# Duopolistic Hiring and Sales Competition - A Theoretical and Experimental Analysis - 

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#### Abstract

Two firms compete on a sales market as well as in hiring four available workers. The duopolists' sales levels of their homogeneous products depend on their workforce. There are two each of two types of workers, mobile and immobile, with differing effort costs. An immobile worker's effort costs are lower when he is employed by the local firm, whereas mobile workers always incur the same effort costs. The principals offer employment contracts to all four workers, each selects an employer. The workers' choices determine production levels and profits.

Our experimental results qualitatively confirm the theoretical predictions: Vanishing cost differences between mobile and immobile workers (homogeneous case) induce monopolistic hiring (one firm hires all workers), but low profits. In contrast, large cost differences between the two (heterogeneous case), which capture immobility of labor or firm specificity of human capital, result in higher profits and allow for various hiring constellations such as one firm hiring only low-cost workers or both firms employing both types.


JEL classification: C72, C90, F16, J21, J24, L10
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## 1 Introduction

Traditional principal-agent analysis (see e.g. Myerson, 1983) focuses solely on intra-firm conflicts and neglects competition on the labor and sales markets. Here, we investigate both, intra-firm and inter-firm conflicts, theoretically as well as experimentally, by extending earlier findings (see Berninghaus et al., 2007, and (less relatedly) Berninghaus et al., 2004). Two firms compete on two related markets, a labor and a sales market. There are two types of workers, mobile and immobile, with two candidates each. We analyze which hiring constellation results when firms can make simultaneous wage offers to all workers, who then choose their employer. We also discuss possible welfare effects in this multiple-principal-multiple-agent game. As Kirstein and Kirstein (2007) observe, the number of papers that simultaneously analyze oligopolistic competition between firms and intra-firm conflicts is rather small. To the best of our knowledge, our approach to capturing sales competition via hiring constellations is new ${ }^{1}$.

One possible interpretation of our model (see Berninghaus et al., 2007) is that firms (and thus principals) are located in different countries and that they can hire internationally on a global labor market. Another interpretation would rely on the usual idea of firm-specific human capital, in which we restrict firm specificity to one type of worker. A mobile worker has the same costs of producing wherever he is employed, whereas immobile workers have lower production costs when they are employed locally.

Our theoretical predictions are confirmed by the experimental observations: When the cost difference between immobile and mobile is negligible, firms offer the same wages to all workers and a monopolistic market structure prevails with a tendency to zero-profits. In contrast, if the cost difference is high, a broader pattern of hiring constellations occurs, such as one principal hiring all low-cost workers or both firms employing both types.

In Section 2, we present the model (analyzed in less detail by Berninghaus et al. (2007)). Section 3 contains the theoretical analysis and details the hypotheses concerning the equilibrium hiring constellations and welfare effects. The experimental protocol is described in Section 4. The experimental observations, discussed in Section 5, widely confirm the hypotheses. Section 6 concludes.

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## 2 Model

We assume a multiple-principal-multiple-agent model with two principals and four agents. Principals $A$ and $B$ own firm $A$ and firm $B$, respectively. Agents (workers) $A_{1}$ and $A_{2}$ are closer to firm $A$, while agents $B_{1}$ and $B_{2}$ are nearer to firm $B$. A worker $i$ has to spend one unit of effort $(e=1)$ to produce exactly one unit of output when employed by principal $j$ for all $i \in\left\{A_{1}\right.$, $\left.A_{2}, B_{1}, B_{2}\right\}$ and $j \in\{A, B\}$.

A principal can hire several agents, but there are no synergy effects. We distinguish between mobile and immobile workers according their proximity to firms or to capture their firm specificity. The immobile workers $A_{1}$ and $B_{1}$ (type 1) have specific skills, meaning that their costs of effort, $e>0$, denoted by $C(e)$, are higher when they are employed by the more distant firm. We have, for $e \geq 0$,

$$
\begin{equation*}
C(e)=\frac{c}{2} \cdot e^{2} \text { with } c>0 \tag{1}
\end{equation*}
$$

when type 1s work for their local employer and

$$
\begin{equation*}
C(e)=\frac{d}{2} \cdot e^{2} \text { with } d>c \tag{2}
\end{equation*}
$$

when they are employed by the more distant firm.
The costs of effort for the type 2 workers are not firm-specific and are assumed to be given by (1) throughout.

The homogeneous sales market is captured by a standardized, linear inverse demand function:

$$
\begin{equation*}
p=a-X \tag{3}
\end{equation*}
$$

with

$$
\begin{equation*}
X=x_{a}+x_{b}, x_{a}, x_{b} \geq 0 \text { and } x_{a}+x_{b} \leq a \tag{4}
\end{equation*}
$$

where $p(\geq 0)$ denotes the uniform sales price, $a$ the reservation price and $X$ the total supply, which is composed of the two individual sales amounts of firm A $\left(x_{a}\right)$ and firm B $\left(x_{b}\right)$.

The decision process consists of three stages, in which common knowledge of the rules as well as complete and perfect information is assumed.

## Contract Offer Stage:

Each principal $j$ offers a fixed wage ${ }^{2} W_{j}^{i} \in \mathbb{R}$ and a piece rate $s_{j}^{i} \geq 0$ to every worker $i$, i.e., a linear contract:

$$
\left(W_{j}^{i}, s_{j}^{i}\right)
$$

for all four potential workers $i$.

## Choice of Employer Stage:

Knowing his offers from both principals, each agent $i$ selects one of the two.

## Choice of Effort:

Being aware of the hiring constellation, which specifies the employer for all four agents, each agent $i$ chooses his effort level $e_{i}$.

Since our focus is on the contract offers in the theoretical as well as in the experimental analysis, we use the optimal decisions of agents in the two latter stages already in the description of our model. Agents select their principal based on what they would earn in the case of optimal efforts for each contract offered. To neglect unemployment, the participation constraint is assumed to be not binding, resulting in full employment throughout. ${ }^{3}$ Thus, there are $2^{4}=16$ possible hiring constellations (HCs), since every agent $\left(A_{1}, A_{2}, B_{1}, B_{2}\right)$ chooses between two principals $(A$ and $B)$ :

- Scenario O: All job candidates work for their local firm.
- Scenarios N1-N4: One firm hires both type 2 workers and one type 1 worker. If principal $A(B)$ hires both type 2 workers and her local type 1 worker, we call this scenario N1 (N2). We refer to this scenario as N3 (N4) if the other (distant) type 1 worker is employed by principal $A(B)$ in addition to the two type 2 workers.
- Scenarios V1-V4: One firm hires both type 1 workers in addition to one type 2 worker. Scenarios V1-V4 are designated analogously to scenarios N1-N4 above with respect to the third worker hired.
- Scenario E: Each firm hires its local type 1 worker and the distant type 2 worker.

[^2]- Scenario I: Both type 2 workers are working for their local principal, while all type 1 workers are employed by the distant firm.
- Scenario T: All workers work for the distant firm.
- Scenarios R1 and R2: Both type 1 workers are working in the same firm and both type 2 workers in the other one. In R1(R2), both type 2 workers are employed by principal $A(B$.)
- Scenarios M1 and M2: In M1 (M2), principal $A(B)$ hires all four workers.

How can the HC be derived? If worker $i$ accepts the contract offer $\left(W_{j}^{i}, s_{j}^{i}\right)$ by firm $j$, his payoff depends on his effort $e_{i}$ according to

$$
\begin{equation*}
\omega_{i}=W_{j}^{i}+s_{j}^{i} \cdot e_{i}-C\left(e_{i}\right) . \tag{5}
\end{equation*}
$$

Thus, the optimal effort is either

- $e_{i}^{*}=s_{j}^{i} / c$ for a type 1 worker in his local form and for both type 2 workers or
- $e_{i}^{*}=s_{j}^{i} / d$ for a type 1 worker in the distant firm.

Inserting this into (5) yields the effort-optimal payoff; i.e. the wage income,

$$
\begin{equation*}
\omega_{i}^{*}=W_{j}^{i}+\frac{\left(s_{j}^{i}\right)^{2}}{2 c} \quad \text { or } \quad \omega_{i}^{*}=W_{j}^{i}+\frac{\left(s_{j}^{i}\right)^{2}}{2 d} \tag{6}
\end{equation*}
$$

All workers $i$ simultaneously compare their payoff from working for either their local principal $h$ or for the distant principal $f$ (with $h, f \in\{A, B\}$ and $h \neq f$ ) and select the principal offering the higher payoff.
Thus, a type 1 worker chooses his home (foreign) firm $h(f)$ if

$$
\begin{equation*}
W_{h}^{i}+\frac{\left(s_{h}^{i}\right)^{2}}{2 c}>(<) W_{f}^{i}+\frac{\left(s_{f}^{i}\right)^{2}}{2 d} \tag{7}
\end{equation*}
$$

Similarly, a type 2 worker selects firm $h(f)$ if

$$
\begin{equation*}
W_{h}^{i}+\frac{\left(s_{h}^{i}\right)^{2}}{2 c}>(<) W_{f}^{i}+\frac{\left(s_{f}^{i}\right)^{2}}{2 c} \tag{8}
\end{equation*}
$$

In the case of equality in (7) or (8), an employee can choose either employers. Anticipating such optimal reactions, employer $j$ 's profit is:

$$
\begin{equation*}
\pi_{j}=R_{j}-\sum_{i} d_{j}^{i} \cdot\left(s_{j}^{i} \cdot e_{i}^{*}+W_{j}^{i}\right) \tag{9}
\end{equation*}
$$

where $d_{j}^{i}=1$ if worker $i$ works for principal $j$ and $d_{j}^{i}=0$ otherwise. The revenue $R_{j}$ of firm $j$ is defined as

$$
\begin{equation*}
R_{j}=p \cdot \sum_{i} d_{j}^{i} \cdot e_{i}^{*}=\left[a-\sum_{k} \sum_{i} d_{k}^{i} \cdot e_{i}^{*}\right] \cdot \sum_{i} d_{j}^{i} \cdot e_{i}^{*}, \tag{10}
\end{equation*}
$$

where $i \in\left\{A_{1}, A_{2}, B_{1}, B_{2}\right\}$ and $j, k \in\{A, B\}$. This shows the crucial aspect of the model, the expression of inter-firm hiring competition via the simultaneous wage offers of both principals to all four workers.

## 3 Inter-Firm Competition

Berninghaus et al. (2007) prove that there are exogenous parameter configurations (for $a, c, d$ ) such that Scenario O - the so-called immobility scenario - can be an equilibrium scenario. They derive conditions such that a unilateral deviation from Scenario O to any of the other fifteen HCs does not pay for either of the two principals $A$ and $B$; provided that workers $A_{1}, A_{2}, B_{1}$, and $B_{2}$ behave optimally and both principals decide symmetrically.

Here we drop the symmetry assumption and extend the analysis by checking for each of the sixteen scenarios whether a unilateral deviation is profitable and if this hiring constellation can result in equilibrium. This leads to non-existence and numerical existence results for pure strategy equilibria. From this, hypotheses for the experimental investigation are derived.

The theoretical analysis is structured into three steps to check which HCs might or might not occur as an equilibrium outcome:

1. For each of the sixteen HCs, we calculate the principals' optimal piece rates for hired workers. All fixed wage offers as well as piece rates of non-successful offers remain so far undetermined variables. Thus the principals' profits for each HC still depend on these variables.
2. For each HC, we derive conditions for these variables such that both principals' unilateral deviations to Scenario M1 or M2 are not profitable. These conditions can be rearranged such that we receive a lower
and an upper bound for a fixed wage offer or a combination of fixed wage offers, depending only on exogenous parameters.
3. Comparing the upper and lower bounds for fixed wages depending on exogenous parameters only, we can exclude some HCs as equilibrium HCs due to profitable deviations to monopoly. Further analysis allows us to exclude even more HCs as possible equilibria. The result is that there are parameter regions, $c=d$ among them, without a pure strategy equilibrium. By numerical examples, we also show that multiple equilibria are possible.

### 3.1 Optimal Piece Rates for Given HCs

Following Berninghaus et al. (2007), we use a substitution principle by which we can substitute the fixed wage offer in principal $j$ 's profit formula (9) by its competitor's strategy variables.

## Lemma 1 (Substitution Principle):

In an equilibrium scenario, the following relation holds:

$$
W_{h}^{i}=W_{f}^{i}+\frac{\left(s_{f}^{i}\right)^{2}}{2 x}-\frac{\left(s_{h}^{i}\right)^{2}}{2 y}
$$

for all $i \in\left\{A_{1}, A_{2}, B_{1}, B_{2}\right\}, h, f \in\{A, B\}, h \neq f$, where cost parameters $x$ and $y$ equal $c$ or $d$ depending on the worker type and the relation between $i$ and $f$, resp. $h$.

Proof: Let w.l.o.g. agent $i$ work for principal $h$ in equilibrium with

$$
W_{h}^{i}+\frac{\left(s_{h}^{i}\right)^{2}}{2 y}>W_{f}^{i}+\frac{\left(s_{f}^{i}\right)^{2}}{2 x} .
$$

Then, $h$ can profitably change his wage offer without changing $i^{\prime} s$ effort level and without losing $i$ as an employee by lowering $W_{h}^{i}$ somewhat - a contradiction to the equilibrium assumption. In the case of equality, neither $f$ nor $h$ can profitably deviate from their wage offers (c.p. in terms of the considered HC and worker $i^{\prime} s$ effort level). q.e.d.

We now turn to the computation of optimal piece rates for all HCs, exemplarily for Scenario O. If Scenario O is realized, principal $A^{\prime}$ profit (after
substituting $W_{A}^{A 1}$ and $W_{A}^{A 2}$ via the substitution principle) is

$$
\begin{aligned}
\pi_{A}\left(s_{A}^{A 1}, s_{A}^{A 2}\right)= & {\left[a-\frac{s_{A}^{A 1}+s_{A}^{A 2}+s_{B}^{B 1}+s_{B}^{B 2}}{c}\right] \cdot \frac{s_{A}^{A 1}+s_{A}^{A 2}}{c}-\frac{\left(s_{A}^{A 1}\right)^{2}+\left(s_{A}^{A 2}\right)^{2}}{c} } \\
& -\left[W_{B}^{A 1}+\frac{\left(s_{B}^{A 1}\right)^{2}}{2 d}-\frac{\left(s_{A}^{A 1}\right)^{2}}{2 c}\right]-\left[W_{B}^{A 2}+\frac{\left(s_{B}^{A 2}\right)^{2}-\left(s_{A}^{A 2}\right)^{2}}{2 c}\right]
\end{aligned}
$$

Principal $B$ 's profit is:

$$
\begin{aligned}
\pi_{B}\left(s_{B}^{B 1}, s_{B}^{B 2}\right)= & {\left[a-\frac{s_{A}^{A 1}+s_{A}^{A 2}+s_{B}^{B 1}+s_{B}^{B 2}}{c}\right] \cdot \frac{s_{B}^{B 1}+s_{B}^{B 2}}{c}-\frac{\left(s_{B}^{B 1}\right)^{2}+\left(s_{B}^{B 2}\right)^{2}}{c} } \\
& -\left[W_{A}^{B 1}+\frac{\left(s_{A}^{B 1}\right)^{2}}{2 d}-\frac{\left(s_{B}^{B 1}\right)^{2}}{2 c}\right]-\left[W_{A}^{B 2}+\frac{\left(s_{A}^{B 2}\right)^{2}-\left(s_{B}^{B 2}\right)^{2}}{2 c}\right] .
\end{aligned}
$$

Maximizing $\pi_{A}$ and $\pi_{B}$ with respect to the two piece rates chosen by a particular principal yields a system of equations (first order conditions for interior solutions) that imply the following equilibrium piece rates for Scenario O (see Appendix A for further details): ${ }^{4,5}$

$$
\begin{equation*}
{ }_{o} s_{A}^{A 1}={ }_{o} s_{A}^{A 2}={ }_{o} s_{B}^{B 1}={ }_{o} s_{B}^{B 2}=\frac{a c}{c+6} . \tag{11}
\end{equation*}
$$

To calculate the principals' profits, we use placeholders for the four piece rates belonging to the unsuccessful contract offers: ${ }^{6}$

$$
\begin{equation*}
{ }_{o} s_{B}^{A 1}=r_{A 1}, \quad{ }_{o} s_{B}^{A 2}=r_{A 2}, \quad{ }_{o} s_{A}^{B 1}=r_{B 1} \quad \text { and } \quad{ }_{o} s_{A}^{B 2}=r_{B 2} \tag{12}
\end{equation*}
$$

With (11) and (12), the principals' profits in Scenario O become

$$
\begin{equation*}
\pi_{A}(o)=\frac{a^{2}(4+c)}{(6+c)^{2}}-\frac{\left(r_{A 1}\right)^{2}}{2 d}-\frac{\left(r_{A 2}\right)^{2}}{2 c}-W_{B}^{A 1}-W_{B}^{A 2} \tag{13}
\end{equation*}
$$

and

$$
\begin{equation*}
\pi_{B}(o)=\frac{a^{2}(4+c)}{(6+c)^{2}}-\frac{\left(r_{B 1}\right)^{2}}{2 d}-\frac{\left(r_{B 2}\right)^{2}}{2 c}-W_{A}^{B 1}-W_{A}^{B 2} . \tag{14}
\end{equation*}
$$

Analogously, piece rates and profit expressions can be derived for all other HCs.

[^3]
### 3.2 Unilateral Deviation Analysis

It is useful to establish Lemma 2 before illustrating the exemplary unilateral deviation analysis for Scenario O.

## Lemma 2:

For any equilibrium candidate $H C$, the condition that unilateral deviations of principals $A$ and $B$ to the monopolistic HC M1 are unprofitable establishes a lower and an upper bound for a fixed wage $W_{j}^{i}$ or the sum of fixed wages. The difference between these bounds is independent of $r_{A 1}, r_{A 2}, r_{B 1}$, and $r_{B 2}$.
Proof: See Appendix B.
For a possible equilibrium Scenario O, suppose a deviation to Scenario M1, where principal $A$ hires all four workers rather than only her local ones. ${ }^{7}$ From A's point of view, all wage offers of principal B are fixed - in particular, ${ }_{o} s_{B}^{B 1}={ }_{o} s_{B}^{B 2}=\frac{a c}{c+6},{ }_{o} s_{B}^{A 1}=r_{A 1}$, and ${ }_{o} s_{B}^{A 2}=r_{A 2}$. Principal A's profit from deviating from Scenario O to Scenario M1 is therefore ${ }^{8}$ :

$$
\begin{aligned}
{ }_{o} \pi_{A}(m 1)= & \frac{a^{2}\left(c^{3}+108 d+c^{2}(8+d)+12 c(3+2 d)\right)}{2(6+c)^{2}(6 d+c(2+d))}-\frac{\left(r_{A 1}\right)^{2}}{2 d} \\
& -\frac{\left(r_{A 2}\right)^{2}}{2 c}-W_{B}^{A 1}-W_{B}^{A 2}-W_{B}^{B 1}-W_{B}^{B 2} .
\end{aligned}
$$

For the unilateral deviation not to be profitable, the following condition must hold:

$$
\pi_{A}(o)-{ }_{o} \pi_{A}(m 1) \geq 0
$$

Via the substitution principle, this becomes

$$
\begin{aligned}
W_{A}^{B 1}+W_{A}^{B 2} \geq \alpha_{L}:= & \frac{a^{2}\left(c^{3}+60 d+c^{2}(8+d)+4 c(5+4 d)\right)}{2(6+c)^{2}(6 d+c(2+d))} \\
& -\frac{\left(r_{B 1}\right)^{2}}{2 d}-\frac{\left(r_{B 2}\right)^{2}}{2 c},
\end{aligned}
$$

where $\alpha_{L}$ is the lower bound for the sum of the fixed wages $W_{A}^{B 1}$ and $W_{A}^{B 2}$. The profit ${ }_{o} \pi_{B}(m 1)$ of $B$ 's unilateral deviation equals zero, because in M1 principal B employs no workers. This leads to the upper bound

$$
W_{A}^{B 1}+W_{A}^{B 2} \leq \alpha_{U}:=\frac{a^{2}(4+c)}{(6+c)^{2}}-\frac{\left(r_{B 1}\right)^{2}}{2 d}-\frac{\left(r_{B 2}\right)^{2}}{2 c}
$$

[^4]The difference between the upper and the lower interval bound, for both unilateral deviations from Scenario O to M1 is

$$
{ }_{o} I_{m 1}:=\alpha_{U}-\alpha_{L}=-\frac{a^{2}\left(c^{3}-c^{2}(d-4)-4 c(d-1)+12 d\right)}{2(6+c)^{2}(6 d+c(2+d))} .
$$

If ${ }_{o} I_{m 1}$ is positive, there exists a generic range for the sum $W_{A}^{B 1}+W_{A}^{B 2}$ such that the equilibrium requirements for Scenario O can be generically guaranteed in terms of deviations to M1.

### 3.3 Equilibrium Scenarios

We now can determine the remaining fifteen differences ${ }_{H C} I_{m 1}$. We list the results in Appendix C.1. From this, we are able to derive the following results:

Proposition 1 (Homogeneous Agents, i.e., $d=c$ ):
a) For all HCs other than M1 and M2, at least one principal has an incentive to deviate to a monopolistic HC (M1 or M2), i.e., no such HC is an equilibrium outcome.
b) Scenarios M1 and M2 are never equilibria (in pure strategies).

Proof: a) All differences ${ }_{H C} I_{m 1}$ in which the HC is not M1 or M2, are negative (see Appendix C.1). Thus, there are no values for the specific (combinations of) fixed wages such that any of the HCs except M1 and M2 is an equilibrium scenario.
b) This is a special case of the more general Proposition 2.a), proved in Appendix C.3.
q.e.d.

## Corollary 1 (No Equilibria for Homogeneous Agents):

For perfectly homogeneous workers $(d=c)$, there exists no equilibrium $H C$ in pure strategies.

Proof: Due to monopolization instability for non-monopolistic HCs, and since M1 and M2 are not equilibria, the claim is obvious.
q.e.d.

Monopolistic HCs could be equilibrium outcomes, since there is no principal gaining from unilaterally deviating to the reverse monopolistic HC. This
follows from ${ }_{m 1} I_{m 2}={ }_{m 2} I_{m 1}=0$ (Appendix C.1). Nevertheless, monopolistic HCs are not equilibria in the four-agent model since other kinds of deviations are profitable.

If there were only one instead of four identical agents with effort cost parameter $c$, principals could only deviate from one monopolistic to the reverse monopolistic HC, because there are no other HCs. One can easily derive from the interval bounds differences ${ }_{m 1} I_{m 2}={ }_{m 2} I_{m 1}=0$ in the homogeneous case that monopolistic HCs are always equilibria for this one-agent model and that principals have zero-profits in these equilibria.

## Corollary 2 (Monopoly Equilibria for only One Agent):

For the simplified one-agent model with cost parameter $c$,
a) both HCs (M1 and M2) are equilibria and
b) both sellers earn zero-profits.

Proof: a) Since ${ }_{m 1} I_{m 2}={ }_{m 2} I_{m 1}=0$, monopolistic HCs are equilibria - there are no other deviations than from one monopoly to the other one.
b) See Appendix C.2.
q.e.d.

According to Corollary 2, in the one-agent scenario, principals are not able to extract any profits when the sales and labor markets are characterized by duopolistic winner-takes-all competition. Equivalently, when principals cannot or do not want to discriminate between agents, what might play a role in practice as well as in experimental settings, in a four-agent model zero-profits result.

In the heterogeneous case, the number of possible equilibrium scenarios increases.

## Proposition 2 (Equilibria for Heterogeneous Agents):

Let

$$
\begin{gathered}
\phi_{1}(c):=\frac{4 c+4 c^{2}+c^{3}}{4 c+c^{2}-12} \\
\phi_{2}(c):=\frac{9 c+24 c^{2}+18 c^{3}+8 c^{4}+c^{5}}{6 c^{2}+8 c^{3}+c^{4}-27-36 c}
\end{gathered}
$$

and

$$
\Omega:=\frac{4}{3} \sqrt{13} \cos \left(-\frac{1}{3} \arctan \left(\frac{9}{103} \sqrt{303}\right)+\frac{1}{3} \pi\right)-\frac{5}{3} \approx 1.96 .
$$

Then, restricting attention to pure strategy equilibria,
a) M1 and M2 are never equilibria.
b) For $c \leq \Omega$, there is no equilibrium $H C$,
for $c>\Omega \wedge d<\phi_{2}$, there is no equilibrium $H C$,
for $c>\Omega \wedge \phi_{2} \leq d<\phi_{1}$, only from N1 and N2 unilateral monopolization is not profitable.
for $c>\Omega \wedge d \geq \phi_{1}$, only from N1 and N2, $O$, and $E$ unilateral monopolization is not profitable.
c) For some $a, c, d$, with $c>\Omega \wedge d \geq \phi_{1}$, the set of equilibria is non-empty and contains more than one element.

Proof: a) See Appendix C.3.
b) All differences ${ }_{H C(-6)} I_{m 1}$ are negative ${ }^{9}$ (see Appendix C.1). Thus, only N1, N2, O, and E can be monopolization stable and equilibria. Further restrictions are given in Appendix C.4.
c) To prove the multiplicity of equilibrium HCs for parameters in the declared parameter region, we give numerical examples based on the numerical generic equilibrium in Berninghaus et al. (2007), who establish HC O as an equilibrium scenario.

We establish Scenario O (and thus E) as an equilibrium scenario by assuming symmetry in the principals' piece rate decision for the other workers:

$$
r_{A 1}=r_{B 1} \quad \text { and } \quad r_{A 2}=r_{B 2}
$$

For the particular parameter values and placeholders

$$
a=15, \quad c=8, \quad d=60, \quad r_{A 1}=1, \quad r_{A 2}=10
$$

O (E) can be established as an equilibrium scenario for generic intervals of fixed wages $W_{o}:=\left(W_{A}^{A 1}, W_{A}^{A 2}, W_{B}^{B 1}, W_{B}^{B 2}\right)$. In the case of $W_{o}=(0,2.42,0,2.42)$, this implies the profits $\pi_{A}(o)=\pi_{B}(o) \approx 2.17$, and for the workers, the positive payoffs $\omega_{A 1}=\omega_{B 1} \approx 4.54$ and $\omega_{A 2}=\omega_{B 2} \approx 7.01$. Consumers' surplus amounts to approximately 9.18.

[^5]We are also able to establish Scenario N1 (N2) as an equilibrium by assuming equality in the placeholders $r_{A 2}$ and $r_{B 2}$ (since workers A 2 and B 2 have the same cost parameters when working for principal B). By fixing the parameters and placeholders at

$$
a=15, \quad c=8, \quad d=60, \quad r_{A 1}=1, \quad r_{A 2}=10, \quad r_{B 1}=5
$$

and choosing the vector of fixed wages as $W_{n 1}:=\left(W_{A}^{A 1}, W_{A}^{A 2}, W_{B}^{B 1}, W_{A}^{B 2}\right)=$ $(0,3.1,-1,3.1)$, the principals' profits are $\pi_{A}(o) \approx 2.54$ and $\pi_{B}(o) \approx 2.45$, and the positive payoffs for the workers are $\omega_{A 1} \approx 3.88, \omega_{A 2}=\omega_{B 2} \approx 6.98$, and $\omega_{B 1} \approx 4.80$. Consumers' surplus is approximately 8.66. q.e.d.

Proposition 2 shows that for a wide range of exogenous parameters, there are no equilibria in pure strategies, which is a fairly typical result in the case of finite games (see Nash, 1951). When the cost difference $c-d$ is high enough, however, even multiple equilibria are possible; the candidates are rather intuitive ones, like the local HC O (which is equivalent to E ) or the HCs N1 and N,2 which are the only HCs in which principals only hire workers with low effort costs. Figure 1 in Appendix C. 4 illustrates the relevant parameter ranges.

### 3.4 Hypotheses

According to the results of the equilibrium analysis, i.e. from Propositions 1 and 2 as well as from Corollaries 1 and 2, we establish the following hypotheses to be tested experimentally.

## Hypothesis 1:

For perfectly homogeneous workers, mainly monopolistic HCs will prevail.
The intuition for this conjecture comes from the one-agent model (Corollary 2). Implicitly, it claims that since principals tend to perceive the four agents as one group, individual agents are not discriminated against. We expect a principal to extend similar offers to all agents.

## Hypothesis 2:

For heterogeneous workers with sufficiently high costs and cost difference, the HCs O, E, N1, and N2 occur more often than all other HCs.

This hypothesis results from Proposition 2.

## Hypothesis 3:

For perfectly homogeneous workers, the principals' profits are lower compared to the heterogeneous case.

This intuition is again based on Corollary 2. In the one-agent case, zeroprofits result for the principals.

Since total production costs are lower in the homogeneous case with no immobile workers, it should increase welfare. Due to principals' decreasing profits, we formulate a further hypothesis.

## Hypothesis 4:

The agents' payoffs and consumers' surplus are higher in the homogeneous case than in the heterogeneous case.

## 4 Experimental Design

We ran the experiment in a reduced version with only the two principals represented by human participants. Their decisions were quite complex: They had to choose four fixed wage and four piece rate offers in 30 successive periods of playing our one-shot model. The employees' optimal choices of employer and effort level were supplemented by the experimental software. Earlier findings justify this simplification. ${ }^{10}$ The experiment was programmed and conducted with the software z-Tree (Fischbacher, forthcoming). In the instructions, each subject was informed in detail about how each principal's payoff would depend on the various HCs and wage offers. ${ }^{11}$

The experiment took place at the computer laboratory of the Max Planck Institute of Economics (Jena). Each session of the experiment was organized into two different treatments. The 32 participants of one session were divided into four matching groups consisting of eight subjects each. Subjects

[^6](students from different faculties at the University of Jena) were recruited using the ORSEE software (see Greiner, 2003). ${ }^{12}$ After the instructions were given out and read aloud to guarantee common knowledge, participants were asked to answer some control questions.

Treatments differ with respect to $d-c$, which measures the difference in cost of type 1 workers when working for their local firm or the distant firm. In Treatment I, we assume $c=d=8$, i.e. all four workers in this treatment are homogeneous; in Treatment II, the workers' cost parameters are set to $c=8$ and $d=60$. The eight members of each matching group were partitioned into four two-person employer groups (firms A and B) that were rematched every period. ${ }^{13}$ We performed two sessions with 32 participants each for both treatments (I and II).

When both players had decided, the resulting payoffs were shown on the screen. Via mouse clicks, participants could also obtain information about market prices, HCs, their own output levels, and their own previous wage offers as well as their own and the competitor's profits in all previous periods. The payoffs for each subject were accumulated over 30 periods and were paid out in cash after the experiment. ${ }^{14}$ Subjects started with an initial financial endowment of 60 CUs (currency units). The mean payoff was 11.31 EUR (with a standard deviation of 5.56). The experiment lasted about 100 minutes on average.

## 5 Experimental Results

According to Corollary 2, the monopolistic HCs M1 and M2 are the only equilibria in the homogeneous treatment with $d=c=8$. Otherwise, there is no equilibrium in pure strategies. For $c=8$ and $d=60$ - the heterogeneous treatment $-d=60>\phi_{1}(c) \approx 9.52>\phi_{2}(c) \approx 9.24>c=8>0$. This implies that N1, N2, O, and E are monopolization stable. In the proof to Proposition 2.c), we showed numerically that these four HCs can be established as equilibria.

[^7]When analyzing the data with regard to Hypotheses 1 to 4, we mainly focus on two aspects:

- the observed HCs and
- the distribution of total welfare, in particular the principals' profits.


### 5.1 Hiring Constellations

The absolute and relative frequencies of the HCs that occurred in Treatment I (homogeneous) and Treatment II (heterogeneous) are provided in Table 1. ${ }^{15}$

|  | Treatment I <br> (homogeneous) |  | Treatment II <br> (heterogeneous) |  |
| :--- | :---: | :---: | :---: | :---: |
| HC | abs. freq. | rel. freq. | abs. freq. | rel. freq. |

Table 1: Absolute and relative frequencies of hiring constellations
In the homogeneous treatment, the monopolistic HCs M1 and M2 each occurred in about one third of all observations. All other HCs were observed

[^8]fewer than 120 times, the expected level if HCs were uniformly distributed $\left(\frac{1920}{16}=120\right)$. The chi-square statistic for uniform distribution of all 16 HCs rejects uniformity, ${ }^{16}$ i.e. the monopolistic scenarios were predominant.

Even when comparing the equilibrium group I (EGI) with M1 and M2 on the one hand and the non-equilibrium group I (N-EGI) with the remaining 14 HCs on the other, the monopolistic scenarios together occurred far more often (1310 times, i.e. with a relative frequency of more than 68 percent). The chi-square statistic rejects equal shares of equilibrium and non-equilibrium scenarios. ${ }^{17}$ The results are robust to separate analyses of the two sessions. Hence, we find Hypothesis 1 confirmed.

## Result 1:

For perfectly homogeneous workers, the prevailing HCs rely on monopoly hiring.

The heterogeneous treatment scenarios N1 and N2, where one participant hires all low-cost workers (both mobile and the immobile worker with lower costs when working for her), occurred most often, with 27 percent and 30 percent, respectively. The next most prominent HC was the immobility scenario O, with about 11 percent. Besides these three equilibrium scenarios, only M2 occurred more often than would be predicted by the uniform distribution. Meanwhile, the last of the four equilibrium candidates, Scenario E, resulted only in about 4 percent of all cases.

The chi-square statistic for uniform distribution of the absolute frequencies of all 16 HCs rejects uniformity, ${ }^{18}$ i.e. Scenarios N1, N2, O, and M2 were played more often than would be expected by chance.

Let us again compare the absolute frequencies of the equilibrium group II (EGII) with Scenarios N1, N2, O, and E, and the non-equilibrium group II (N-EGII), consisting of all other HCs. Here, uniform distribution would predict 480 occurrences of EGII and 1440 of N-EGII. But we observed EGII 1376 times, in line with Hypothesis 2. The chi-square statistic rejects uniformity. ${ }^{19}$ We again obtain similar results when both sessions are analyzed separately.

[^9]
## Result 2:

For heterogeneous workers and a large cost difference, the combination of the HCs O, E, N1, and N2 occurred more often than all other HCs combined. Furthermore, N1, N2, O, and M2 were the most prominent HCs.

The first part of Result 2 confirms Hypothesis 2. Considering the high frequency of M2, a further analysis of the hiring strategies that the participants intended to play indicates that monopolization was most likely not their aim. The same result also occurred when two participants with divergent wage offers were matched. ${ }^{20}$ The relatively rare occurrence of E can be explained by the equivalence to O , which is more focal.

### 5.2 Welfare Analysis

Let us now compare the distribution of total welfare between the homogeneous and the heterogeneous treatments (see Table 2).

|  | Treatment I <br> (homogeneous) |  | Treatment II |  |
| :---: | :---: | :---: | :---: | :---: |
| (heterogeneous) |  |  |  |  |
| Group | value | fraction | value | fraction |
| Principals | 2404.09 | $7.56 \%$ | 4357.43 | $16.46 \%$ |
| Agents | 23218.57 | $72.96 \%$ | 18067.83 | $68.24 \%$ |
| Consumers | 6197.39 | $19.48 \%$ | 4049.7 | $15.30 \%$ |
| Welfare | 31820.05 | $100 \%$ | 26474.96 | $100 \%$ |

Table 2: Total welfare by groups - Treatments I and II
The values in each column of Table 2 are the aggregates resulting from the decisions of all 64 (principal) participants in a treatment; the consumers' surplus in Treatment I, for example, is the sum of 64 ( 2 sessions $\times 32$ participants) individual 30-period aggregates.

[^10]The most striking point is that the principals' profits were much lower in Treatment I than in Treatment II, although total welfare was larger in Treatment I than in Treatment II. Their fraction of welfare declined from about 16.5 percent in Treatment II to about 7.5 percent in Treatment I, while the agents' as well as the consumers' fractions of welfare increased (from about 68 percent to about 73 percent for the former and from about 15.5 percent to about 19.5 percent for the latter).

Since we did not find significant differences between the principal's profits in the two sessions of one treatment when a $U$-test is applied, ${ }^{21}$ we decided to pool the data for both treatments. ${ }^{22}$ A $U$-test confirms that the difference between the principals' profits between the two treatments is statistically significant. ${ }^{23}$

Let us interpret this finding with the help of Corollary 2: The solution predicts that principals are not able to extract profits in the monopolistic HCs M1 and M2, which are the only equilibria in the homogeneous one-agent treatment. These HCs were also played more frequently in this treatment, which probably caused a decline in the principals' profits. The principals' total profits are higher in the heterogeneous treatment for which HCs M1 and M2 were observed less frequently as predicted by Proposition 2. Hence, we consider Hypothesis 3 as qualitatively confirmed:

## Result 3:

For homogeneous workers, the principals' profits are lower than in the heterogeneous case.

Finally, we want to take a brief look at total welfare as well as the agents' and consumers' surplus. It is no surprise that welfare was significantly larger in the homogeneous treatment than in the heterogeneous one, ${ }^{24}$ since in the

[^11]homogeneous case all workers incur the lower costs of producing ( $c=d=8$ ).
Concerning the agents' payoffs and consumers' surplus in Treatments I and II, we can state: ${ }^{25}$ Agents and consumers benefit significantly from the homogeneous setting compared to the heterogeneous one. ${ }^{26}$ Our findings confirm Hypothesis 4 and can be summarized by Result 4.

## Result 4:

The agents' payoffs and consumers' surplus are lower in the heterogeneous case than in the homogeneous case.

Let us have a look at the four types of agents (see Table 3). For the immobile workers (A1 and B1) as well as for the mobile ones (A2 and B2), the sum of profits was higher in the homogeneous treatment than in the heterogeneous treatment. For the former, the increase is significant, ${ }^{27}$ which is not surprising, since workers A1 and B1 become more productive when their productivity is no longer locality- or firm-specific. The increase for the mobile workers whose per-unit effort costs did not change was probably caused by fiercer competition in the homogeneous case, but is not significant. ${ }^{28}$
selves to list the $p$-values of the corresponding $U$-tests: homogeneous treatments, $p$-value: $=0.732$; heterogeneous treatments, $p$-value: $=0.856$.
${ }^{25}$ Consumers are, of course, only captured by their aggregate demand function and not personally presented in the experiment.
${ }^{26}$ Agents: Treatments I and II: two-tailed $U$-test: observations: 64, 64; mean agent's payoff: $362.79,282.31$; test-statistics: $3364 ; p$-value: $<0.001$. We again pooled the data; the $p$-values of the corresponding $U$-tests are: homogeneous treatments, $p$-value: $=0.995$; heterogeneous treatments, $p$-value: $=0.541$. Consumers: Treatments I and II: two-tailed $U$-test: observations: 64,64 ; mean consumers' surplus: $96.83,63.28$; test-statistics: 2522 ; $p$-value: $<0.001$. We again pooled the data; the $p$-values of the corresponding $U$-tests are: homogeneous treatments, $p$-value: $=0.280$; heterogeneous treatments, $p$-value: $=0.524$.
${ }^{27}$ Treatments I and II: two-tailed $U$-test: observations: 64,64 ; mean payoff of agent A1: $90.25,60.55$; test-statistics: $3258 ; p$-value: $<0.001$. Treatments I and II: two-tailed $U$-test: observations: 64,64 ; mean payoff of agent B1: 90.90, 63.34 ; test-statistics: 3455 ; $p$-value: $=0.001$.
${ }^{28}$ Treatments I and II: two-tailed $U$-test: observations: 64, 64; mean payoff of agent A2: $90.43,79.82$; test-statistics: 3258; $p$-value: $=0.114$. Treatments I and II: two-tailed $U$-test: observations: 64, 64; mean payoff of agent B2: 91.20, 78.60; test-statistics: 3455; $p$-value: $=0.084$.

| Agent | Treatment I <br> (homogeneous) | Treatment II <br> (heterogeneous) |
| :---: | :---: | :---: |
| A1 | 5776.20 | 3875.24 |
| A2 | 5787.71 | 5108.56 |
| B1 | 5817.59 | 4053.73 |
| B2 | 5837.07 | 5030.29 |
| $\sum$ | 18067.83 | 23218.57 |

Table 3: Agents' welfare by subgroups - Treatments I and II

## 6 Conclusion

In the social sciences, theoretical studies have to abstract some aspects from the usually rich scenarios in the field. It is thus understandable that principalagent theory initially concentrated on intra-firm conflicts to contrast these with the unitary actor-firm model in general equilibrium models of production economies. But abstracting the fact that firms face not only intra-firm but also inter-firm conflicts can easily generate misleading intuitions. The usual recommendation of using incentive pay when monitoring is not too costly is at least empirically questionable when considering top managers (see the econometric study of Jensen and Murphy, 1998). A poorly performing firm will not be able to hire skillful employees if it offers less attractive contracts than its more successful competitors.

With our study (see also Berninghaus et al., 2004, and Berninghaus et al., 2007), we hoped to contribute to as well as initiate an integrative analysis of intra-firm and inter-firm conflicts that would hopefully steer clear of misleading conclusions. In our quest to do so, however, we might ourselves have fallen prey to studying an overly specific situation, e.g. the extreme case of homogeneous markets (see the discussion of Brennan, Güth, and Kliemt, 2004). Our justification for this is that the theoretical analysis was already challenging enough and that a similar analysis for heterogeneous markets will now be much easier given our experiences of analyzing hiring and sales competition in the simpler case of homogeneous sales markets.

In addition to our game theoretic analysis of pure strategy equilibrium hiring constellations, we performed parallel experiments confirming (at least qualitatively) the main equilibrium predictions. We thereby wanted to highlight the importance of developing not only the theory of intra- and inter-firm
conflict but also the empirical exploration of this kind of integrative conflict analysis. Additional field studies, similar to those conducted by Jensen and Murphy (1998) and Schwalbach and Brenner (2003), are an important next step.

Note that our model can be considered as both:
(i) a two-country model with partly less mobile workers who suffer when they leave their home country to work abroad,
(ii) a model where firms compete on both their labor market and their sales market.

In view of interpretation (i), we feel that we are contributing to the study of the effects of a global economy, especially one entailing the international migration of workers. Per interpretation (ii), we simply hope to enrich the theory of principal-agent conflicts as well as the body of Industrial Organization literature with our integrative analysis of competition in hiring and selling.

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## A Equilibrium Piece Rates in Scenario O

To determine the equilibrium piece rates in Scenario O, we derive the first order conditions of the profit functions, which, for an interior optimum with respect to $s_{A}^{A 1}$ and $s_{A}^{A 2}$, yields

$$
\frac{\partial \pi_{A}}{\partial s_{A}^{A 1}}=0 \Rightarrow \quad s_{B}^{B 1}+s_{B}^{B 2}=10 c-2 \cdot s_{A}^{A 2}-(2+c) \cdot s_{A}^{A 1}
$$

and

$$
\frac{\partial \pi_{A}}{\partial s_{A}^{A 2}}=0 \Rightarrow \quad s_{B}^{B 1}+s_{B}^{B 2}=10 c-2 \cdot s_{A}^{A 1}-(2+c) \cdot s_{A}^{A 2}
$$

which implies

$$
s_{A}^{A 1}=s_{A}^{A 2} .
$$

The analogous conditions for $\pi_{B}$ are symmetric to those for $\pi_{A}$ and also induce $s_{B}^{B 1}=s_{B}^{B 2}$. Hence, the equilibrium piece rates for principals $A$ and $B$ in Scenario O are

$$
{ }_{o} s_{A}^{A 1}={ }_{o} s_{A}^{A 2}={ }_{o} s_{B}^{B 1}={ }_{o} s_{B}^{B 2}=\frac{a c}{c+6} .
$$

## B Proof of Lemma 2

To avoid too much formalism, we prove Lemma 2 by presenting the main considerations for a particular HC and then generalizing these considerations.

Suppose that we are analyzing the HC in which workers $A 1, A 2$, and $B 1$ are working for principal $A$ and worker $B 2$ for principal $B$ (Scenario V1). This means that we have to introduce placeholders for the piece rate offers of principal $B$ to $A$ 's workers $A 1, A 2$, and $B 1$ and for the piece rate offer of principal $A$ to $B$ 's worker $B 2$ :

$$
{ }_{v 1} s_{B}^{A 1}=r_{A 1}, \quad{ }_{v 1} s_{B}^{A 2}=r_{A 2}, \quad{ }_{v 1} s_{B}^{B 1}=r_{B 1}, \quad \text { and } \quad{ }_{v 1} s_{A}^{B 2}=r_{B 2}
$$

Let us now present the principals' profits in V1 in a reduced form, where we concentrate only on the placeholders and the fixed wages: ${ }^{29}$

[^12]$$
\pi_{A}(v 1)=\ldots-W_{B}^{A 1}-\mu\left(r_{A 1}\right)-W_{B}^{A 2}-\mu\left(r_{A 2}\right)-W_{B}^{B 1}-\mu\left(r_{B 1}\right)
$$
and
$$
\pi_{B}(v 1)=\ldots-W_{A}^{B 2}-\mu\left(r_{B 2}\right)
$$

The substitution principle allows us to replace a principal's fixed wage offer(s) by its opponent's fixed wage and piece rate offer(s) (the placeholder(s)).

If principal A deviates unilaterally to Scenario M1 (wherein she employs all workers), her profit will be:

$$
{ }_{v 1} \pi_{A}(m 1)=\ldots-W_{B}^{A 1}-\mu\left(r_{A 1}\right)-W_{B}^{A 2}-\mu\left(r_{A 2}\right)-W_{B}^{B 1}-\mu\left(r_{B 1}\right)-W_{B}^{B 2} .
$$

Her profit ${ }_{v 1} \pi_{A}(m 1)$ is independent from a placeholder $r_{B 2}$, because from her point of view principal B's piece rate offer to worker B 2 is fixed.

To analyze if the deviation from V1 to M1 $\left({ }_{v 1} D m 1\right)$ is not beneficial for principal A , the following condition must hold:

$$
{ }_{v 1} D m 1_{A}:=\pi_{A}(v 1)-{ }_{v 1} \pi_{A}(m 1)=\ldots+W_{B}^{B 2} \geq 0
$$

With the help of the substitution principle, this becomes a lower interval bound for the fixed wage $W_{A}^{B 2}$ :

$$
\beta_{L}:=W_{A}^{B 2} \geq \ldots-r_{B 2} .
$$

If principal B deviates unilaterally to M 1 , her profit will of course be equal to zero, because she does not hire anybody. Thus,

$$
{ }_{v 1} D m 1_{B}:=\pi_{B}(v 1)-0=\ldots-W_{A}^{B 2}-r_{B 2} \geq 0 .
$$

This directly leads to an upper interval bound for $W_{A}^{B 2}$ :

$$
\beta_{U}:=W_{A}^{B 2} \leq \ldots-r_{B 2} .
$$

It is now obvious that the difference of the the upper and the lower interval bounds is independent of the installed placeholders.

It is furthermore easy to see that the same logic applies to all other HCs. The deviation variable ${ }_{H C} D m 1_{A}$ always depends on the fixed wage(s) that principal B offered her worker(s) in the HC , while ${ }_{H C} D m 1_{B}$ depends on the fixed wage(s) that principal A offered B's worker(s) and the corresponding placeholder(s). By using the substitution principle, ${ }_{H C} D m 1_{A}$ can be transformed such that it depends on the same fixed wage(s) and placeholder(s) as ${ }_{H C} D m 1_{B}$. The difference of upper and lower bounds is thus independent of all placeholders.
q.e.d.

## C Proofs to Equilibrium Analysis

## C. 1 Differences of Upper and Lower Interval Bounds

The upper and lower interval bounds that we need for the propositions in the remainder are:

Note that numerators of ${ }_{n 3} I_{m 1},{ }_{n 4} I_{m 1},{ }_{u} I_{m 1}$, and ${ }_{t} I_{m 1}$ are larger than zero. When $c+x$ is substituted for
$d$, this becomes immediately obvious.

## C. 2 Proof of Corollary 2.b)

We want to show that

$$
(\mathrm{C} .2-1) \pi_{A}(m 1)=\pi_{B}(m 2)=0
$$

and furthermore learn that

$$
(\mathrm{C} .2-2) \pi_{A}(m 2)=\pi_{B}(m 1)=0, \quad(\mathrm{C} .2-3) \bar{m}_{B}(\bar{n})=0
$$

for $c=d$ and all combinations of $\bar{m}, \bar{n} \in\{m 1, m 2\}$.
Equations (C.2-2) are obviously true, since in Scenario M2 (M1), principal A (B) has no employees.

Furthermore, the deviation from Scenario M2 to M1 is - ex ante - not beneficial for either principals if

$$
\text { (C.2-4) } \quad \Pi_{A}:=\pi_{A}(m 2)-{ }_{m 2} \pi_{A}(m 1) \geq 0
$$

and

$$
\text { (C.2-5) } \quad \Pi_{B}:=\pi_{B}(m 2)-{ }_{m 2} \pi_{B}(m 1) \geq 0
$$

Conditions (C.2-4) and (C.2-5) lead to an upper and lower interval bound for a fixed wage expression $\hat{W}$,

$$
\text { (C.2-6) } \quad \hat{W} \leq \gamma_{U} \quad \text { and } \quad \text { (C.2-7) } \quad \hat{W} \geq \gamma_{L}
$$

Furthermore, we know - ex post - that

$$
\text { (C.2-8) }{ }_{m 2} I_{m 1}=\gamma_{U}-\gamma_{L}=0
$$

and thus

$$
\text { (C.2-9) } \quad \hat{W}=\gamma_{U}=\gamma_{L}
$$

Lemma C.2-1: Ex post, the following equations hold:

$$
\Pi_{A}:=\pi_{A}(m 2)-{ }_{m 2} \pi_{A}(m 1)=0
$$

and

$$
\Pi_{B}:=\pi_{B}(m 2)-{ }_{m 2} \pi_{B}(m 1)=0
$$

Proof: Note that because of (C.2-9), there is exactly one $\hat{W}$ that affects $\Pi_{A}$ as well as $\Pi_{B}$ in different directions (positive and negative slope). Suppose the following different cases:

- $\Pi_{A}>0, \Pi_{B}=0$ and rising $\hat{W}$ increases $\Pi_{A}$. This would imply an at least marginal decrease of $\hat{W}$ to $\hat{W}_{d}$, such that $\Pi_{A} \geq 0$ and $\Pi_{B}>0$ is possible. So, $\hat{W}_{d}$ represents a second value apart from $\hat{W}$ with $\hat{W}_{d} \neq \hat{W}$ that would contradict equation (9).
- For $\Pi_{A}>0, \Pi_{B}=0$ and $\Pi_{A}$ decreasing in $\hat{W}$, a marginal increase to $\hat{W}_{i}$ would also result in a contradiction.

With analogous arguments, it is possible to rule out $\Pi_{A}=0$ and $\Pi_{B}>0$ as $\Pi_{A}>0, \Pi_{B}>0$, respectively.
q.e.d.

Lemma C.2-1 proves equality in conditions (C.2-4) and (C.2-5). Since $\pi_{A}(m 2)$ and ${ }_{m 2} \pi_{B}(m 1)$ equal zero, we have

$$
\text { (C.2-10) } 0-{ }_{m 2} \pi_{A}(m 1)=0 \Rightarrow{ }_{m 2} \pi_{A}(m 1)=0
$$

and

$$
\text { (C.2-11) } \quad \pi_{B}(m 2)-0=0 \Rightarrow \pi_{B}(m 2)=0
$$

Analogously, the deviation from Scenario M1 to M2 is - ex ante - not beneficial for either of the principals if

$$
(\mathrm{C} .2-12) \quad \pi_{A}(m 1)-{ }_{m 1} \pi_{A}(m 2) \geq 0
$$

and

$$
(\mathrm{C} .2-13) \quad \pi_{B}(m 1)-{ }_{m 1} \pi_{B}(m 2) \geq 0
$$

Considerations similar to those in Lemma C.2-1 lead to equality in conditions (C.2-12) and (C.2-13). Furthermore, $\pi_{B}(m 1)$ and ${ }_{m 1} \pi_{A}(m 2)$ equal zero. Thus

$$
(\mathrm{C} .2-14) \quad \pi_{A}(m 1)-0=0 \Rightarrow \pi_{A}(m 1)=0
$$

and

$$
\text { (C.2-15) } \quad 0-{ }_{m 1} \pi_{B}(m 2)=0 \Rightarrow{ }_{m 1} \pi_{B}(m 2)=0
$$

## C. 3 Proof of Proposition 2.a)

To establish a HC as an equilibrium scenario (for $d>c$ or $d=c$ ), we have to analyze all 16 possible deviations from this HC for each principal. These deviations lead to a system of 32 conditions for (combinations of) the fixed wages. The HC can only be an equilibrium scenario for specific parameter values, if all of these conditions are fulfilled.

For the monopoly scenario M1, it is easy to find a general contradiction for all possible parameter values in these conditions.

For principal B to deviate unprofitably from Scenario M1 to Scenario N1, a lower interval bound for $W_{A}^{B 1}$ results:

$$
\begin{equation*}
{ }_{m 1} D n 1_{B}:=\pi_{B}(m 1)-{ }_{m 1} \pi_{B}(n 1) \geq 0 \Rightarrow W_{A}^{B 1} \geq X_{1} \tag{15}
\end{equation*}
$$

with

$$
X_{1}:=\frac{a^{2}\left(-c^{3} d+9 d^{2}+6 c d(2+d)+c^{2}\left(4+2 d+d^{2}\right)\right)}{2(2+c)(6 d+c(2+d))^{2}}
$$

Furthermore, considering the deviation by principal A from M1 to M2 and by principal B from M1 to N4 requires

$$
\begin{equation*}
W_{A}^{A 1}+W_{A}^{A 2}+W_{A}^{B 1}+W_{A}^{B 2} \leq X_{2} \tag{16}
\end{equation*}
$$

and

$$
\begin{equation*}
W_{A}^{A 1}+W_{A}^{A 2}+W_{A}^{B 2} \geq X_{3} \tag{17}
\end{equation*}
$$

with

$$
X_{2}:=\frac{a^{2}(c+3 d)^{2}}{6 d+c(2+d))^{2}}
$$

and

$$
X_{3}:=\frac{a^{2}\left(72 d^{3}+c^{3}(1+d)^{2}+12 c d^{2}(5+d)+c^{2} d\left(14+10 d-d^{2}\right)\right)}{2(4 d+c(2+d))(6 d+c(2+d))^{2}}
$$

for an M1-equilibrium. This implies (with $\Delta X_{3} \geq 0$ )

$$
\begin{equation*}
W_{A}^{A 1}=X_{3}+\Delta X_{3}-W_{A}^{A 2}-W_{A}^{B 2} \tag{18}
\end{equation*}
$$

inducing

$$
\begin{equation*}
X_{1} \leq W_{A}^{B 1} \leq X_{2}-X_{3}-\Delta X_{3} \leq X_{2}-X_{3} \tag{19}
\end{equation*}
$$

It is furthermore easy to show generally that

$$
\begin{equation*}
X_{1}>X_{2}-X_{3} \tag{20}
\end{equation*}
$$

Hence, conditions (1), (2), and (3) cannot be fulfilled simultaneously. The same holds true for Scenario M2.

## C. 4 Proof of Proposition 2.b)

We can subdivide the quadrant $c, d \geq 0$ into three parameter regions, depending on when the HCs O, E, N1, and N2 offer profitable unilateral deviations to monopolization.

- In order to eliminate O and E as possible equilibrium scenarios due to profitable deviations, the following condition must hold (see Appendix C.1):

$$
\begin{equation*}
-\frac{a^{2}\left(c^{3}-c^{2}(d-4)-4 c(d-1)+12 d\right)}{2(6+c)^{2}(6 d+c(2+d))}<0 \tag{C.4-1}
\end{equation*}
$$

Condition (C.4-1) holds if

$$
\text { (C.4-2) } \quad d \cdot D N_{1}:=d \cdot\left(4 c+c^{2}-12\right)<4 c+4 c^{2}+c^{3}
$$

The only root of the denominator $D N_{1}$ for positive values of $c$ is $c=2$. If $c<2$, condition (C.4-1) is equivalent to:

$$
d>\frac{4 c+4 c^{2}+c^{3}}{4 c+c^{2}-12}=: \phi_{1}(c),
$$

which is always true; if $c>2$, it is equivalent to:

$$
d<\phi_{1}(c) .
$$

Condition (C.4-2) is obviously fulfilled for $c=2$. Altogether, this means that O and E cannot be equilibrium scenarios if either $c \leq 2$ or if $c>2$ and $d<\phi_{1}(c)$.

- Using the same logic, one can analyze HCs N1 and N2. The condition for these two HCs to be eliminated as equilibrium scenarios is:
(C.4-3) $-\frac{a^{2}\left(c^{5}+c^{3}(18-8 d)-c^{4}(d-8)-6 c^{2}(d-4)+27 d+9 c(1+4 d)\right)}{2\left(9+8 c+c^{2}\right)^{2}(6 d+c(2+d))}<0$.

Condition (C.4-3) holds if

$$
\begin{equation*}
d \cdot D N_{2}:=\quad d \cdot\left(6 c^{2}+8 c^{3}+c^{4}-27-36 c\right)<9 c+24 c^{2}+18 c^{3}+8 c^{4}+c^{5} \tag{C.4-4}
\end{equation*}
$$

The denominator $D N_{2}$ has four roots, but only one of them leads to a positive value of $c:{ }^{30}$

$$
c=\Omega:=\frac{4}{3} \sqrt{13} \cos \left(-\frac{1}{3} \arctan \left(\frac{9}{103} \sqrt{303}\right)+\frac{1}{3} \pi\right)-\frac{5}{3} \approx 1.96
$$

If $c<\Omega$, condition (C.4-3) is equivalent to:

$$
d>\frac{9 c+24 c^{2}+18 c^{3}+8 c^{4}+c^{5}}{6 c^{2}+8 c^{3}+c^{4}-27-36 c}=: \phi_{2}(c)
$$

which is always true. If $c>\Omega,(\mathrm{C} .4-3)$ is equivalent to:

$$
d<\phi_{2}(c) .
$$

Condition (C.4-4) is obviously fulfilled for $c=\Omega$. Altogether, this means that N1 and N2 cannot be equilibrium scenarios for $c \leq \Omega$ or if $c>\Omega$ and $d<\phi_{2}(c)$.

- The last step is to divide $c, d>0$ into ranges in term of a comparison of $\phi_{1}(c)$ and $\phi_{2}(c)$.

$$
\Phi:=\phi_{2}-\phi_{1}=-\frac{4 c^{3}\left(c^{2}+5 c-9\right)}{\left(c^{2}+4 c-12\right)\left(c^{4}+8 c^{3}+6 c^{2}-36 c-27\right)}
$$

which is not defined for $c=\Omega$ and $c=2$, the poles of $\phi_{2}(c)$ and $\phi_{1}(c)$. It is easy to show that

$$
\Phi=0 \Leftrightarrow \phi_{2}=\phi_{1} \quad \Leftrightarrow c=\frac{1}{2}(-5+\sqrt{61}):=\Lambda \approx 1.41 .
$$

$\phi_{1}(c)>\phi_{2}(c)$ for $\Lambda<c<\Omega$ or $c>2$. Conversely, $\phi_{1}(c)<\phi_{2}(c)$ for $c<\Lambda$ or $\Omega<c<2$. Note that for $\phi_{1}(c)>0$, we also have $\phi_{1}(c)>c$. Analogously, for $\phi_{2}(c)>0$, we also have $\phi_{2}(c)>c$.

Combining these three considerations, the claim follows easily if we take the following ranges of $c$ and $d: c<\Lambda, \Lambda<c<\Omega, \Omega<c<2$ and $c>2$ as well as the corresponding bounds into account.

We illustrate our findings by Figure 1 . Note that only $c \geq d$ matters. Since for $\phi_{1}(c), \phi_{2}(c)>0$, we have $\phi_{1}(c)>\phi_{2}(c)>c>0$, we have three


Figure 1: Parameter ranges for the heterogeneous case
"layers" for all $c>2$ : If $d<\phi_{2}(c)$, there is no equilibrium; if $d$ is of medium size, i.e. $\phi_{2}(c) \leq d<\phi_{1}(c)$, at least N1 and N2 are monopolization stable; for $d \geq \phi_{1}(c), \mathrm{O}, \mathrm{E}, \mathrm{N} 1$, and N 2 are monopolization stable.

## D Instructions for the Heterogeneous Treatment

You and other participants are taking part in a decision-making experiment. You have the chance to earn cash credits, which will be paid off in euros at the end of the experiment. The calculation unit in this experiment is the so-called currency unit, CU, where 10 CUs equal one euro. Each participant makes decisions, isolated from the other participants, at his or her computer

[^13]terminal. Communication between participants is not allowed. Please bear in mind that all your decisions will be handled confidentially and evaluated anonymously.

The experiment extends over 30 periods. In every period, you and another participant are matched in a two-person group. At the beginning of every period, all two-person groups are re-matched randomly. There is no interaction between your group and other groups within a period. You and all the other participants are confronted with the same decision-making situation, which is described in the remainder.

Each of you represents an employer. At the beginning of the experiment, you will be randomly assigned the role of either employer $A$ or $B$. Accordingly, the other participant in your two-person group is either employer $B$ or $A$ and therefore your competitor. Employer $A$ has its domicile in country $\mathcal{A}$, her competitor, employer $B$, in country $\mathcal{B}$. Each employer is given 60 CUs at the beginning of the experiment. Both of you are able to manufacture a homogeneous product with the help of up to four employees, $A 1, B 1, A 2$, and $B 2$. Employees $A 1$ and $A 2$ are from country $\mathcal{A}$, while $B 1$ and $B 2$ are from country $\mathcal{B}$. Each employee will be working for exactly one employer. In every period, you and your competitor provide wage offers to all four employees simultaneously. The four employees are simulated by the computer system. Each simulated employee chooses the wage offer that is the best from his perspective. Then the employee produces for his employer. When he does so, production costs arise that have to be paid by the employees themselves. The market price adapts in a way such that the total output is sold on the market, which is solely supplied by the two employers.

A wage offer provided by employer $i \in\{A, B\}$ for employee $j \in\{A 1, B 1, A 2, B 2\}$ consists of two components (in CU): a fixed wage $W_{i}^{j} \in[0 ; 7]$ and a piece rate $s_{i}^{j} \in[0 ; 12]$. An accepted wage offer generates a fixed wage $W_{i}^{j}$ for employee $j$ as well as an additional piece rate $s_{i}^{j}$ for every produced unit. Consequently, as employers, you and your competitor as an employer have to offer four fixed wages and four piece rates in total.

Each of the four employees chooses exactly one of the two proposed wage offers in every period, i.e., he produces for a particular employer in the current period. Two types of employees exist:

- Type 1 workers: Employees $A 1$ and $B 1$ are more productive in their homeland, i.e. working for the employer with the corresponding letter. This means that for employee $A 1$, the costs of producing one unit of output are lower when working for employer $A$ than when working for employer $B$; vice versa for employee $B 1$, whose costs of producing are lower when working for employer $B$.
- Type 2 workers: Employees $A 2$ and $B 2$ are equally productive in their homeland as well as abroad, i.e. with either employer. The costs of producing are equal regardless for which employer a type 2 employee works.

The total costs for $x$ units of output that an employee (columns) produces for an employer (rows) are given in the following table, where $d>c>0$.

| costs in CU | $A 1$ | $A 2$ | $B 1$ | $B 2$ |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | $\frac{1}{2} c x^{2}$ | $\frac{1}{2} c x^{2}$ | $\frac{1}{2} d x^{2}$ | $\frac{1}{2} c x^{2}$ |
| $B$ | $\frac{1}{2} d x^{2}$ | $\frac{1}{2} c x^{2}$ | $\frac{1}{2} c x^{2}$ | $\frac{1}{2} c x^{2}$ |

For each of the two wage offers, the virtual employees determine their optimal output. It is easy to show that, depending on the costs, $\frac{s_{i}^{j}}{c}$ and $\frac{s_{i}^{j}}{d}$, resp., units of output would be produced by the employee. These amounts of output, along with the fixed wage and after considering the costs, would generate the payoffs $W_{i}^{j}+\frac{\left(s_{i}^{j}\right)^{2}}{2 c}$ CUs and $W_{i}^{j}+\frac{\left(s_{i}^{j}\right)^{2}}{2 d}$ CUs, resp., for the employee. Each employee decides in favor of the employer promising the higher payoff. If two wage offers generate the same payoff, the employee chooses his employer at random with equal probability.

The production costs, here equal to the wage costs that result from an accepted wage offer, are $W_{i}^{j}+\frac{\left(s_{i}^{j}\right)^{2}}{c}$ CUs and $W_{i}^{j}+\frac{\left(s_{i}^{j}\right)^{2}}{d}$ CUs, resp., for the employer. The total output of an employer ( $x_{A}$ and $x_{B}$, resp.) is the sum of the single amounts produced by her hired employees. The price (in CU) on the market decreases in the total supply. It is $p=a-x_{A}-x_{B}$, with

$$
a=15 .
$$

In the experiment, the cost parameters of producing one unit of output are:

$$
\begin{array}{r}
c=8 \\
d=60 .
\end{array}
$$

Remember that your period profits as an employer, depending on the hiring constellation and the wage offers, are described in the table on the following pages. Your competitor's period profits are calculated analogously. Your total income at the end of the experiment is calculated by summing up all your period profits (and losses) plus your original funds. If your total income at the end of the game is negative, your monetary payoff will be 0 euros.

## Information

At the end of each period, you can review the hiring constellations as well as your individual output, the market price, and your wage offers to the four employees that have been realized in previous periods. You can also review your and your competitors' periodic incomes.
(The payoff table for all 16 hiring constellations is left out.)


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[^1]:    ${ }^{1}$ For simultaneous consideration of oligopoly and oligopsony by means of aggregate supply and demand functions, see Chang and Tremblay (1991, 1994) in a conjectural variations approach.

[^2]:    ${ }^{2}$ Negative fixed wage rates $W_{j}^{i}$ are not only a formal simplification for the theoretical analysis, it also can be easily interpreted as not to rule out an "entrance fee" a worker has to pay for a particular job.
    ${ }^{3}$ See Berninghaus et al. (2004) for an analysis allowing for voluntary unemployment.

[^3]:    ${ }^{4}$ The Hessian of the second order conditions proves that these solutions are maxima. This also holds for all other scenarios. We will not discuss second order conditions in the remainder; they are always fulfilled.
    ${ }^{5}$ For the sake of readability, we denote the scenarios by lower case letters when needed.
    ${ }^{6}$ Berninghaus et al. (2007) assumed $r_{A 1}=r_{B 1}$ and $r_{A 2}=r_{B 2}$ by relying on symmetry arguments. This is quite intuitive but not necessary for Scenario O (with its symmetric allocation of workers) and not at all convincing for most other HCs.

[^4]:    ${ }^{7} A$ marginally increases her fixed wage offers for the new workers.
    ${ }^{8}$ From now on, the left index of $\pi$ denotes the candidate for an equilibrium scenario that is challenged by unilateral deviation.

[^5]:    ${ }^{9}$ with $H C(-6) \in\{N 3, N 4, V 1, V 2, V 3, V 4, U, T, R 1, R 2\}$

[^6]:    ${ }^{10}$ See Berninghaus et al., 2004. In their experiment agents also had to decide between two contract offers in a setting similar to ours. The authors did not find any significant deviations from optimal choices for these agents. In particular, firm loyalty did not occur.
    ${ }^{11}$ Appendix D contains an English translation of the instructions for the heterogeneous treatment; participants also received a table showing the payoffs for all 16 hiring constellations and wage offers. The original German instructions are available on request.

[^7]:    ${ }^{12}$ The instructions had to be slightly changed in between due to a minor typo; see Berninghaus et al. (2007).
    ${ }^{13}$ Subjects were not informed about the restricted matching groups. Since they were only told that they would not be matched with the same competitor firm in the next period, they should have expected any of the remaining 30 participants.
    ${ }^{14}$ The conversion rate was 1 EUR per 10 CUs (currency units).

[^8]:    ${ }^{15}$ Absolute frequencies sum up to 1920 ( $=64$ participants $\times 30$ periods).

[^9]:    ${ }^{16}$ Treatment I: chi-square test; test statistics: 5522.6; $p$-value $<0.005$. Throughout this paper, we use the $5 \%$ significance level.
    ${ }^{17}$ Treatment I: chi-square test; test statistics: $5451.9 ; p$-value $<0.005$.
    ${ }^{18}$ Treatment II: chi-square test; test statistics: 3958.7 ; $p$-value $<0.005$.
    ${ }^{19}$ Treatment II: chi-square test; test statistics: $360.53 ; p$-value $<0.005$.

[^10]:    ${ }^{20}$ To classify the participants' wage decisions into different strategy profiles, a player's strategy can, for instance, be defined as an indifference strategy if he offered approximately the same wage income to all four workers. Let us assume that the indifference strategy indicates the intention to establish a monopoly. In the heterogeneous treatment, the indifference strategy was played in only 12 of 1920 observations. In comparison, we observed it in 1098 out of 1920 observations in the homogeneous treatment, where monopolization was the most frequent result.

[^11]:    ${ }^{21}$ Normally, we use Mann-Whitney Rank Sum tests since we cannot always exclude heteroscedasticity or assume normality. $T$-tests led to the qualitatively same results throughout.
    ${ }^{22}$ Homogeneous treatments, 2 sessions: two-tailed $U$-test: observations: 32, 32; mean profits: 41.95, 33.18; test-statistics: 1104; p-value: $=0.394$. Heterogeneous treatment, 2 sessions: two-tailed $U$-test: observations: 32,32 ; mean profits: $77.40,58.77$; test-statistics: 1130; $p$-value: $=0.229$.
    ${ }^{23}$ Treatments I and II: two-tailed $U$-test: observations: 64, 64; mean profits: 37.56 , 68.09; test-statistics: 4758; $p$-value: $=0.003$.
    ${ }^{24}$ Treatments I and II: two-tailed $U$-test: observations: 64, 64; mean welfare: 497.19, 413.67; test-statistics: 4786; $p$-value: $=0.002$. We pooled the data again and limit our-

[^12]:    ${ }^{29}$ Note that $\mu\left(r_{A 1}\right)=\frac{r_{A 1}^{2}}{2 d}$ represents a short notation for the surplus the corresponding piece rate offer would generate for an agent. Furthermore, we omit summands by ... that do not contain fixed wages and placeholders, since they are irrelevant for the proof.

[^13]:    ${ }^{30}$ The others are at $c \approx-6.22, c=-3$, and $c \approx-0.74$.

