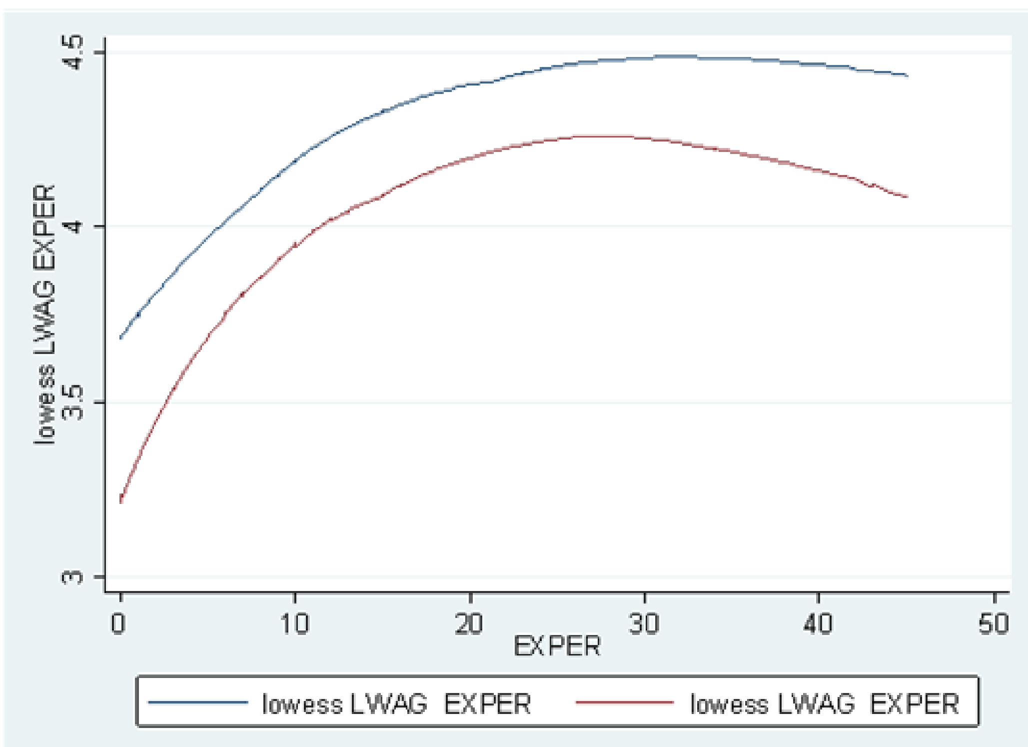


Figure 6: Smoothed log wage over experience span for Germany 2000 (red – retail trade, blue – wholesale trade)

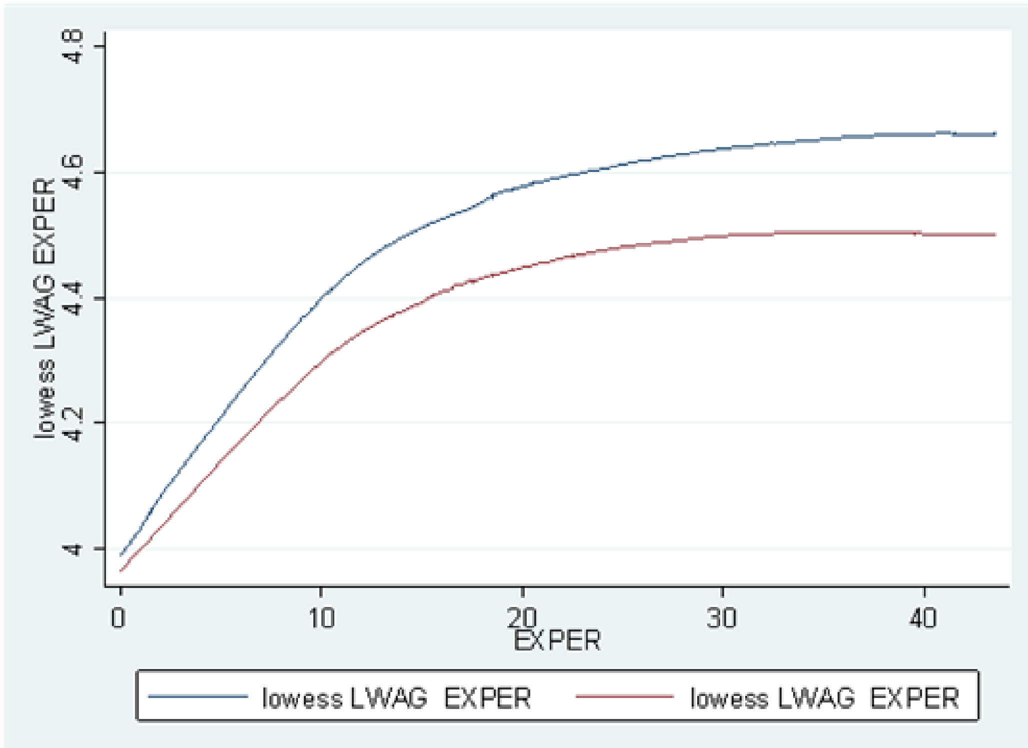
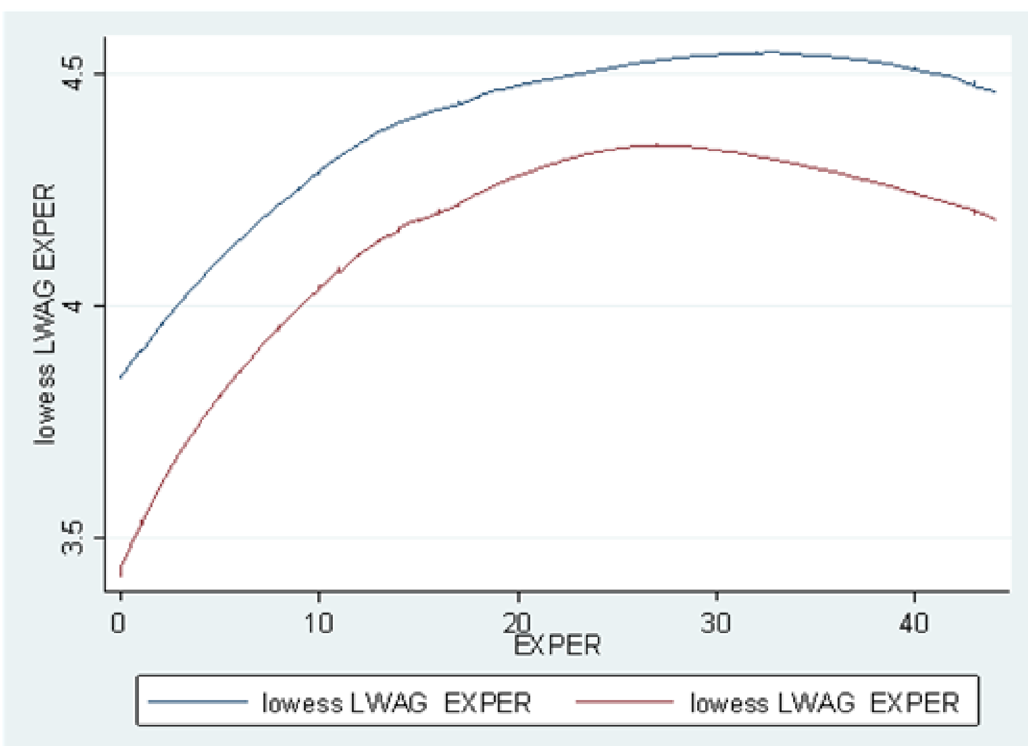


Figure 7: Smoothed log wage over experience span for the United States 2000 (red – retail trade, blue – wholesale trade)



Appendix 2

Figure 8: Mincerian regression (green dots) vs. fractional polynomial (blue line); lowess smoothing as benchmark (orange line) for the Germany 1995 for retail and wholesale trade.

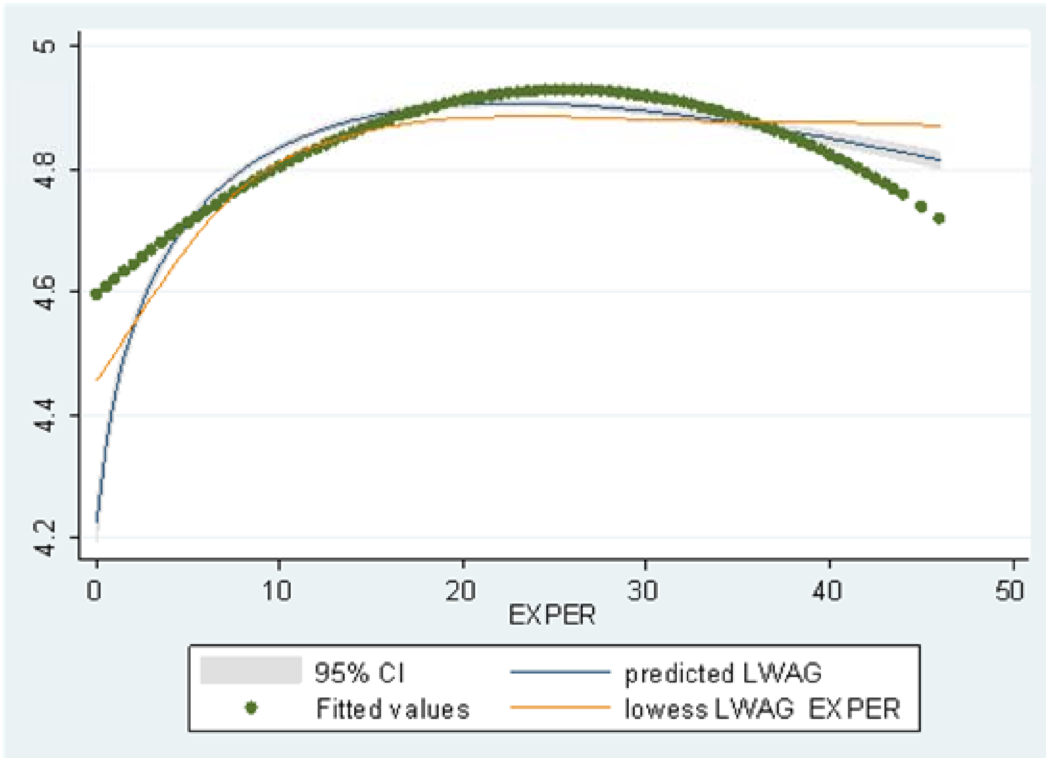


Figure 9: Wage profiles across quantiles (blue – 2nd decile, pink – 4th decile, green – 6th decile, orange – 8th decile). US 2000, retail.

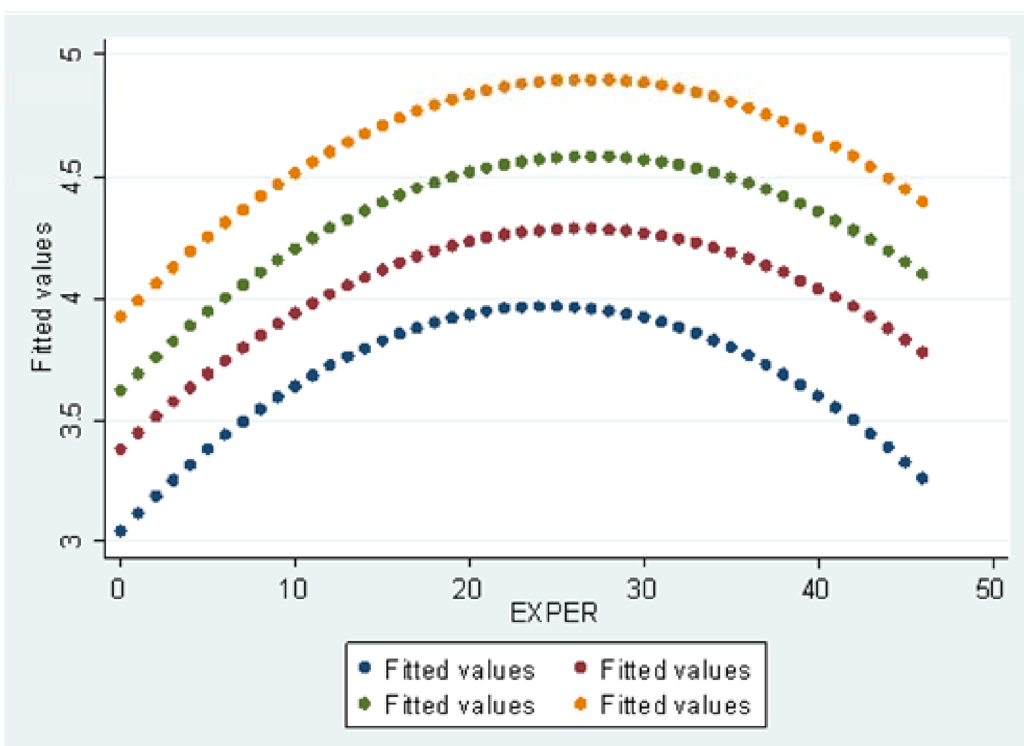


Figure 10: Interquantile differences, wholesale trade, USA, 1990. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

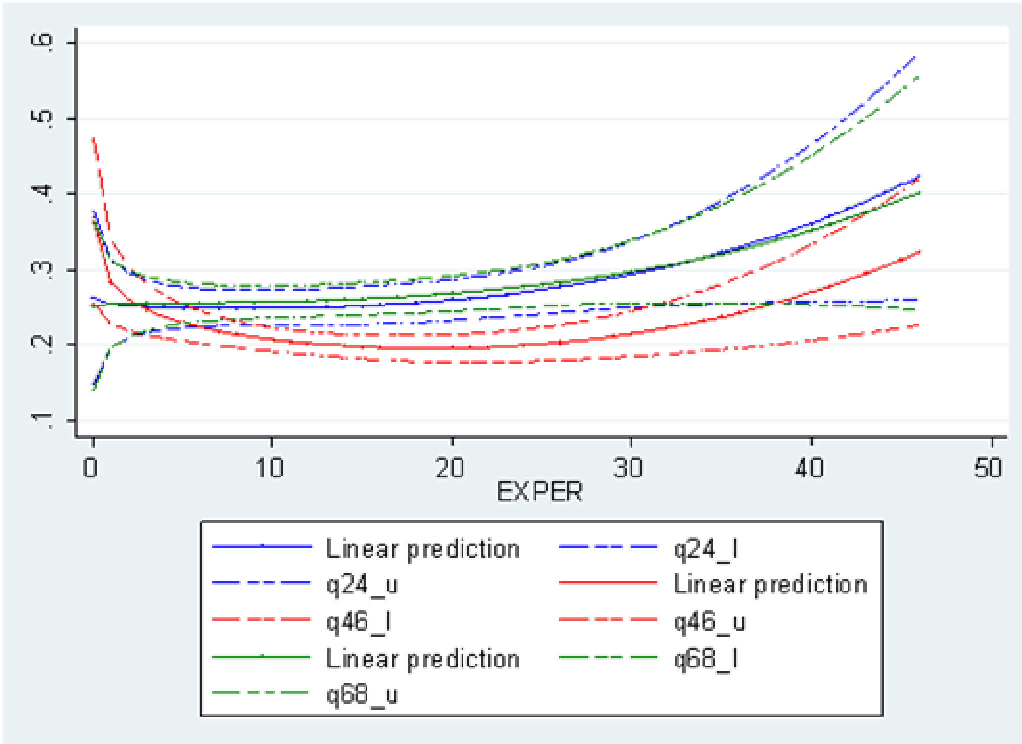


Figure 11: Interquantile differences, wholesale trade, Germany, 1990. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

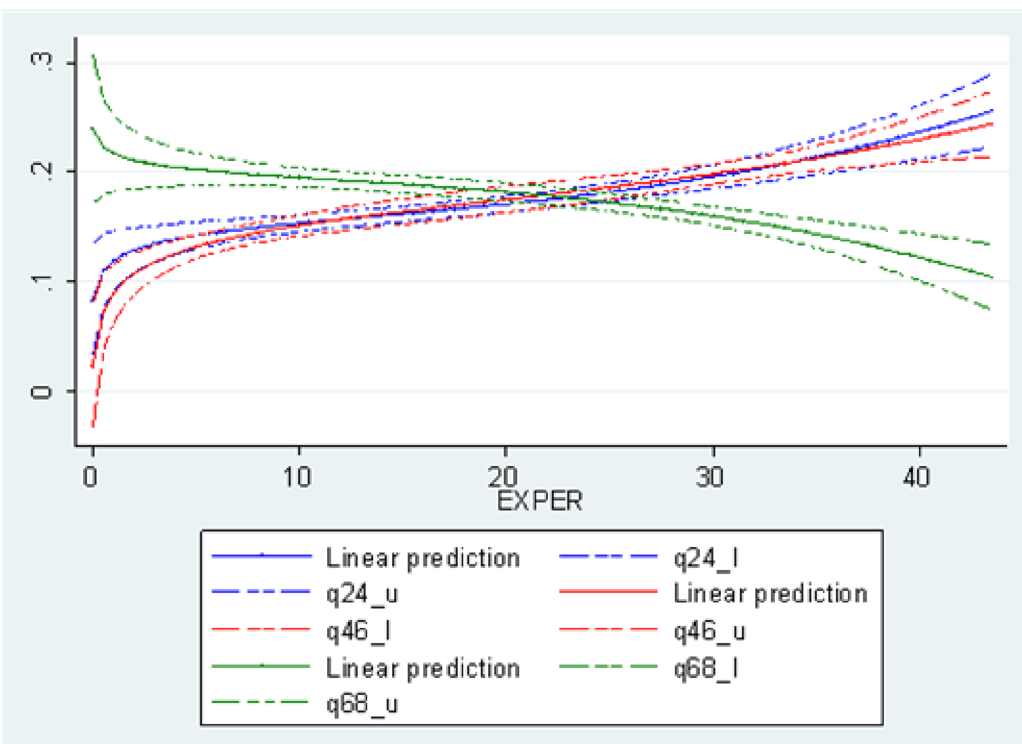


Figure 12: Interquantile differences, retail trade, USA, 1990. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

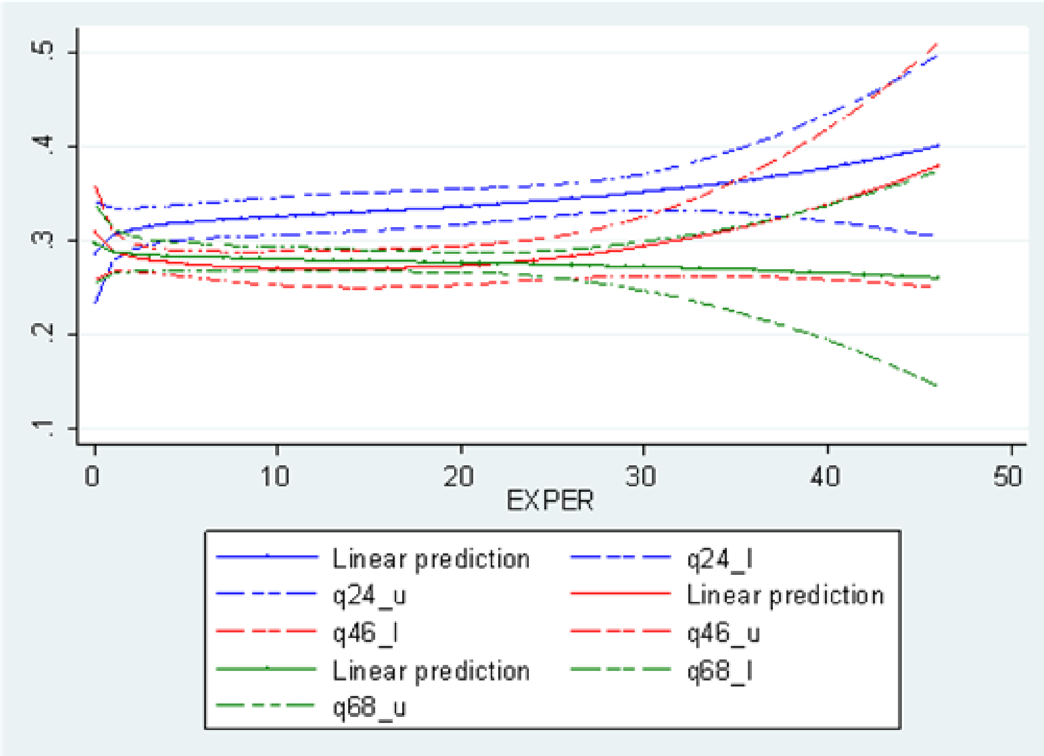


Figure 13: Interquantile differences, retail trade, Germany, 1990. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

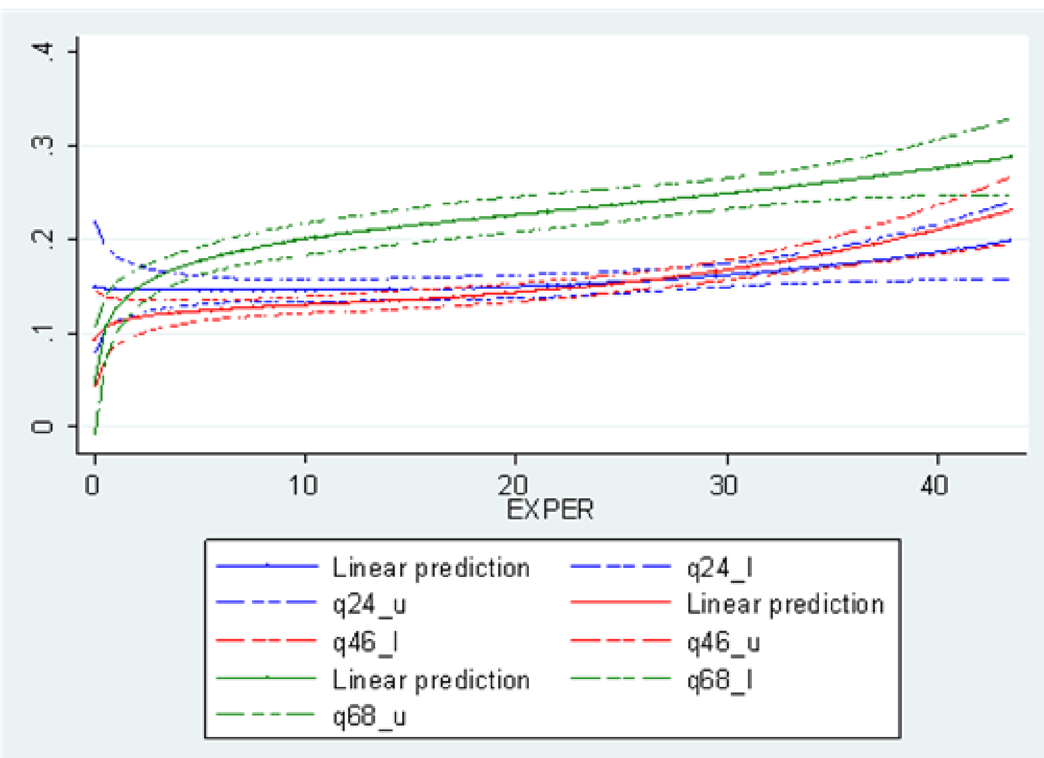


Figure 14: Interquantile differences, wholesale trade, USA, 1995. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

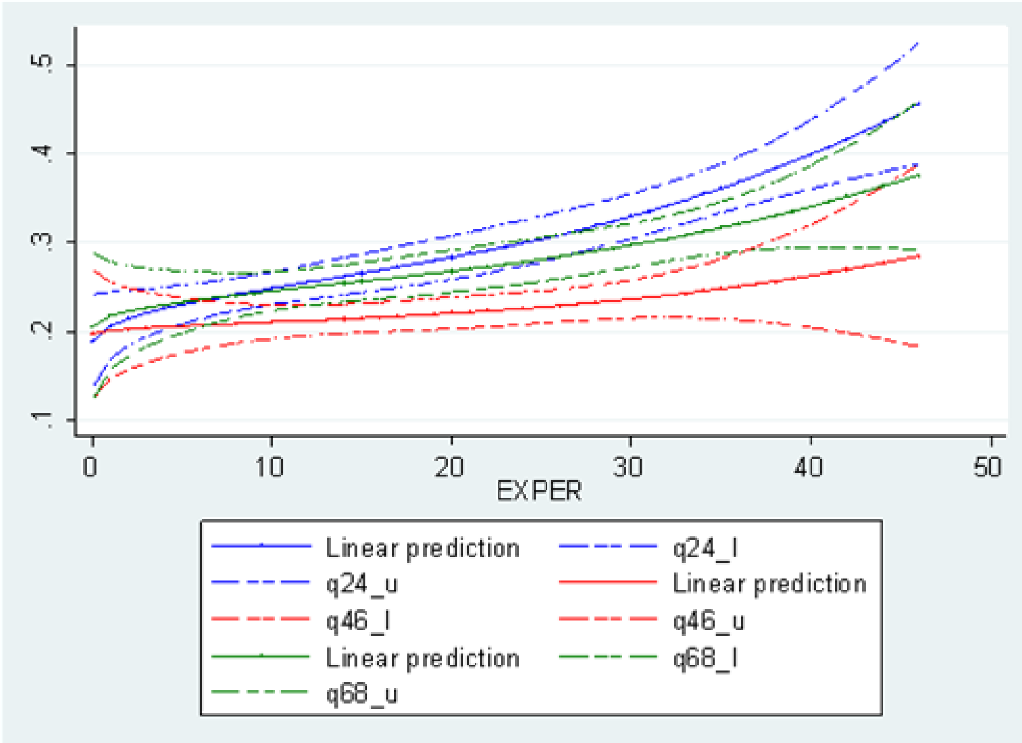


Figure 15: Inter-quantile differences, wholesale trade, Germany, 1995. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

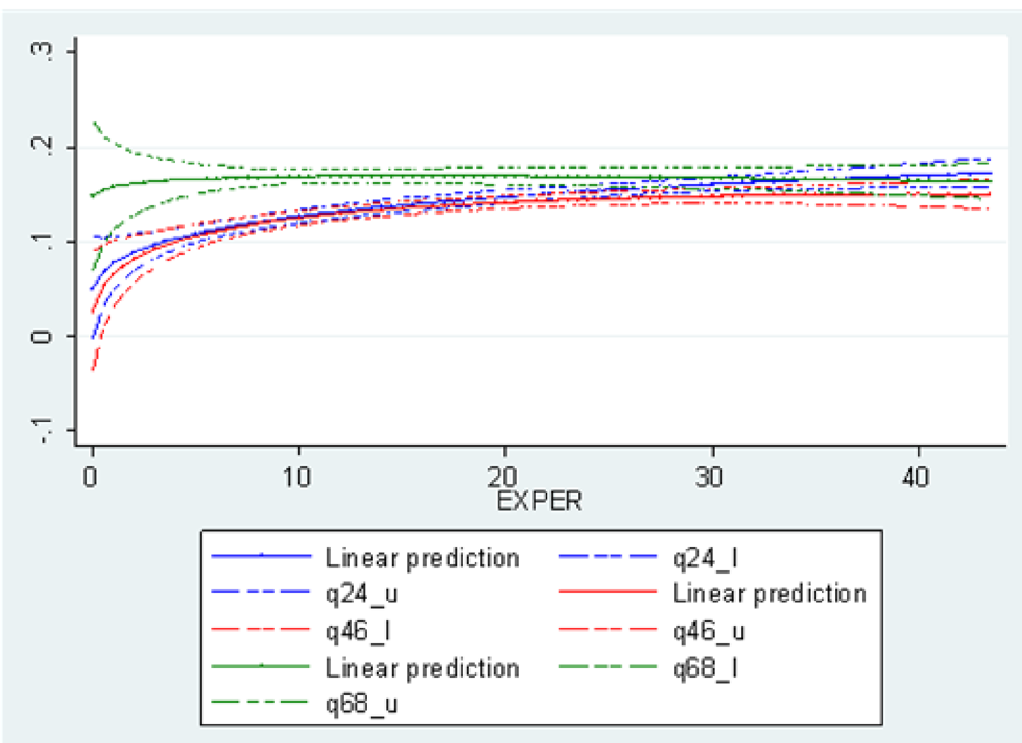


Figure 16: Interquantile differences, retail trade, USA, 1995. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

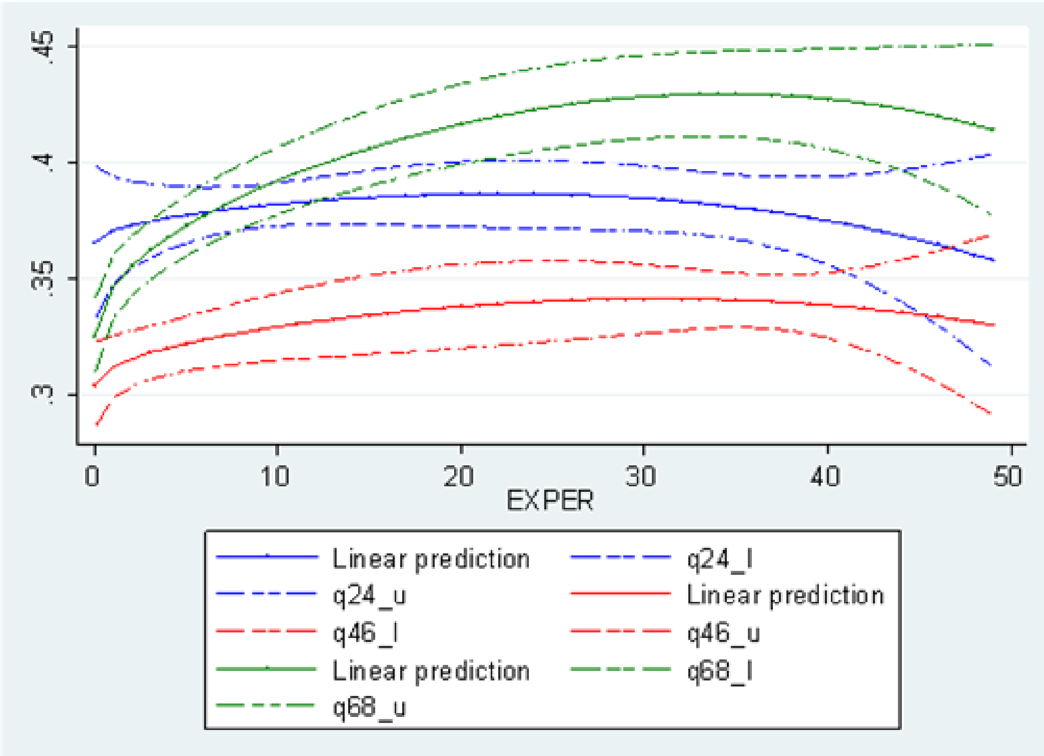


Figure 17: Interquantile differences, retail trade, Germany, 1995. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

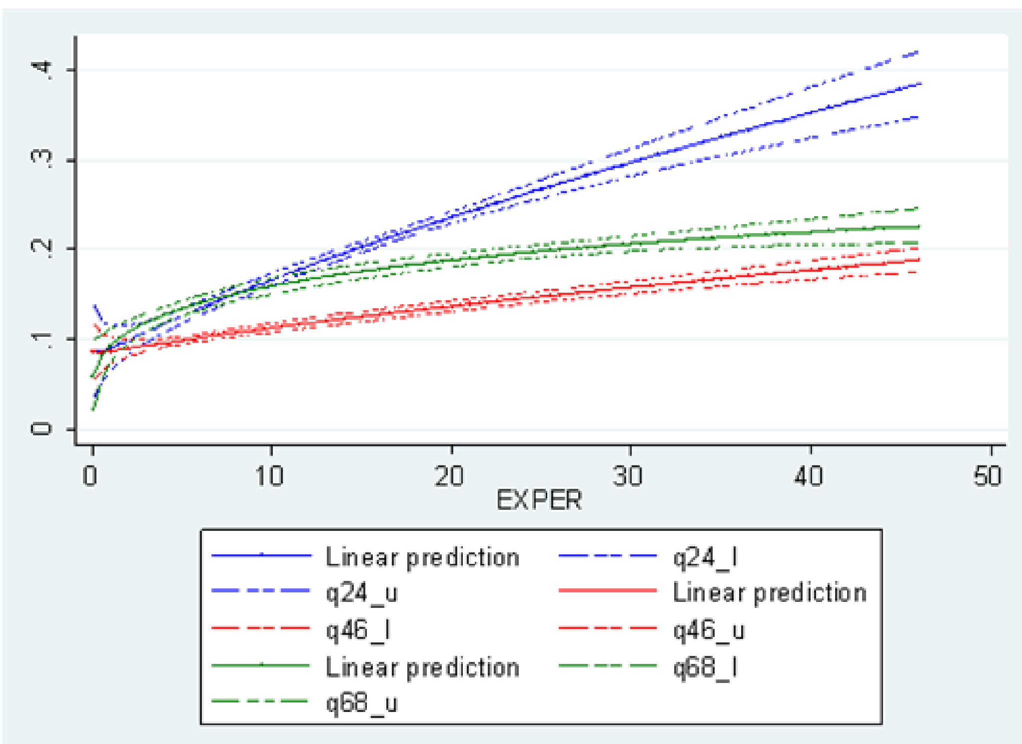


Figure 18: Interquantile differences, wholesale trade, USA, 2000. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

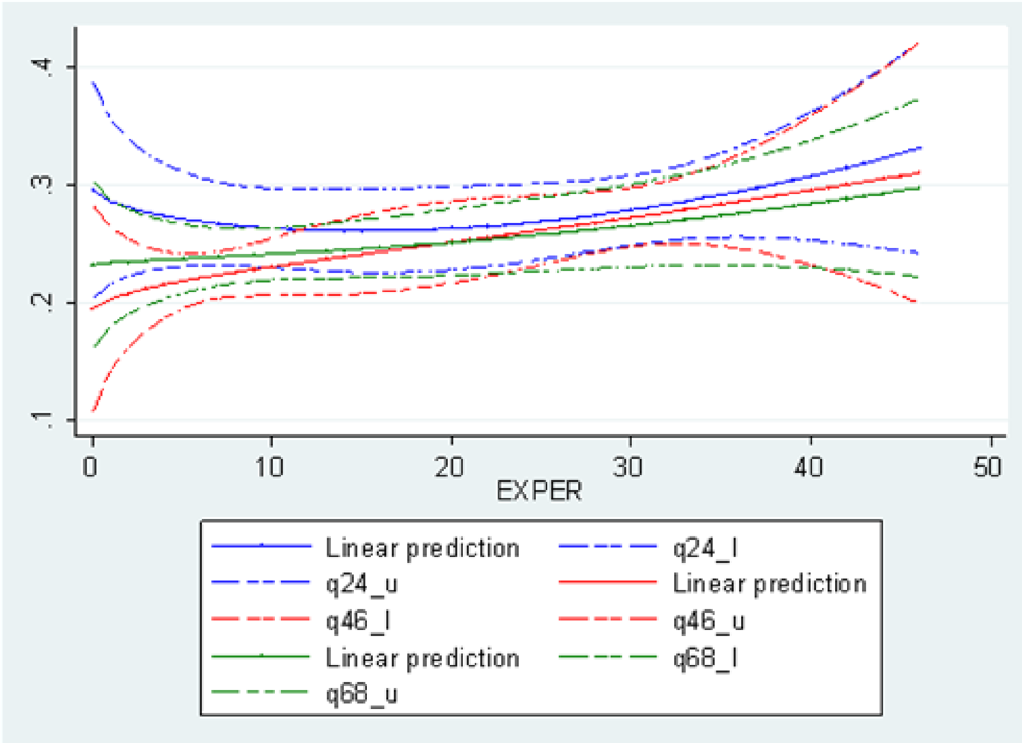


Figure 19: Interquantile differences, wholesale trade, Germany, 2000. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

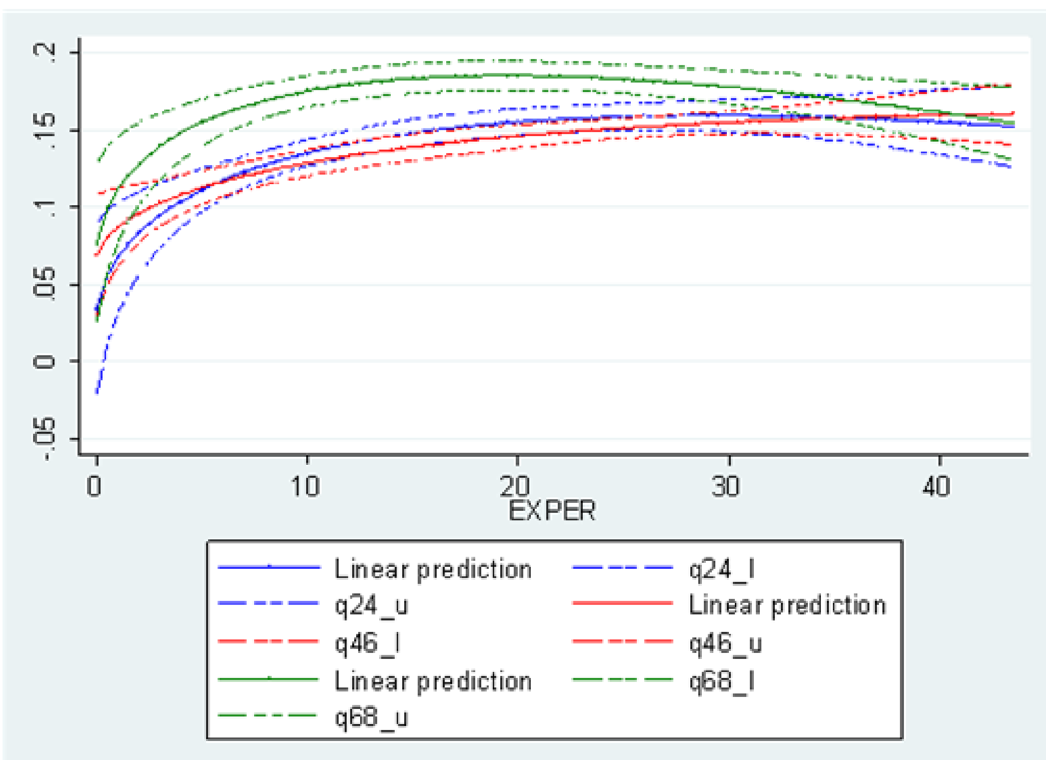


Figure 20: Interquantile differences, retail trade, USA, 2000. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.

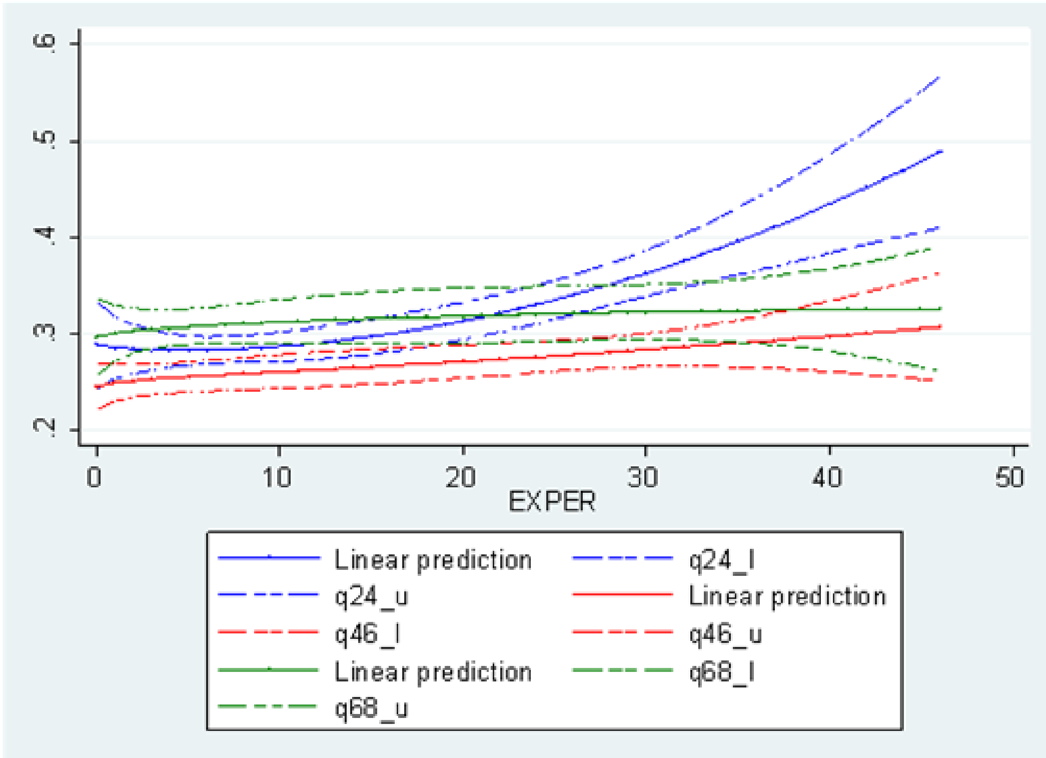


Figure 21: Interquantile differences, retail trade, Germany, 2000. Blue line – difference between 2nd and 4th decile, red – 4th and 6th, green – 6th and 8th. Dashed line represent 95% confidence intervals.