# Employment fluctuations in a dual labor market

James Costain, Juan F. Jimeno, and Carlos Thomas
Banco de España

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**Abstract**

The recent economic downturn has highlighted a remarkable heterogeneity in the response of unemployment across OECD countries. This paper analyzes the role of labor market duality — meaning the coexistence of "temporary" contracts with low firing costs and "permanent" contracts with high firing costs — in explaining employment and unemployment volatility.

We address this question in a version of the Mortensen-Pissarides (1994) model of job creation and destruction. Calibrating the model to match Spanish labor market flows implies a large amount of inefficient churning, with temporary jobs disappearing four times as fast as permanent jobs, and with productivity of the marginal temporary job destroyed due to contract expiry more than double that of the marginal permanent job destroyed.

Recessions and booms have highly asymmetric effects in this framework, because employment builds up gradually during booms, due to matching frictions, whereas the beginning of a recession causes a sudden burst of firing of low productivity jobs. While both contract types suffer in this wave of firing, the fact that temporary employment converges much faster in booms than permanent employment does implies that fluctuations in temporary jobs play a disproportionate role in explaining unemployment volatility.

We compare labor market volatility under dual contracting with volatility in an otherwise-identical economy with just a single type of employment contract. For our Spanish calibration, unemployment fluctuates 22% more in the dual economy than in a single-contract economy with the same total flow of firing costs; it fluctuates 35% more in the dual economy than in a single-contract economy with the same average unemployment rate. While labor market duality thus appears to be one important factor to explain the high volatility of the Spanish economy, we find that the high level of protection

offered to the unemployed also plays a large role in amplifying Spanish labor market fluctuations.

# 1 Introduction[1]

The unemployment response to GDP fluctuations seems to vary significantly across countries and across periods. Looking at the recent experience during the crisis, the change in unemployment for each percentage point of fall in GDP ranges from 0.1 in Germany to 2.2 in Spain.[2] And, in contrast with the US, in European countries the unemployment rate seems to have been more responsive to GDP fluctuations in recent periods than previously. For instance, Bertola (2009) shows that, while in the US the unemployment rate have increased by about 0.4 percentage points for each percentage point of GDP growth slowdown, both during the 1962-82 and the 1983-2007 periods, in France this ratio rose from 0.14 in the earlier period to 0.4 in the most recent one. Furthermore, during the current crisis, both in the US and in France, the rise in the unemployment rate per percentage point of GDP growth slowdown seems even higher.

There could be many reasons explaining cross-country differences in unemployment volatility. First, GDP fluctuations may be caused by different types of shocks, in terms of sources (preferences, productivity, etc.) and sectoral composition (more or less concentrated in labor-intensive activities), and the response of unemployment needs not to be the same regardless of the fundamental and sectoral natures of the shocks. Secondly, labor market institutions constraining labor market flows (i.e., firing and hiring costs, ), wage determination procedures (i.e, nominal and real wage rigidities) and unemployment benefits and other social policies also determine how (un)employment fluctuates. In this regard, the main institutional change in most European countries has been the "liberalization" of "atypical" employment contracts (temporary contracts) that in many countries have become so prevalently used that, indeed, the labor market has a dual structure. Finally, and regarding the most recent downturn, countries have differed significantly in the employment policies put in place to fight the rise in unemployment, with Germany giving a strong push to subsidies to short-term work schemes and other expanding significantly income support for job losers and income earners.[3]

[1]We thank Laura Hospido and Aitor Lacuesta for providing us with data, and seminar participants at the Banco de España for helpful comments. The views expressed in this paper are those of the authors and do not necessarily coincide with those of the Banco de España or the Eurosystem.

[2]Babecký, van der Cruijsen-Knoben, and Fahr (2009).

[3]For a summary of the employment policies put in place in OECD countries to deal with the

This paper focuses on the role of some labor market institutions at explaining the volatility of (un)employment. Specifically, we analyze the cyclical consequences of a dual labor market, in which workers may be hired under two different employment contracts, temporary and permanent. Temporary contracts have a determined duration and, upon expiration, the firm has to decide whether to keep the worker under a permanent contract or dismiss her at no cost. Permanent contracts are open-ended contracts and dismissals entail strictly positive firing costs. Under this dual structure, firms face three relevant decisions: i) hirings under each type of contract, ii) upgrading of temporary workers into permanent positions, and iii) firings of permanent and temporary workers. All these decisions are very much affected by the gap in firing costs between both types of employment contracts, and, hence, this gap also determine to a great extent the responses to shocks of employment and unemployment inflows and outflows.

Our instrument to perform this analysis is an extended version of the Mortensen-Pissarides (1994) model, with endogenous job creation and job destruction. The new features of the model are required by the analysis of the impact of firing costs in a dual labor market: i) the coexistence of the two types of employment contracts, ii) contract-specific hiring and firing behavior, and iii) conversion of temporary employees into permanent ones. Hiring, firing, and conversion are driven both by economic and legal considerations. First, matching frictions constrain job creation; once matches are formed, productivity shocks lead to job creation and job destruction. Second, legal constraints on the use of temporary contracts are modelled by forcing incumbent temporary employees either to be upgraded to permanent positions or be dismissed at a given rate. To account for business cycle fluctuations, productivity shocks have an aggregate component and a match-specific component, both displaying some degree of persistence.

Within this framework, and assuming flexible wages, hiring and firing decisions can be summarized in terms of the productivity of the match. In our model, all new jobs (endogenously) start under temporary contracts when the productivity of the match is above a "hiring threshold". A temporary workers is dismissed whenever the match productivity falls below the "hiring threshold". Additionally, in each period, a certain fraction of temporary contracts reach the end of their contracts. Of these workers, those whose productivity is above a "conversion threshold" are upgraded to permanent, while those in a match with productivity below that threshold are dismissed. Finally, permanent workers are dismissed when the productivity of the match is below a "firing threshold". It turns out that these three thresholds can be unambiguously ordered: the "conversion threshold" is above the "hiring threshold"

recent economic downturn, see OECD (2009).

which, in turn, is above the "firing threshold". Moreover, the distance between the thresholds depend on the state of the economy and the level of firing costs, with all three thresholds collapsing into one when firing costs are nil. This has interesting implications not only for the determination of stock and flow variables in the steady state, but also for their cyclical properties. For instance, the fact that the "conversion threshold" is above the "firing threshold" has negative consequences for aggregate productivity, as lower productivity permanent matches are kept in place while higher productivity temporary matches are destroyed.

To assess the impact of firing costs on permanent contract on employment volatility we calibrate the model to the Spanish labor market, the most typical example of a dual one, and then perform several simulation exercises, comparing the employment volatility in the dual labor market with that of labor markets with single employment contracts of various firing costs. Our baseline parameterization reproduces the average stocks and flows of the Spanish labor market quite successfully. Its volatility is too low overall, but in relative terms it does a good job of capturing the volatility effects of temporary and permanent jobs. Temporary employment is more volatile in relative terms (that is, in terms of its coefficient of variation) than permanent employment. But moreover, it also explains a larger part of total employment fluctuation, in spite of the fact that it represents a smaller stock.

There have been many papers on the macroeconomic implications of dual labor markets, athough most of them have focused on the determination of unemployment and other labor market variables at the steady state equilibrium. Blanchard and Landier (2002) take temporary contracts as contracts of a given duration that can be terminated at little or no cost and that, if converted into permanent ones, the contracts become subject to regular firing costs. They show that introducing such type of contracts may increase turnover, which results into higher, not lower, unemployment. Cahuc and Postel-Vinay (2002) embed this conversion decision into a Mortensen-Pissarides (1994) framework and assume that a constant fraction of new hires should be done under permanent contracts due to legal restrictions. Dolado, Jansen, and Jimeno (2007) is another version of the Mortensen-Pissarides model to analyze dual labor market, taking into account another feature of temporary contracts, namely that they are targeted to low-productivity workers so that, under some circumstances, only workers of specific characteristics are entitled to be hired under temporary contracts.

Despite this proliferation of studies of the implications of dual labor markets for (un)employment determination at the steady state, to the best of our knowledge, only Sala and Silva (2009) and Sala, Silva, and Toledo (2009) have investigated the cyclical consequences of dual labor markets, focusing on their implications for (un)employment volatility. They conclude that a labor market with limited flexibility in the use of temporary contracts ought to display less unemployment volatility than a fully flexible labor market. In a similar vein, Cahuc, Le Barbanchon, Bentolila and Dolado and (2009) use this framework to compare employment adjustments in France and Spain during the crisis, but they treat this as a comparison of steady states, instead of calculating the model's dynamics.

Sala et al. (2009) is the paper most closely related to our own, since it studies cyclical dynamics in the context of endogenous separation, and their model shares much of the structure of ours. However, there is an important difference, with non-trivial quantitative implications. While Sala et al (2009) assume that the match-specific component of productivity is i.i.d., we instead allow for persistence both in aggregate and match-specific productivity. Assuming i.i.d. match-specific productivity simplifies calculations, because it eliminates the problem of solving for the equilibrium distribution of productivities. Unfortunately, it implies a counterfactual parameterization, in which there is no persistence in the quality of a given job relationship, greatly changing the incentives involved in promoting a worker to permanence. Furthermore, persistent match-specific productivity substantially changes business cycle dynamics. If idiosyncratic productivity is persistent, then economic expansions lead to the accumulation of temporary workers in "fragile jobs" which are destroyed *en masse* as soon as the state of the economy worsens. This makes employment fluctuations highly asymmetric, a feature of the business cycle already highlighted by Mortensen and Pissarides (1994). It may also make them larger, and we show that it tends to make temporary jobs relatively more volatile, because the stock of fragile temporary workers converges faster than the stock of fragile permanent workers. The cost of allowing for persistence is that we must keep track of the distribution of productivity over time in our dynamic simulation; our computational method follows Costain and Jansen (2009).

The paper is laid out in four more sections. We first present the model and describe some of its basic implications for hiring and firing behavior (Section 2). Next, we discuss the model's steady state, and analyze how it is affected by labor market policy parameters (Section 3). Then, we discuss its dynamics under aggregate shocks that follow a two-state Markov process (Section 4; the $N$-state case is studied in Appendix 1). Then we perform some simulation exercises to compare (un)employment volatility in different scenarios (Section 5). The final section contains some concluding remarks.

# 2 The model

Here we define a version of the Mortensen-Pissarides (1994) model of job creation and destruction with two classes of contracts, temporary and permanent.

## 2.1 Productivity of matches

The productivity of a matched worker-firm pair is assumed to be the sum of two components, an idiosyncratic component $z$ and an aggregate component $y$. The idiosyncratic productivity distribution for new jobs is $G_0(z)$. In each period that a match continues, a worker receives a new idiosyncratic shock with probability $\lambda$. New values of productivity are then drawn from the distribution $G(z)$. For simplicity, we focus on the case where the two distributions are the same: $G_0 = G$.[4]

Total productivity is $z + y$, where $y$ is an aggregate random variable with mean $\overline{y}$ which takes $N$ possible values $y_1 < y_2 < ... < y_N$. In each period, there is a new aggregate productivity shock with probability $\mu$. The probability that the next state is $y_j$ conditional on current state $y_i$ is written $M_{y_j|y_i}$, which we can arrange into a Markov matrix as follows:

$$M = \begin{pmatrix} M_{y_1|y_1} & & M_{y_1|y_N} \\ & ... & \\ M_{y_N|y_1} & & M_{y_N|y_N} \end{pmatrix}$$

Here column $j$ represents the probabilities of the $N$ possible successor states of the current state, so the columns of $M$ must sum to one. For concise notation we will sometimes abbreviate $\mu_{j|i} \equiv \mu M_{y_j|y_i}$. In this notation, we have $\sum_{k=1}^{N} \mu_{k|i} = \mu$.

## 2.2 Matching process

The total labor force is normalized to one. In each unit of time, a mass $\rho$ of new workers is born, and fraction $\rho$ of existing workers (employed or unemployed) retire and exit the labor pool.

Firms may open any number of vacant jobs; keeping a job open costs $c$ per unit of time. The total number of vacant jobs is $v$. Unemployed workers produce output $b$. We assume some jobs are more productive than unemployment; that is, $G(b - \overline{y}) < 1$. Only unemployed workers can search. Search *per se* is costless. Newly

[4] It would be straightforward to allow for accumulation of match-specific experience by assuming $G$ dominates $G_0$ in the sense of first-order stochastic dominance. See for example Mortensen and Nagypal (Scand JE 2008).

matched worker-firm pairs can separate costlessly, which implies that in equilibrium, the value of unemployment is less than or equal to the value of being newly matched. Therefore, in equilibrium, all unemployed workers search.

Searching workers $u$ and vacant jobs $v$ meet according to the matching function

$$m(u, v).$$

Assuming constant returns to scale, the instantanous meeting probability for vacancies is given by

$$\frac{m(u, v)}{v} = m\left(\frac{1}{v/u}, 1\right) \equiv q(\theta),$$

where $\theta \equiv v/u$ is labor market tightness. The meeting probability for unemployed workers equals $\theta q(\theta)$.

Both workers and firms can decide to separate from their current matches, subject to legal costs which will be discussed below. There is no recall of matches. That is, if either agent chooses to separate, both agents become unmatched, and can only become matched again with a new partner by means of the matching function.

## 2.3 Labor market policy

A firm that creates a new job may choose to hire a worker under two types of contract: a fixed-term contract we will call "temporary", or an open-ended contract we will call "permanent". Temporary contracts can be freely destroyed at any time. However, if a contract is initially of the temporary type, this contract status expires with probability $\delta$ per period. Upon expiry the firm must decide whether to fire the worker or promote him/her to a permanent contract.

Firing a worker who has a permanent contract requires the firm to pay a firing cost $F$. We assume $F$ represents a loss of income to the matched pair, that is,.it is a "red-tape" cost instead of a transfer of income from the firm to the worker.

## 2.4 Match surplus and wage bargaining

The productivity processes $y$ and $z$ are the only shocks in our model. We conjecture that agents' values are functions of productivity only, as in Mortensen-Pissarides (1994). Therefore we write the values of unemployed workers and vacant jobs as $U(y)$ and $V(y)$, respectively, in terms of aggregate productivity only. We define firms' values of temporary and permanent jobs as $J^T(z, y)$ and $J^P(z, y)$, and workers' values of temporary and permanent jobs as $W^T(z, y)$ and $W^P(z, y)$. We postpone statement of the associated Bellman equations to Sections 3 and 4.

Since pairs with temporary contracts can separate costlessly, the surplus of a worker in a temporary job is $W^T(z,y) - U(y)$; the firm's surplus for this job is $J^T(z,y) - V(y)$; and the total surplus of a temporary job is

$$S^T(z,y) = W^T(z,y) - U(y) + J^T(z,y) - V(y)$$

A worker can also separate costlessly from a permanent job, so the worker's surplus from this job is $W^P(z,y) - U(y)$. However, when a permanent job separates, the firm must pay the firing cost $F$. Thus the outside option of a firm with a permanent job is $-F$, and its surplus relative to this outside option is $J^P(z,y) - V(y) + F$. Therefore the total surplus of a permanent job is

$$S^P(z,y) = W^P(z,y) - U(y) + J^P(z,y) - V(y) + F$$

We assume that the wage is determined by Nash bargaining between a firm and its new hires, treating separation as the outside option. In addition, the wage is updated whenever new information arrives that affects the value of the match, so the surplus sharing equations hold at all times. The worker's bargaining share is $\eta$. These assumptions imply that the surplus-sharing rule for temporary contracts is

$$J^T(z,y) - V(y) = (1-\eta)\, S^T(z,y),$$

whereas the rule for permanent workers is given by

$$J^P(z,y) - V(y) + F = (1-\eta)\, S^P(z,y),$$

since the firm's outside option is destroying the job and paying $F$. Hence there are distinct wage functions for temporary and permanent jobs, $w_T(z,y)$ and $w_P(z,y)$.

## 2.5 Job creation and job destruction

Vacancies are opened until their value $V(y)$ is driven down to zero, which implies the following job creation equation:

$$V(y) = 0 \qquad \rightarrow \qquad \frac{c}{q(\theta(y))} = \int_{R^T(y)}^{\infty} J^T(z,y)dG(z) = (1-\eta)\int_{R^T(y)}^{\infty} S^T(z,y)dG(z), \tag{1}$$

where $c$ is the flow cost of an open vacancy, and $R^T(y)$ is the threshold level of productivity sufficient to justify hiring the worker.

Separations are determined by three threshold levels of productivity above which matches continue in each aggregate state $y$, depending on the current state of the contract. First, as we just mentioned, there is a threshold productivity for temporary jobs, $R^T(y)$, such that any job eligible for a temporary contract continues as long as $z \geq R^T(y)$. This threshold is determined by

$$J^T(R^T(y), y) = 0 \qquad \rightarrow \qquad S^T(R^T(y), y) = W^T(R^T(y), y) - U(y) = 0 \tag{2}$$

Note therefore that hiring and continuation of temporary contracts is jointly optimal: it occurs if and only if both parties benefit.

Second, there is a threshold productivity at the moment temporary status expires, $R^C(y)$, such that any job which is no longer eligible for temporary status is converted to permanence if $z \geq R^C(y)$. This threshold is determined by

$$J^P(R^C(y), y) = 0 \qquad \rightarrow \qquad S^P(R^C(y), y) = \frac{F}{1-\eta} > 0 \tag{3}$$

That is, the firm must be indifferent between making the worker permanent or destroying the job at zero cost.

Note, therefore, that the promotion decision is not bilaterally efficient. If a firm were to promote a worker with productivity $z = R^C(y) - \varepsilon$, with $\varepsilon$ infinitesimally small, the firm's value would become infinitesimally negative, while the worker's value would become strictly positive, implying a net gain for the pair. This Pareto improvement would be possible if matched pairs could sign binding wage contracts prior to promotion. The optimal wage contract would commit the worker to a lower wage, implicitly sharing the expected cost of firing between the worker and the firm. Since we instead assume wages are constantly reset by Nash bargaining, such commitment is impossible. The firm expects a permanent worker to bargain up the wage by taking advantage of the firm's unfavorable threat point, and therefore may choose to fire a worker even when that worker would have strictly positive surplus after promotion.

Finally, there is a threshold productivity $R^P(y)$ for firing of permanent jobs, such that jobs with permanent status continue as long as $z \geq R^P(y)$. This last threshold is determined by

$$J^P(R^P(y), y) + F = 0 \qquad \rightarrow \qquad S^P(R^P(y), y) = W^P(R^P(y), y) - U(y) = 0 \tag{4}$$

Note that from the matched pair's perspective, firing of permanent contracts is jointly efficient; it occurs only if both parties benefit. But this comment takes as given and sunk the cost $F$, which is a policy parameter. So even though separation is jointly efficient from the pair's perspective conditional on policy, it is not socially efficient.

## 2.6 Equilibrium

In equations (1)-(4), we see that the job creation and destruction decisions define four equations for each aggregate state $y$ that determine job tightness $\theta(y)$ and the reservation productivities $R^T(y)$, $R^C(y)$, and $R^P(y)$ for each aggregate state $y$. Moreover, (1)-(4) show that all these conditions can be stated in terms of the surplus functions $S^T(z, y)$ and $S^P(z, y)$. Later we will see how to calculate the surplus functions in terms of the reservation productivities, which will allow us to substitute the surplus functions out of equations (1)-(4).

Therefore, equations (1)-(4) implicitly form a system of $4N$ equations to determine the $4N$ unknowns $\theta(y)$, $R^T(y)$, $R^C(y)$, and $R^P(y)$ for the $N$ aggregate states $y$. A solution of this system of equations is an equilibrium of our model.

## 2.7 Characterizing the reservation thresholds

Calculating the ordering of all the reservation thresholds is nontrivial in general, but we can deduce several key facts from first principles. To understand the ordering, it helps to reason on the basis of the joint payoff to the pair, which is just a discounted flow of output $z + y$ minus the worker's cost of employment (later we will see that this cost equals $b + \frac{\eta c\theta(y)}{1-\eta}$), ending with a lump sum payment of 0 (if the contract is temporary) or $F$ (if the contract is permanent).

First, compare the expected payoff to a matched pair in a temporary contract with that to a matched pair in a permanent contract. Considering all possible future realizations of the process for $z + y$, the expected flow of income in these two pairs is the same up to the moment of separation. The only difference is that upon separation, the pair in a permanent contract loses $F$. Therefore the expected payoff to the pair is lower in the case of a permanent contract, that is, $W^P(z, y) + J^P(z, y) \leq W^T(z, y) + J^T(z, y)$. Moreover, offering the worker a permanent contract lowers the firm's threat point from 0 to $-F$. Since offering a permanent contract diminishes the pair's joint payoff, and also lowers the firm's threat point, a firm always prefers to offer a temporary contract if legally permitted to do so. That is:

**Proposition 1** *If a firm can choose between hiring a worker on a temporary contract and hiring the same worker on a permanent contract, it chooses the temporary contract.*

Next, note that a higher current value of $z$ raises the payoff to the match until a new idiosyncratic shock arrives, or until the match separates. If a new idiosyncratic shock arrives, its value is uncorrelated with the current $z$. And separation is *less* likely to occur if the current $z$ is higher (since separation occurs only when surplus falls sufficiently low; we prove this more formally in Appendix 2). Therefore match surplus is an increasing function of $z$. Second, suppose the process for $y$ exhibits first-order stochastic dominance, in the sense of Assumption 2, so that a higher $y$ now makes a higher $y$ more likely in the future too.

**Assumption 2** *$M$ is a Markov matrix, with all elements strictly positive. $M$ has the property that for any two nonnegative vectors $v$ and $v'$, if $v \geq v'$, then $(I+M)v \geq (I+M)v'$, where $I$ is the $N$-by-$N$ identity matrix.*

Then a higher current value of $y$ raises the payoff to the match until a new aggregate shock arrives, or until the match separates; moreover, it predicts a higher $y$ when the next shock arrives, and makes separation less likely. Therefore, we conclude that match surplus is also an increasing function of the aggregate shock $y$.

**Lemma 3** *The surplus functions associated with temporary and permanent matches are increasing functions of $z$ and $y$:*

$$\text{for all } y,\ z_1 \leq z_2 \text{ implies } S^T(z_1, y) \leq S^T(z_2, y) \text{ and } S^P(z_1, y) \leq S^P(z_2, y)$$

$$\text{for all } z,\ y_1 \leq y_2 \text{ implies } S^T(z, y_1) \leq S^T(z, y_2) \text{ and } S^P(z, y_1) \leq S^P(z, y_2)$$

**Proof.** See Appendix 2.

All three types of reservation thresholds are determined by equating surplus to a constant: $S^T(R^T(y), y) = 0$, $S^P(R^C(y), y) = \frac{F}{1-\eta}$, and $S^P(R^P(y), y) = 0$. Geometrically, this means a higher $y$ requires a *lower* reservation threshold $R^i(y)$ for each type of threshold $i \in \{T, C, P\}$. Likewise, since $R^C(y)$ is associated with a higher level of surplus than $R^P(y)$, we conclude that $R^P(y) \leq R^C(y)$ for any $y$.

To determine where the temporary hiring threshold lies relative to the other two thresholds, note that firms are initially able to choose between hiring on a temporary and permanent basis, and we have argued they strictly prefer temporary hiring (assuming $z$ is sufficiently high; otherwise they prefer to let the worker go). Expiry of a temporary contract simply shrinks the firm's choice set, eliminating its preferred choice, requiring it instead to hire on a permanent basis (or to let the worker go). Thus expiry of a temporary contract makes a match strictly less valuable to the firm; and therefore a firm is less willing to promote than it is to hire, that is, $R^T(y) \leq R^C(y)$.

Finally, consider the relation between $R^P(y)$ and $R^T(y)$. We already know a matched pair has a lower expected payoff in a permanent contract than in a temporary contract: $W^P(z, y) + J^P(z, y) \leq W^T(z, y) + J^T(z, y)$. This occurs because some permanent relationships continue, in order to avoid paying the cost $F$, even when $W^P(z,y)+J^P(z,y) \leq U(y)$. But therefore separation occurs whenever the loss exceeds $F$, implying $W^P(z,y)+J^P(z,y)+F \geq U(y)$ as long as a match continues. Thus, considering all future paths starting from a given state $(z,y)$, the payoff to a permanent contract is lowered along some realizations by an amount that never exceeds $F$, implying $W^P(z,y)+J^P(z,y)+F \geq W^T(z,y)+J^T(z,y)$. But therefore the surplus of a permanent contract, which includes $F$, is higher than that of a temporary contract evaluated in the same state: $S^P(z,y) \geq S^T(z,y)$. Thus given Lemma 2, together with the definition of the hiring thresholds $S^P(R^P(y),y) = S^T(R^T(y),y) = 0$, we must have $R^P(y) \leq R^T(y)$.

For notational simplicity we will often abbreviate $R^i(y_j) \equiv R^i_j$ for $i \in \{T,C,P\}$ and $j \in \{1,2,...,N\}$. Therefore we can summarize what we know about the reservation productivities as follows:

**Proposition 4** *Assume $y$ exhibits FOSD in the sense of Definition 1. Then each type $i \in \{T,C,P\}$ of reservation threshold is a decreasing function of aggregate productivity:*

$$R^i_N \leq R^i_{N-1} \leq ... \leq R^i_1.$$

*Moreover, for each aggregate state $y_j$, $j \in \{1,2,...,N\}$, the firing threshold for permanent contracts lies below the hiring/firing threshold for temporary contracts, which lies below the promotion threshold:*

$$R^P_j \leq R^T_j \leq R^C_j.$$

## 2.8 Employment and productivity dynamics

Once $R^P(y)$, $R^C(y)$, $R^T(y)$, and $\theta(y)$ are known, we can simulate employment dynamics. In state $y$, unemployed workers become employed at rate $(1-G(R^T(y)))\theta(y)q(\theta(y))$. Conditional on idiosyncratic productivity shocks or the expiry of temporary contracts, continuation is determined by the reservation productivities. Also, whenever the aggregate state *decreases* $(y(t) = y_i > y(t+dt) = y_j)$, there is a noninfinitesimal mass of firing, as all temporary employees in the interval $[R^T_i, R^T_j)$ and all permanent employees in the interval $[R^P_i, R^P_j)$ suddenly separate.

Note that the probability of promotion and/or separation of a match with state $(z,y)$ does not depend on the exact value of $z$; it only depends on where $z$ lies relative to the reservation thresholds. We state this formally as Proposition 5a:

**Proposition 5** *Consider an interval $I = [R^a(y_j), R^b(y_k))$ formed by two adjacent reservation thresholds, that is, $a,b \in \{T,C,P\}$ and $j,k \in \{1,2,...,N\}$, with no other*

*reservation threshold between these two.*
***(a.)*** *Consider two temporary matches $i$ and $j$ with productivities $z_{it}$ and $z_{jt}$ at time $t$. If $z_{it} \in I$ and $z_{jt} \in I$, then matches $i$ and $j$ face the same probabilities of separation and promotion and of drawing any new productivity shock $z'$.*

*Let the number of temporary matches in interval $I$ at time $t$ be $n_t^T(I) > 0$. Then, in the limit as $t \to \infty$, the probability distribution of productivity among temporary matches has the following properties:*
***(b.)*** *the density over $z$ for temporary matches satisfying $z \in I$ at $t$ is $G'(z)/n_t^T(I)$;*
***(c.)*** *the average productivity of temporary matches in $I$ at $t$ is $\int_{R^a(y_j)}^{R^b(y_k)} z \frac{G'(z)}{n_t^T(I)} dz$.*

*Formulas analogous to (a), (b), and (c) hold for permanent matches as well.*

Parts (b) and (c) of the proposition show the simplest way to keep track of the distribution of employment and productivity over time. The probabilities of any given change in the state of a given match depend only on which pair of reservation produtivities match productivity lies between. Therefore to know how the productivity distribution is evolving it suffices to keep track of the mass of employment on each interval defined by two adjacent reservation thresholds.

Of course, we could analyze the dynamics of the model from any arbitrary initial productivity distribution; in this case there will initially be transition dynamics as the productivity distribution gradually converges to its long run form. But in the long run, the productivity distribution converges to a very simple form, as stated in Prop. 5a: the distribution of $z$ on the interval between two adjacent reservation thresholds is just a truncated version of the *ex ante* productivity distribution $G(z)$.

The reason this proposition holds is that temporary matches entering any interval of this form are initially drawn from distribution $G$; thereafter all transitions in employment status are conditional on $z$ only insofar as they depend on which interval $z$ lies in. Thus, while the overall distribution of productivity among job matches changes over time, due to the effects of aggregate shocks, nonetheless the form of the productivity distribution in the interval between any two adjacent reservation thresholds is always just a truncation of $G$. Keeping track of the mass of employment on each interval of this type therefore suffices to know the full productivity distribution at all times.

# 3 Steady state

Before addressing the full dynamics or our model, we next study steady state general equilibrium. Thus aggregate productivity takes a fixed value $\overline{y}$, and only idiosyncratic productivity $z$ is subject to shocks. We indicate steady state quantities by eliminating the argument $y$ and adding the subscript $ss$.

## 3.1 Value functions

### 3.1.1 Jobs

We begin by deriving the Bellman equations that govern the value functions of workers and firms, and analyze the form of the surplus functions. The value of a temporary job, $J^T_{ss}(z)$, must satisfy the Bellman equation

$$(r+\rho)J^T_{ss}(z) \;=\; z+\overline{y}-w^T_{ss}(z)+\delta\left[\mathbf{1}(z\geq R^C_{ss})J^P_{ss}(z)-J^T_{ss}(z)\right]+\lambda\left[\int_{R^T_{ss}} J^T_{ss}(x)dG(x)-J^T_{ss}(z)\right]$$

where $\mathbf{1}(x)$ is an indicator function that equals one if $x$ is true and zero otherwise. Note that the job value is discounted both by the pure rate of time preference $r$ and by the rate $\rho$ of retirement (which is simply treated as exit from the model). Besides earning income net of wages $z+\overline{y}-w^T_{ss}(z)$ per period, the firm also anticipates that temporary contracts will expire with probability $\delta$ per period, in which case the job becomes permanent if $z$ exceeds the promotion threshold $R^C_{ss}$; otherwise the job separates and has value $V_{ss}=0$. Also, the firm expects idiosyncratic shocks to arrive at rate $\lambda$; if the new level of productivity exceeds the threshold $R^T_{ss}$ the match continues; otherwise it separates, yielding value $V_{ss}=0$.

The intuition for the remaining Bellman equations is similar. The value of a permanent job satisfies

$$(r+\rho)J^P_{ss}(z) \;=\; z+\overline{y}-w^P_{ss}(z)+\lambda\left[\int_{R^P_{ss}} J^P_{ss}(x)dG(x)-G(R^P_{ss})F-J^P_{ss}(z)\right].$$

*Firms' match surplus*

We allow for free entry, so the the value of a vacancy is $V_{ss}=0$, and therefore the firm's surplus from a temporary job is just the value of that job, $J^T_{ss}(z)$. Simplifying our earlier equation,

$$(r+\rho+\delta+\lambda)\,J^T_{ss}(z) \;=\; z+\overline{y}-w^T_{ss}(z)+\delta\mathbf{1}(z\geq R^C_{ss})J^P_{ss}(z)+\lambda\int_{R^T_{ss}} J^T_{ss}(x)dG(x).$$

Since the outside option of a firm with a permanent contract is the payment of the firing cost (i.e. the value $-F$), the surplus associated with a permanent job is

$J^P_{ss}(z) + F$. Rearranging our earlier equation, we obtain

$$(r+\rho+\lambda)\left(J^P_{ss}(z)+F\right) \;=\; z+\overline{y}-w^P_{ss}(z)+\lambda\int_{R^P_{ss}} J^P_{ss}(x)dG(x)+\left[r+\rho+\lambda(1-G(R^P_{ss}))\right]F$$

where $(1-G(R^P_{ss})) = prob(z' \leq R^P_{ss})$ is the probability of continuation after a new idiosyncratic shock $z'$.

### 3.1.2 Workers

A worker's value of employment under a temporary contract must satisfy

$$\begin{aligned}(r+\rho)W^T_{ss}(z) \;&=\; w^T_{ss}(z) \;+\; \delta\left[\mathbf{1}(z\geq R^C_{ss})W^P_{ss}(z)+\mathbf{1}(z<R^C_{ss})U-W^T_{ss}(z)\right]\\ &\quad+\lambda\left[\int_{R^T_{ss}} W^T_{ss}(x)dG(x)+G(R^T_{ss})U_{ss}-W^T_{ss}(z)\right]\end{aligned}$$

where

$$(r+\rho)W^P_{ss}(z) \;=\; w^P_{ss}(z)+\lambda\left[\int_{R^P_{ss}} W^P_{ss}(x)dG(x)+G(R^P_{ss})U_{ss}-W^P_{ss}(z)\right]$$

is the value of permanent employment, and

$$(r+\rho)U_{ss} \;=\; b+\theta_{ss}q(\theta_{ss})\int_{R^T_{ss}}^{\infty}\left(W^T_{ss}(z)-U_{ss}\right)dG(z)$$

is the value of unemployment.

*Workers' match surplus*

A worker's surplus from a temporary job is $W^T_{ss}(z)-U_{ss}$. Rearranging the previous equations, we obtain

$$\begin{aligned}(r+\rho+\delta+\lambda)\left(W^T_{ss}(z)-U_{ss}\right) \;&=\; w^T_{ss}(z)-b-\theta_{ss}q(\theta_{ss})\int_{R^T_{ss}}^{\infty}\left(W^T_{ss}(x)-U_{ss}\right)dG(x)\\ &\quad+\delta\mathbf{1}(z\geq R^C_{ss})\left(W^P_{ss}(z)-U_{ss}\right)+\lambda\int_{R^T_{ss}}\left(W^T_{ss}(x)-U_{ss}\right)dG(x)\end{aligned}$$

Likewise, the surplus from a permanent job is $W^P_{ss}(z)-U_{ss}$, which satisfies

$$(r+\rho+\lambda)\left(W^P_{ss}(z)-U_{ss}\right) \;=\; w^P_{ss}(z)-b-\theta_{ss}q(\theta_{ss})\int_{R^T_{ss}}^{\infty}\left(W^T_{ss}(x)-U_{ss}\right)dG(x)+\lambda\int_{R^P_{ss}}\left(W^P_{ss}(x)-U_{ss}\right)dG(x)$$

## 3.2 Surplus functions

It now simplifies the analysis to combine all these Bellman equations and focus only on total match surplus. We can also use the zero-profit condition (1) on vacancies to substitute as follows:

$$\theta_{ss}q(\theta_{ss})\int_{R^T_{ss}}^{\infty}\left(W^T_{ss}(x)-U_{ss}\right)dG(x)=\theta_{ss}q(\theta_{ss})\eta\int_{R^T_{ss}}^{\infty}S^T_{ss}(x)dG(x)=\eta c\theta_{ss}/(1-\eta).$$

Summing our previous expressions for firms' and workers' surplus, the Bellman equations governing total match surplus for temporary and permanent jobs are

$$(r+\rho+\lambda+\delta)\,S^T_{ss}(z)\;=\;z+\overline{y}-b-\frac{\eta c\theta_{ss}}{1-\eta}+\delta\mathbf{1}(z\geq R^C_{ss})\left(S^P_{ss}(z)-F\right)+\lambda\int_{R^T_{ss}}S^T_{ss}(x)dG(x), \tag{5}$$

$$(r+\rho+\lambda)\,S^P_{ss}(z)\;=\;z+\overline{y}-b+(r+\rho)F-\frac{\eta c\theta_{ss}}{1-\eta}+\lambda\int_{R^P_{ss}}S^P_{ss}(x)dG(x). \tag{6}$$

The key point to notice here is that we can take derivatives through (5)-(6) with respect to $z$ at most points, except at $R^C_{ss}$, where (5) implies a sudden change in slope. Differentiating, we find that $S^P_{ss}(z)$ is linear, and $S^T_{ss}(z)$ is piecewise linear. The slopes are

$$\frac{dS^T_{ss}(z)}{dz}\;=\;\begin{cases}\frac{1}{r+\rho+\lambda+\delta}, & z<R^C_{ss}\\ \frac{1}{r+\rho+\lambda}, & z\geq R^C_{ss}\end{cases}$$

$$\frac{dS^P_{ss}(z)}{dz}\;=\;\frac{1}{r+\rho+\lambda}$$

In addition to a change in slope, (5) shows that the surplus of a temporary match is discontinuous at $z=R^C_{ss}$. Note that $J^P_{ss}(R^C_{ss})=0$ implies $S^P_{ss}(R^C_{ss})-F=\frac{\eta}{1-\eta}F$. Plugging this formula into (5), the jump in $S^T_{ss}(z)$ at $z=R^C_{ss}$ equals $(r+\rho+\lambda+\delta)^{-1}\frac{\delta\eta}{1-\eta}F$. This discontinuity represents the sudden decrease in the pair's joint value as $z$ falls below $R^C(y)$, because of the inefficient elimination of promotion below this value of $z$.

Putting all this information together, and setting $S^P_{ss}(R^P_{ss})=S^T_{ss}(R^T_{ss})=0$, we can write the surplus functions explicitly conditional on the reservation productivities:

$$S^T_{ss}(z)\;=\;\begin{cases}\frac{z-R^T_{ss}}{r+\rho+\lambda+\delta}, & z<R^C_{ss}\\ \frac{R^C_{ss}-R^T_{ss}+\delta\eta F/(1-\eta)}{r+\rho+\lambda+\delta}+\frac{z-R^C_{ss}}{r+\rho+\lambda}, & z\geq R^C_{ss}\end{cases} \tag{7}$$

$$S^P_{ss}(z)\;=\;\frac{z-R^P_{ss}}{r+\rho+\lambda} \tag{8}$$

## 3.3 Steady state equilibrium

Equilibrium requires that the job creation and destruction equations (1)-(4) be satisfied when we plug in the Bellman equations (5)-(6) that define the surplus. The steady state job creation equation is simply

$$\frac{c}{q(\theta_{ss})} = (1-\eta)\int_{R^T_{ss}} S^T_{ss}(x)dG(x) \tag{9}$$

Next, since we know that $R^T(y) < R^C(y)$ for any $y$, we have $\mathbf{1}(z \geq R^C_{ss}) = 0$ at $z = R^T_{ss}$. Therefore the $\delta$ term cancels out of the job destruction condition for temporary workers, leaving

$$0 = R^T_{ss} + \overline{y} - b - \frac{\eta c\theta_{ss}}{1-\eta} + \lambda \int_{R^T_{ss}} S^T_{ss}(x)dG(x). \tag{10}$$

The steady state job destruction condition for permanent workers is

$$0 = R^P_{ss} + \overline{y} - b + (r+\rho)\,F - \frac{\eta c\theta_{ss}}{1-\eta} + \lambda \int_{R^P_{ss}} S^P_{ss}(x)dG(x). \tag{11}$$

Finally, the equation for the promotion threshold can be written as

$$(r+\rho+\lambda)\frac{F}{1-\eta} = R^C_{ss} + \overline{y} - b + (r+\rho)\,F - \frac{\eta c\theta_{ss}}{1-\eta} + \lambda \int_{R^P_{ss}} S^P_{ss}(s)dG(x). \tag{12}$$

but it is simpler to subtract this equation from (11) and thus replace it by

$$R^C_{ss} = R^P_{ss} + (r+\rho+\lambda)\frac{F}{1-\eta} \tag{13}$$

These equations can be simplified further by plugging the explicit surplus formulas (7)-(8) into the integrals on the right-hand side, leaving just four unknowns $R^T_{ss}$, $R^C_{ss}$, $R^P_{ss}$, and $\theta_{ss}$. Thus steady state equilibrium can be calculated by solving the system of four equations in four unknowns (9)-(10).[5]

[5] It might seem easier to plug (7)-(8) directly into the job destruction equations (2)-(4). However, by doing this, all information about the post-hiring productivity distribution $G(z)$ is lost. That is, $G(z)$ enters the Bellman equations (5)-(6) but not the explicit formulas (7)-(8). Therefore (7)-(8) are necessary but not sufficient conditions to determine the equilibrium surplus functions.

### 3.4 Steady state employment

Given $R^P_{ss}$, $R^C_{ss}$, $R^T_{ss}$, and $\theta_{ss}$, we can also calculate employment. By Prop. 5, we conclude that steady state productivity distribution for temporary workers is just the underlying distribution $G$, truncated at $R^T_{ss}$. Using this fact, the transitional dynamics in the absence of aggregate shocks are:

$$\begin{aligned}\dot{n}^T_t &= \theta_{ss} q(\theta_{ss})(1-G(R^T_{ss}))u_t - (\rho+\delta+\lambda G(R^T_{ss}))n^T_t \\ \dot{n}^P_t &= \delta\frac{1-G(R^C_{ss})}{1-G(R^T_{ss})}n^T_t - \left(\rho+\lambda G(R^P_{ss})\right)n^P_t\end{aligned}$$

where

$$u_t = 1 - n^T_t - n^P_t$$

at all times.

In steady state, the first two equations imply

$$n^T_{ss} = \frac{\theta_{ss} q(\theta_{ss})(1-G(R^T_{ss}))}{\rho+\delta+\lambda G(R^T_{ss})}u_{ss}$$

$$n^P_{ss} = \frac{\delta(1-G(R^C_{ss}))}{(\rho+\lambda G(R^P_{ss}))\,(1-G(R^T_{ss}))}n^T_{ss}$$

Note that in principle we may find $R^P_{ss} < 0$, so that $G(R^P_{ss}) = 0$. Thus the last equation shows that the stock of permanent employees can be infinitely larger than the stock of temporary employees *unless* there is a nonzero flow of retirement ($\rho > 0$). Therefore considering $\rho > 0$ allows us to explore a larger parameter space— in particular, it implies a well-defined steady state even with large values of $F$ which firms never or almost never choose to pay. Finally, plugging these equations into the identity $u_{ss} = 1 - n^T_{ss} - n^P_{ss}$, steady state unemployment is

$$u_{ss} = \frac{\rho+\delta+\lambda G(R^T_{ss})}{\rho+\delta+\lambda G(R^T_{ss}) + \theta_{ss} q(\theta_{ss})\left[1-G(R^T_{ss}) + \frac{\delta(1-G(R^C_{ss}))}{\rho+\lambda G(R^P_{ss})}\right]}$$

### 3.5 Calibration

Parameters are given in Table 1. We calibrate our model on a monthly frequency. The real interest rate is set to 2% per annum, or $r = 0.0017$ per month. The exogenous retirement rate, $\rho$, is set to 0.0021, which implies that a worker who does not experience endogenous separations can expect to stay on the same job for 40

| *Description* | *Parameter* | *Value* |
|---|---|---|
| Real interest rate | $r$ | 0.0017 |
| Rate of retirement and rebirth | $\rho$ | 0.0021 |
| *Matching and bargaining* | | |
| Vacancy posting cost | $c$ | $0.3Ez$ |
| Unemployment elasticity of matching | $\epsilon$ | 0.5 |
| Coefficient of matching function | $\chi$ | 0.3985 |
| Worker bargaining power | $\eta$ | 0.5 |
| *Aggregate productivity* | | |
| Unemployment productivity | $b$ | $0.8Ez$ |
| Mean aggregate productivity | $Ey$ | 0 |
| Transition rate to recession from boom | $\mu_{1\|2}$ | 0.05 |
| Transition rate to boom from recession | $\mu_{2\|1}$ | 0.1 |
| Productivity decrement in recession | $y_1 - Ey$ | -0.04 |
| Productivity increment in boom | $y_2 - Ey$ | 0.02 |
| *Idiosyncratic productivity* | | |
| Arrival rate of idiosyncratic shocks | $\lambda$ | 0.0203 |
| Standard deviation of $\log z$ | $\sigma_z$ | 0.0785 |
| Mean of $\log z$ | $\mu_z$ | 0 |
| *Policy* | | |
| Firing cost for permanent jobs | $F$ | $1.9366Ez$ |
| Temporary contract expiry rate | $\delta$ | 0.0417 |

Table 1: Baseline parameterization

years. For most of our sample period, the Spanish labor legislation established that a certain worker could not stay in the same firm under a succession of temporary contracts for more than two years. We thus set the expiry rate, $\delta$, to 1/24.

In the absence of direct evidence on the Spanish matching function, we draw from estimates for other European countries and set the elasticity of the matching function with respect to unemployment, $\varepsilon$, to 0.5.[6] Following standard practice, we assume that the Hosios (1990) condition for efficient job creation holds, which implies setting the workers' bargaining power parameter, $\eta$, equal to $\varepsilon$.

Parameters $c$ and $b$ are set relative to the steady-state equilibrium cross-sectional average of worker productivity, $Ez$. We set the cost of posting a vacancy, $c$, to 0.30 of average worker productivity, which is roughly the midpoint of estimates suggested

[6] See e.g. Petrongolo and Pissarides (2001).

in the literature.[7] Regarding the income flow in unemployment, $b$, the consensus estimate for the US is 70% of average worker productivity.[8] Since unemployment protection is more generous in Spain than in the US, we instead set b to 80% of average worker productivity.[9]

No direct microeconomic evidence exists for the remaining five parameters, namely the non-transfer component of firing costs ($F$), the scale parameter in the matching function ($\chi$), and the parameters governing the arrival rate, mean and standard deviation of idiosyncratic productivity shocks ($\lambda$, $\mu$ and $\sigma$, respectively). Following standard practice, $\mu = E(\log(z))$ is normalized to 0. The remaining four parameters are calibrated using macroeconomic data. In particular, we use quarterly data from the Spanish Encuesta de Población Activa (EPA) to construct series for the stocks of temporary and permanent employment as fractions of the active population, as well as for the quarterly transition probabilities between unemployment and temporary employment, and between permanent employment and unemployment. Our sample period is 2001:Q1-2008:Q3.[10] We then take sample averages of our four series and find the values of $F$, $\chi$, $\lambda$ and $\sigma$ for which the steady state values $n_{ss}^T$, $n_{ss}^P$, $\theta_{ss}q(\theta_{ss})[1-G(R_{ss}^T)]$ and $\lambda G(R_{ss}^P)$ are all exactly equal to the sample average of their corresponding empirical counterpart.[11] This method delivers values of $F = 3.1$ times average monthly worker productivity (i.e. about one fourth of average annual worker productivity), $\chi = 0.315$, $\lambda = 0.02$ (which implies that idiosyncratic shocks

[7] Shimer (2005) proposes a value of 0.213, whereas Hall and Milgrom (2008) use a value of 0.43, in both cases as a fraction of average worker productivity.

[8] See e.g. Hall and Milgrom (2008) and Pissarides (2009). As in those papers, we refer to the average productivity in equilibrium among employed workers, not the mean of the *ex ante* distribution $G$.

[9] Unemployment protection in general includes not only statutory benefits, but also other social mechanisms, such as extended family networks, which Bentolila and Ichino (2008) argue provide higher protection in Mediterranean countries.

[10] The EPA divides the active population in four groups: non-salaried workers, temporary salaried workers, permanent salaried workers, and unemployed workers. Since our model does not include the first group, we assign them to the second and third groups using the same weights as those of temporary and permanent workers in total salaried employment. This way, our empirical rates of unemployment and temporary employment (the latter defined as the share of temporary workers in total salaried employment) remain unchanged.

Also, as is well known, quarterly data on transition rates suffer from aggregation bias (see e.g. Shimer 2008), such that monthly rates are considerably higher than what results from dividing quarterly rates by three. For this reason, in order to obtain estimates of monthly transition rates we rescale the quarterly transition rates by 2/3, rather than simply by 1/3.

[11] Notice that, given the latter four steady state values, the other two transition probabilities in our model (temporary employment to unemployment and to permanent employment, respectively) are pinned down by the steady state laws of motion in our model.

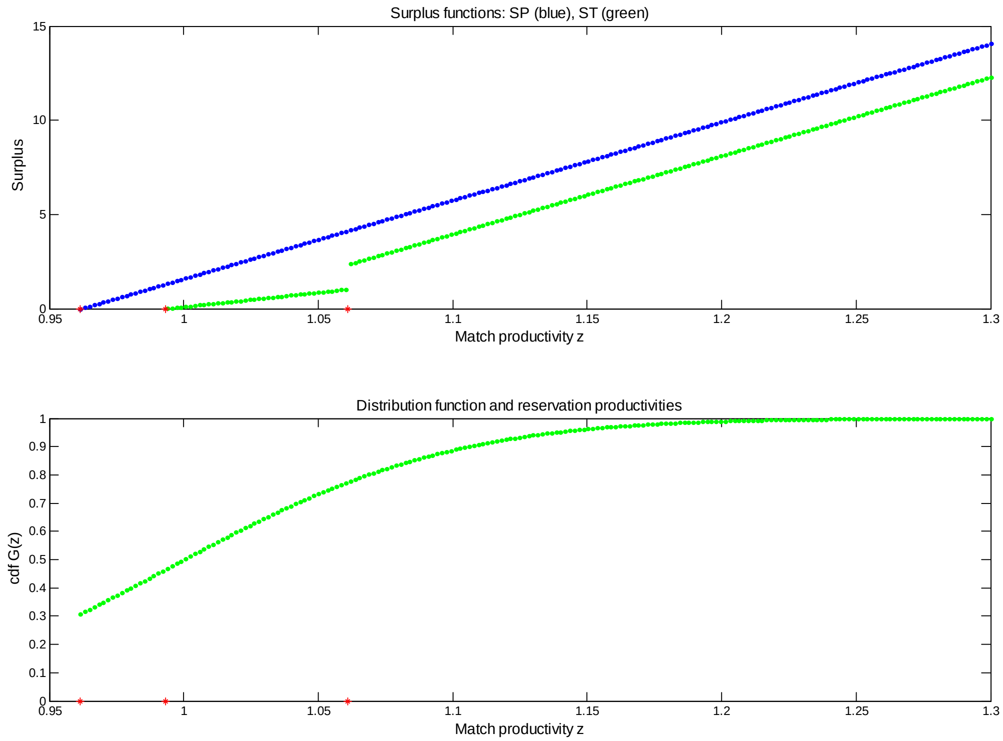


Figure 1: Steady state: surplus functions and distribution function

arrive approximately every four years on average) and $\sigma = 0.126$.

## 3.6 Steady state effects of dual labor markets

### 3.6.1 Surplus functions

Figure 1 illustrates the steady state surplus function under the baseline calibration. The surplus for permanent workers is shown in blue; that of temporary workers is in green. Permanent workers' surplus function lies above that of temporary workers; even though permanent contracts have a lower expected payoff, their surplus is higher since it is calculated relative to a lower outside option for the firm.

The reservation thresholds are highlighted with red stars. As we showed earlier, the order of the thresholds is $R^P < R^T < R^C$; also, we see a discontinuity in the surplus for temporary workers at $R^C$, due to the pairwise inefficiency of separation. For comparison, the lower panel shows the cumulative distribution function of idiosyncratic shocks $z$. We see that somewhat more than half of new matches result in a hire ($G(R^T) = 0.465$). On the other hand, promotion to permanence is much more selective: promoted workers come from the top fifth of the unconditional distribution of idiosyncratic productivity ($G(R^C) = 0.774$), which is roughly the top two fifths of the distribution of productivity among temporary employees ($\frac{1-G(R^C)}{1-G(R^T)} = 0.422$).

### 3.6.2 Comparative statics

Figures 2 and 3 show how the steady state equilibrium is affected by the two main policy parameters, $F$ and $\delta$, and also how these policies interact with the arrival rate $\lambda$ of idiosyncratic shocks. Dots differ by 10%; the graphs show the effects of changing $F$ and $\delta$ by $\pm 30\%$ around their baseline levels. Changes in $\lambda$ range from $-20\%$ (blue) to $+20\%$ (magenta); red dots represent the baseline value of $\lambda$.

Several aspects of Figure 2 illustrate the familiar finding that increased firing costs make the labor market more “sclerotic”, slowing down labor market flows, but with an ambiguous effect overall on the unemployment rate. In the second row, we see that $R^P$ decreases with $F$, whereas $R^C$ increases— firms are less willing to fire permanent workers when firing costs are high, but they are also less willing to promote them to permanence. Therefore the overall flow into and out of permanent jobs is much slower when $F$ is large. Sclerosis can also be seen in the effect on $q(\theta)$: higher firing costs lower vacancy formation and labor market tightness (and hence $q(\theta)$ increases).

On the other hand, since higher firing costs make firms less willing to contract permanent workers, they also become less selective about which temporary workers they hire. Therefore $R^T$ decreases with $F$. This effect is strong enough so that unemployed workers' probability of reemployment, $\theta q(\theta)(1 - G(R^T))$, rises even as workers' matching probability $\theta q(\theta)$ falls. Thus, the flip side of greater "sclerosis" of permanent jobs is greater "churning" of temporary jobs, as both the rate of creation and destruction of temporary jobs increases with $F$.

The overall result, at the baseline calibration of $\lambda$, is that changing $F$ has little effect on unemployment. However, with lower $\lambda$ (blue dots), higher firing costs raise unemployment, as an increasing fraction of total employment is shifted into temporary contracts with little prospect of eventual promotion. At the opposite extreme, with a higher $\lambda$, the current value of idiosyncratic productivity is less important, making firms less selective about all contract types. In particular, with high $\lambda$ the fraction of permanent workers fired after an idiosyncratic shock falls from 30% to

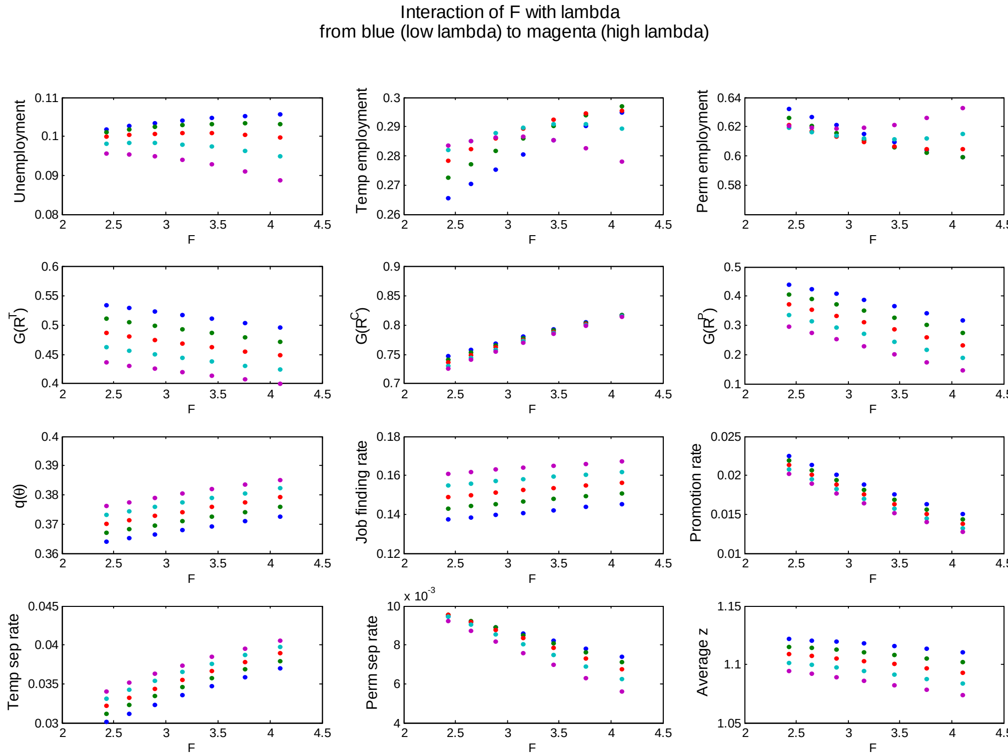


Figure 2: Comparative statics: firing cost

15% as $F$ rises, so in this case unemployment decreases with $F$.

While firing costs have an ambiguous effect on unemployment, over this parameter range they unambiguously reduce productivity, as the last panel of Figure 2 shows. Intuitively, while firing costs make firms more selective about which matches to promote, they also makes firms less selective about the permanent workers they retain, and prompts them to rely more on rapid hiring and firing of relatively low-productivity temporary workers. Thus while an increase in $F$ implies that those workers who have just been promoted to permanence will have higher productivity, it also implies that temporary workers and old permanent workers will have lower productivity. The overall effect is roughly a 1% fall in average worker productivity as we increase $F$ by 60% in Figure 2.

Figure 3 shows the effect of changing the duration $1/\delta$ of temporary contracts,

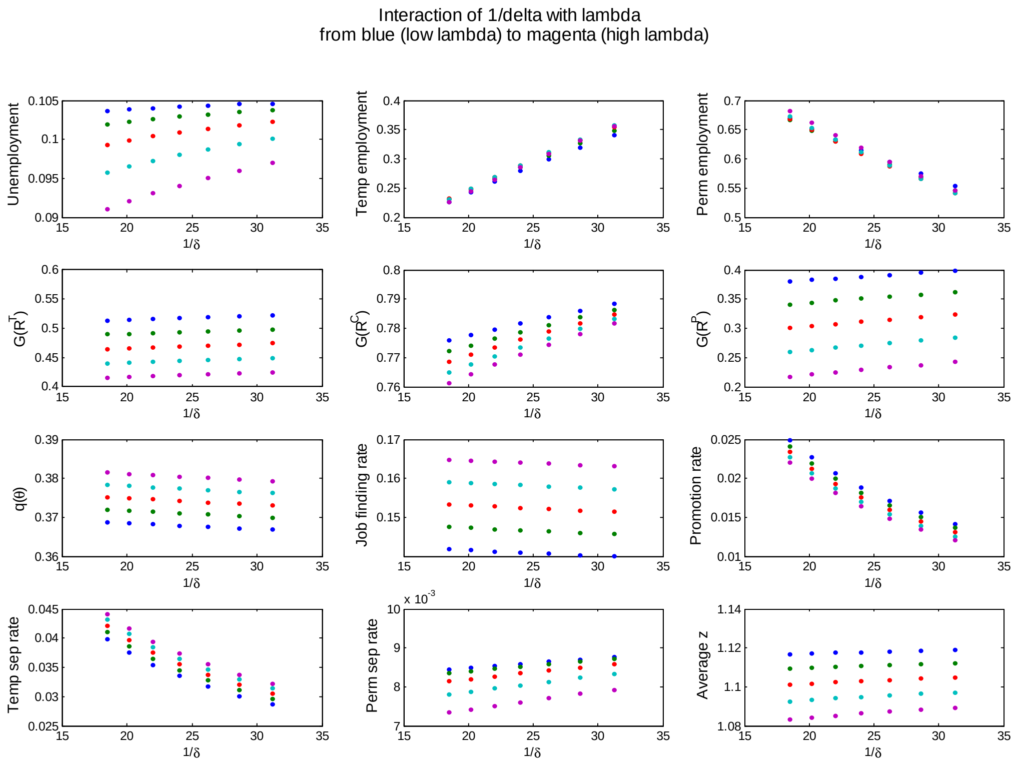


Figure 3: Comparative statics: temporary contract duration

interacted as before with the arrival rate $\lambda$ of idiosyncratic shocks. An increase in $1/\delta$ makes firms moderately more selective at all the reservation thresholds, but the main impact is the direct one: as temporary contracts expire more slowly, they are a sharply increasing fraction of the labor force. In percentage terms, the largest impact on the reservation thresholds comes through the promotion margin, where the fraction promoted falls by roughly 8% (from a 23.6% promotion rate to a 21.7% promotion rate) as the duration of temporary contracts increases from 18 to 33 months. That is, increasing $1/\delta$ causes firms to rely more on "churning" their temporary workforce instead of promotion to permanence, and as a result at the baseline calibration it causes a small increase in the unemployment rate.

# 4 Dynamics

Next, we allow for aggregate shocks. For simplicity, in the main text we assume that there are only two aggregate states: recessions, with aggregate productivity $y_1$, and booms, with aggregate productivity $y_2$. (The case of $N$ states is quite similar, but the parts of the analysis that require more notation are left for Appendix 1.) Hence the transition matrix simplifies to

$$M = \begin{pmatrix} M_{y_1|y_1} & M_{y_1|y_2} \\ M_{y_2|y_1} & M_{y_2|y_2} \end{pmatrix}$$

Note that given the abbreviation $\mu_{j|i} \equiv \mu M_{y_j|y_i}$, we have $\mu_{1|1} + \mu_{2|1} = \mu_{1|2} + \mu_{2|2} = \mu$.

With two states, there are six relevant productivity cutoffs, three for recessions: $R_1^P \leq R_1^T \leq R_1^C$, and three for booms: $R_2^P \leq R_2^T \leq R_2^C$. We also know that $R_2^i \leq R_1^i$ for $i \in \{T, C, P\}$. Furthermore, for the Spanish case that interests us, firing costs are large. Therefore we look for an equilibrium in which $F$ is large enough compared to $y_2 - y_1$ so that $R_1^C$ and $R_2^C$ are both strictly greater than $R_1^T$ and $R_2^T$, which in turn are strictly greater than $R_1^P$ and $R_2^P$.

This known ordering simplifies the job of calculating the surplus functions, and employment and productivity. As Prop. 5 showed, to know the distribution of employment and productivity at all times it suffices to keep track of employment on each interval defined by two adjacent reservation thresholds. Here, there are seven intervals to keep track of, which we will call $I_7 = [r_7, r_6)$, $I_6 = [r_6, r_5)$, $I_5 = [r_5, r_4)$, $I_4 = [r_4, r_3)$, $I_3 = [r_3, r_2)$, $I_2 = [r_2, r_1)$, and $I_1 = [r_1, r_0]$. The bounds on these intervals are $r_7 = 0$, $r_6 = R_2^P$, $r_5 = R_1^P$, $r_4 = R_2^T$, $r_3 = R_1^T$, $r_2 = R_2^C$, $r_1 = R_1^C$, and $r_0 = \infty$. Thus all matches separate in interval $I_7$, whereas all matches continue in interval $I_1$.

## 4.1 Value functions

In the steady state analysis of Section 3, the Bellman equations governing the value functions contained capital gains terms driven by idiosyncratic shocks at rate $\lambda$. Now, they also contain capital gains from aggregate shocks at rate $\mu$. The value functions for temporary and permanent jobs, $J^T(z,y)$ and $J^P(z,y)$, satisfy:

$$\begin{aligned}(r+\rho)\, J^T(z,y) \;=\;& z + y - w_T(z,y) + \delta\left[\mathbf{1}(z \geq R^C(y)) J^P(z,y) - J^T(z,y)\right] \\ &+ \lambda\left[\int_{R^T(y)} J^T(x,y)dG(x) - J^T(z,y)\right] + \mu\left[\sum_{y':R^T(y')\leq z} M_{y'|y} J^T(z,y') - J^T(z,y)\right]\end{aligned}$$

$$
\begin{aligned}
(r+\rho)\,J^P(z,y) &= z+y-w_P(z,y)+\lambda\left[\int_{R^P(y)} J^P(x,y)dG(x)-G(R^P(y))F-J^P(z,y)\right] \\
&\quad +\mu\left[\sum_{y':R^P(y')\le z} M_{y'|y}J^P(z,y')-\sum_{y':R^P(y')>z} M_{y'|y}F-J^P(z,y)\right]
\end{aligned}
$$

A worker's value of employment under a temporary contract is given by

$$
\begin{aligned}
(r+\rho)\,W^T(z,y) &= w_T(z,y)+\delta\left[\mathbf{1}(z\ge R^C(y))W^P(z,y)+\mathbf{1}(z<R^C(y))U-W^T(z,y)\right] \\
&\quad +\lambda\left[\int_{R^T(y)} W^T(x,y)dG(x)+G(R^T(y))U(y)-W^T(z,y)\right] \\
&\quad +\mu\left[\sum_{y':R^T(y')\le z} M_{y'|y}W^T(z,y')+\sum_{y':R^T(y')>z} M_{y'|y}U(y')-W^T(z,y)\right]
\end{aligned}
$$

where

$$
\begin{aligned}
(r+\rho)\,W^P(z,y) &= w_P(z,y)+\lambda\left[\int_{R^P(y)} W^P(x,y)dG(x)+G(R^P(y))U(y)-W^P(z,y)\right] \\
&\quad +\mu\left[\sum_{y':R^P(y')\le z} M_{y'|y}W^P(z,y')+\sum_{y':R^P(y')>z} M_{y'|y}U(y')-W^P(z,y)\right]
\end{aligned}
$$

is the annuity value of being employed in the job with productivity $z$ under a permanent contract, and

$$
(r+\rho)\,U(y)=b+\theta(y)q(\theta(y))\left(W^T(\underline{z},y)-U(y)\right)+\mu\sum_{y'} M_{y'|y}(U(y')-U(y))
$$

is the annuity value of unemployment.

## 4.2 Surplus functions

By combining the Bellman equations for workers' and firms' surplus from employment and search, like we did for the steady state model, we can now obtain a single Bellman equation governing a job's match surplus. The Bellman equation for the surplus associated with permanent jobs is

$$
\begin{aligned}
(r+\rho+\lambda+\mu)\,S^P(z,y) &= z+y-b+(r+\rho)\,F-\frac{\eta c\theta(y)}{1-\eta} \\
&\quad +\lambda\int_{R_i^P} S^P(x,y)dG(x)+\mu\sum_{y':R^P(y')\le z} M_{y'|y}S^P(z,y')
\end{aligned}
$$

Note that this equation is only defined at $z \geq R^P(y)$. Therefore, in state $y = y_i$, the term $\mu_{i|i} S^P(z, y_i)$ can be cancelled from both sides. In the special case of just two aggregate states, where $\mu_{i|i} + \mu_{-i|i} = \mu$, the Bellman equation thus simplifies to[12]

$$\begin{aligned}
\left(r + \rho + \lambda + \mu_{-i|i}\right) S^P(z, y_i) \quad &= \quad z + y_i - b + (r + \rho) F - \frac{\eta c \theta(y_i)}{1 - \eta} \\
&\quad + \lambda \int_{R_i^P} S^P(x, y_i) dG(x) + \mu_{-i|i} \mathbf{1}(R_{-i}^P \leq z) S^P(z, y_{-i})
\end{aligned} \tag{14}$$

As in Section 3.2, we can now inspect (14) to see how $S^P(z, y)$ varies with $z$. First, note that (14) has no discontinuities. Looking at the right-hand side, it seems that there might be a discontinuity at $z = R_{-i}^P$, but by definition $S^P(R_{-i}^P, y_{-i}) = 0$, so the discontinuity vanishes. We therefore conclude that $S^P(z, y)$ is a continuous function.

Differentiating (14) with respect to $z$, the slope of $S^P$ in recessions and booms satisfies

$$\left(r + \rho + \lambda + \mu_{-i|i}\right) \sigma_i^P = 1 + \mu_{-i|i} \mathbf{1}(R_{-i}^P \leq z) \sigma_{-i}^P$$

where we have used the shorthand $\sigma_i^P \equiv \frac{\partial S^P}{\partial z}(z, y_i)$. Evidently, $S^P$ is piecewise linear. Using the fact that $R_2^P < R_1^P$, the slopes in different intervals are:

| | $R_2^P < z < R_1^P$ | $R_1^P < z$ |
|---|---|---|
| $\sigma_1^P$ | n.a. | $(r + \rho + \lambda)^{-1}$ |
| $\sigma_2^P$ | $\left(r + \rho + \lambda + \mu_{1\|2}\right)^{-1}$ | $(r + \rho + \lambda)^{-1}$ |

Finally, using $S^P(R_1^P, y_1) = S^P(R_2^P, y_2) = 0$, the surplus function for permanent jobs can be written explicitly in terms of the reservation productivities as

$$S^P(z, y_1) \quad = \quad \frac{1}{r + \rho + \lambda} \left(z - R_1^P\right) \tag{15a}$$

$$S^P(z, y_2) \quad = \quad \begin{cases} \frac{1}{r+\rho+\lambda+\mu_{1|2}} \left(z - R_2^P\right), & R_2^P \leq z \leq R_1^P \\ S^P(R_1^P, y_2) + \frac{1}{r+\rho+\lambda} \left(z - R_1^P\right), & R_1^P < z \end{cases} \tag{15b}$$

The procedure to calculate the surplus function for temporary workers is similar, but has a few more steps. The Bellman equation for $S^T(z, y)$ is

$$\left(r + \rho + \lambda + \delta + \mu_{-i|i}\right) S^T(z, y_i) = z + y_i - b - \frac{\eta c \theta(y_i)}{1 - \eta} + \delta \mathbf{1}(z \geq R_i^C) \left(S^P(z, y_i) - F\right)$$

[12] Here $-i$ refers to the state that is not state $i$.

$$+\lambda \int_{R_i^T} S^T(x,y_i)dG(x) + \mu_{-i|i}\mathbf{1}(R_{-i}^T \le z)S^T(z,y_{-i}) \tag{16}$$

where we have again simplified by cancelling $\mu_{i|i}S^T(z,y_i)$ from both sides.

We have focused on the case in which $F$ is sufficiently large compared to $y_2 - y_1$ so that the two promotion thresholds $R_2^C$ and $R_1^C$ are both strictly greater than all the other thresholds. Thus (16) implies that functions $S^T(z,y_1)$ and $S^T(z,y_1)$ both have discontinuities both at $R_2^C$ and at $R_1^C$. We write these jumps as

$$\Delta(R_j^C, y_i) \equiv \lim_{dz\to 0}\left[S^T(R_j^C + dz, y) - S^T(R_j^C - dz, y)\right]$$

We can take this limit on both sides of (16) if we note that that $S^P(z,y_i) - F = \frac{\eta}{1-\eta}F$ at $z = R_i^C$, and that all the $T$-thresholds are below all the $C$-thresholds since $F$ is assumed large. We obtain the following formula for the jumps at the promotion thresholds:

$$\left(r+\rho+\lambda+\delta+\mu_{-i|i}\right)\Delta(R_j^C, y_i) = \delta\mathbf{1}(R_j^C = R_i^C)\frac{\eta}{1-\eta}F + \mu_{-i|i}\Delta(R_j^C, y_{-i})$$

These are four equations to determine the jumps $\Delta(R_1^C, y_1)$, $\Delta(R_1^C, y_2)$, $\Delta(R_2^C, y_1)$, and $\Delta(R_2^C, y_2)$. The solution is:

| | at $z = R_2^C$ | at $z = R_1^C$ |
|---|---|---|
| $\Delta(z,y_1)$ | $\mu_{2\|1}\varepsilon F$ | $\left(r+\rho+\lambda+\delta+\mu_{1\|2}\right)\varepsilon F$ |
| $\Delta(z,y_2)$ | $\left(r+\rho+\lambda+\delta+\mu_{2\|1}\right)\varepsilon F$ | $\mu_{1\|2}\varepsilon F$ |

where $\varepsilon = (r+\rho+\lambda+\delta)^{-1}(r+\rho+\lambda+\delta+\mu_{1|2}+\mu_{2|1})^{-1}\frac{\delta\eta}{1-\eta}$.

We now turn to the slopes of $S^T(z,y)$. Using (16), and defining $\sigma_i^T \equiv \frac{\partial S^T}{\partial z}(z,y_i)$, we have

$$\left(r+\rho+\lambda+\delta+\mu_{-i|i}\right)\sigma_i^T = 1 + \delta\mathbf{1}(z \ge R_i^C)\sigma_i^P + \mu_{-i|i}\mathbf{1}(R_{-i}^T \le z)\sigma_{-i}^T$$

We see that the slopes change at the points $R_2^T < R_1^T < R_2^C < R_1^C$. Solving each of this pair of equations (for $i = 1,2$) equations on each relevant interval, we can summarize the slopes as follows:

| | $R_2^T < z < R_1^T$ | $R_1^T < z < R_2^C$ | $R_2^C < z < R_1^C$ | $R_1^C < z$ |
|---|---|---|---|---|
| $\sigma_1^T$ | n.a. | $(r+\rho+\lambda+\delta)^{-1}$ | $\frac{\omega_1}{r+\rho+\delta+\lambda} + \frac{1-\omega_1}{r+\rho+\lambda}$ | $(r+\rho+\lambda)^{-1}$ |
| $\sigma_2^T$ | $\left(r+\rho+\lambda+\delta+\mu_{1\|2}\right)^{-1}$ | $(r+\rho+\lambda+\delta)^{-1}$ | $\frac{\omega_2}{r+\rho+\delta+\lambda} + \frac{1-\omega_2}{r+\rho+\lambda}$ | $(r+\rho+\lambda)^{-1}$ |

Here we have defined the weights $\omega_1 \equiv \frac{r+\rho+\delta+\lambda+\mu_{1|2}}{r+\rho+\delta+\lambda+\mu_{1|2}+\mu_{2|1}}$ and $\omega_2 \equiv \frac{\mu_{1|2}}{r+\rho+\delta+\lambda+\mu_{1|2}+\mu_{2|1}}$. Note that the slope of $S^T$ increases with $z$ (since $\omega_2 < \omega_1$, we find that $\sigma_1^T < \sigma_2^T$ on interval $I_2$).

We can now write down an explicit formula for the surplus function for temporary jobs. Since $S^T$ is discontinuous at some points, it helps to define the notation

$$S_*^T(z,y) \equiv \lim_{x \uparrow z} S^T(x,y)$$

that is, the limit of the surplus function as we approach the point $z$ from below. This notation will help us see where the surplus functions are discontinuous, and how large the jumps are. Taking as given the reservation productivities, the surplus from a temporary job is given by

$$S^T(z,y_1) = \begin{cases} \frac{1}{r+\rho+\delta+\lambda}\left(z - R_1^T\right), & R_1^T \leq z < R_2^C \\ S_*^T(R_2^C, y_1) + \mu_{2|1}\varepsilon F + \left(\frac{\omega_1}{r+\rho+\delta+\lambda} + \frac{1-\omega_1}{r+\rho+\lambda}\right)\left(z - R_2^C\right), & R_2^C \leq z < R_1^C \\ S_*^T(R_1^C, y_1) + \left(r+\rho+\lambda+\delta+\mu_{1|2}\right)\varepsilon F + \frac{1}{r+\rho+\lambda}\left(z - R_1^C\right), & z \geq R_1^C, \end{cases} \tag{17}$$

$$S^T(z,y_2) = \begin{cases} \frac{1}{r+\rho+\delta+\lambda+\mu_{1|2}}\left(z - R_2^T\right), & R_2^T \leq z < R_1^T \\ S^T(R_1^T, y_2) + \frac{1}{r+\rho+\delta+\lambda}\left(z - R_1^T\right), & R_1^T \leq z < R_2^C \\ S_*^T(R_2^C, y_2) + \left(r+\rho+\lambda+\delta+\mu_{2|1}\right)\varepsilon F + \left(\frac{\omega_2}{r+\rho+\delta+\lambda} + \frac{1-\omega_2}{r+\rho+\lambda}\right)\left(z - R_2^C\right), & R_2^C \leq z < R_1^C \\ S_*^T(R_1^C, y_2) + \mu_{1|2}\varepsilon F + \frac{1}{r+\rho+\lambda}\left(z - R_1^C\right), & z \geq R_1^C. \end{cases} \tag{18}$$

Figure 4 shows the dynamic surplus functions (15a)-(15b) and (17)-(18), in equilibrium under our benchmark parameterization. Like the steady state surplus functions, they are piecewise linear with discountinuities in $S^T$ at thresholds $R_2^C$ and $R_1^C$. Now, though, we see four functions, since we are plotting both for recessions and booms; from top to bottom the functions are $S^P(z,y_2)$, $S^P(z,y_1)$, $S^T(z,y_2)$, and $S^P(z,y_1)$. The top two and the bottom two each lie close together, because the difference in surplus between recessions and booms is much smaller than the difference in surplus between temporary and permanent employment status.

Red stars indicate the six reservation thresholds (from left to right) $R_2^P$, $R_1^P$, $R_2^T$, $R_1^T$, $R_2^C$, and $R_1^C$. One effect of passing from boom to recession is the immediate firing of all permanent workers with productivity in the interval $[R_2^P, R_1^P)$, and all temporary workers with productivity in the interval $[R_2^T, R_1^T)$; then when the economy returns to its expansive phase, new stocks of these "fragile" jobs gradually build up. The size of the wave of firing that occurs at the beginning of a recession depends on the width of the interval of fragile jobs, which is wider in the case of temporary

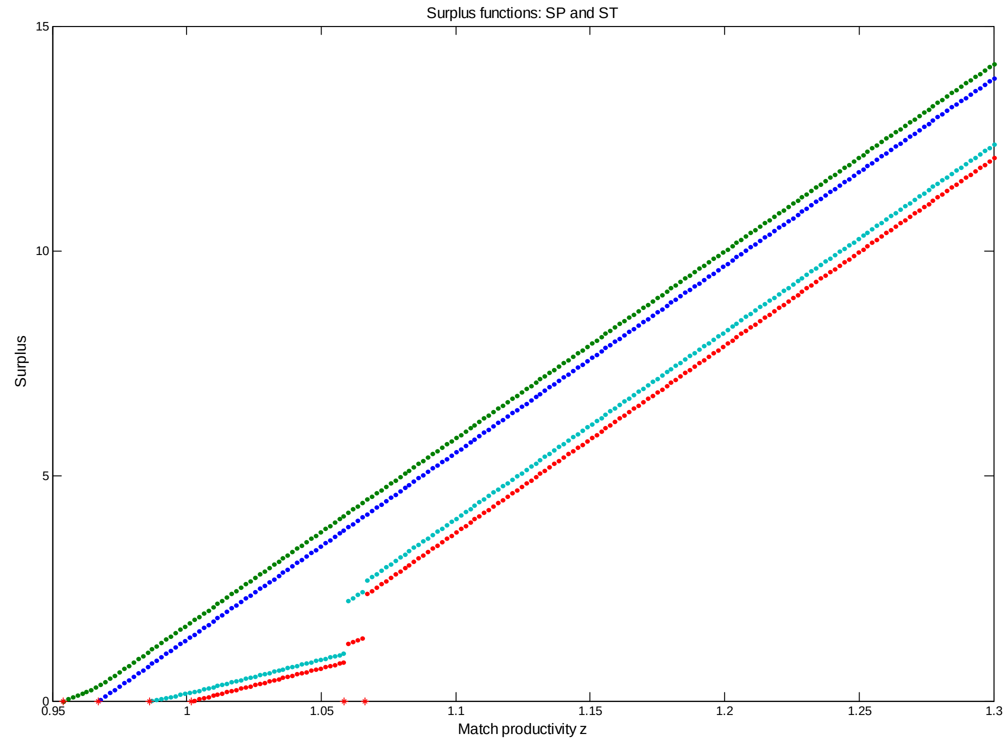


Figure 4: Dynamic surplus functions

workers ($R_1^T - R_2^T$) than it is for permanent workers ($R_1^P - R_2^P$). Also, the interval $[R_2^T, R_1^T)$ is closer to the mode of the distribution $G(z)$, which tends to create a larger stock of jobs in this interval. Both these effects make temporary firing more volatile (in relative terms) than firing of permanents. On the other hand, the fact that we calibrate the overall stock of temporary jobs to be smaller than that of permanent jobs may make temporary firing less volatile in absolute terms.

Finally, the fact that the stocks of fragile jobs in $[R_2^P, R_1^P)$ and $[R_2^T, R_1^T)$ build up gradually over the course of a boom means that the size of the burst of firing at the beginning of a recession depends on the length of the preceding boom. If this boom has been longer, there will be more jobs accumulated that are subsequently subject to firing.

## 4.3 Dynamic equilibrium

In aggregate state $y_i$, the job creation condition is

$$\frac{c}{q(\theta(y_i))} = (1-\eta)\int_{R_i^T} S^T(x, y_i)dG(x) \tag{19}$$

We now rewrite the job destruction conditions using the Bellman equations. The job destruction condition (2) for temporary jobs becomes

$$0 = R_i^T + y_i - b - \frac{\eta c\theta(y_i)}{1-\eta} + \lambda\int_{R_i^T} S^T(x, y_i)dG(x) + \mu_{-i|i}\mathbf{1}(R_{-i}^T \leq z)S^T(z, y_{-i}) \tag{20}$$

For permanent jobs, the job destruction condition (4) is

$$0 = R_i^P + y_i - b + (r+\rho)F - \frac{\eta c\theta(y_i)}{1-\eta} + \lambda\int_{R_i^P} S^P(x, y_i)dG(x) + \mu_{-i|i}\mathbf{1}(R_{-i}^P \leq z)S^P(z, y_{-i}). \tag{21}$$

Finally, the promotion threshold in state $y_i$ can be determined by:

$$(r+\rho+\lambda+\mu)\frac{F}{1-\eta} = R_i^C + y_i - b + (r+\rho)F - \frac{\eta c\theta(y_i)}{1-\eta} + \lambda\int_{R_i^P} S^P(x, y_i)dG(x) + \mu_{-i|i}\mathbf{1}(R_{-i}^P \leq z)S^P(z, y_{-i}), \tag{22}$$

Given hypothetical values of $R^P(y)$, $R^C(y)$, and $R^T(y)$, we can now plug the the surplus formulas (15a)-(15b) and (17)-(18) into terms on the right-hand side of (19)-(22) for states $i \in \{1, 2\}$. The result is eight equations to determine the eight unknowns $\theta(y_1)$, $\theta(y_2)$, $R_1^P$, $R_1^T$, $R_1^C$, $R_2^P$, $R_2^T$, and $R_2^C$ which together describe a dynamic equilibrium.

## 4.4 Employment and productivity dynamics

Once $R^P(y)$, $R^C(y)$, $R^T(y)$, and $\theta(y)$ are known, we can simulate employment over time, by keeping track of temporary and permanent on the productivity intervals $I_j = [r_j, r_{j-1})$, $j \in \{1, 2, ..., 3N\}$ in which employment may occur. First, let $n_j^T(t)$ be the stock of temporary employment of matches with productivity $z$ in interval $I_j = [r_j, r_{j-1})$. Second, define total temporary employment as $n^T(t) = \sum_{j=0}^{3N} n_j^T(t)$. Next, let $n_j^P(t)$ and $n^P(t) = \sum_{j=0}^{3N} n_j^P(t)$ be the corresponding stocks of permanent employment. Finally, define unemployment as $u(t) = 1 - n^T(t) - n^P(t)$.

Over a short time interval $dt$, in which aggregate productivity is $y(t)$, employment of each type evolves according to

$$\begin{aligned} dn_j^T(t) &= \mathbf{1}\left(R^T(y(t)) < r_{j-1}\right)\left[\theta(y(t))q(\theta(y(t)))u(t) + \lambda n^T(t)\right]\left[G(r_{j-1}) - G(r_j)\right]dt \\ &\quad - (\rho + \delta + \lambda)\, n_j^T(t)dt - \mathbf{1}\left(R^T(y(t+dt)) \geq r_{j-1}\right) n_j^T(t) \end{aligned} \tag{23}$$

$$\begin{aligned} dn_j^P(t) &= \mathbf{1}\left(R^P(y(t)) < r_{j-1}\right)\left\{\lambda\left[G(r_{j-1}) - G(r_j)\right] n^P(t) + \delta n_j^T(t)\right\} dt \\ &\quad - (\rho + \lambda)\, n_j^P(t)dt - \mathbf{1}\left(R^P(y(t+dt)) \geq r_{j-1}\right) n_j^P(t) \end{aligned} \tag{24}$$

Equation (23) describes the dynamics of temporary jobs in interval $I_j$ as the sum of four terms. The first two terms represent formation of new jobs from the pool of unemployed $u(t)$, who have probability $\theta q(\theta)$ of finding a job per unit of time, as well as a flow of temporary workers who have received idiosyncratic shocks (which occur with probability $\lambda$ per unit of time). Both these flows fall into interval $I_j$ with probability $[G(r_{j-1}) - G(r_j)]$. These flows of workers remain employed as long as $z \in I_j$ implies $R^T(y(t)) < z$, which can be written more succinctly as $R^T(y(t)) < r_{j-1}$. The third term represents flows out of the stock $n_j^T(t)$, which may occur because the temporary contract expires (with probability $\delta$ per unit of time), or because an idiosyncratic shock arrives (with probability $\lambda$ per unit of time), or because of retirement (with probability $\rho$ per unit of time). Finally, the fourth term is the lump of firing that occurs if there is a decrease in the aggregate state $y(t + dt) < y(t)$ so severe that jobs in $I_j$ must separate, that is, $r_{j-1} \leq R^T(y(t + dt))$. When such a large negative aggregate shock occurs, the whole mass of employment $n_j^T(t)$ suddenly separates, which is why this term is not proportional to $dt$.[13]

The intuition of (24) is similar. Jobs flow into the employment stock $n_j^P(t)$ either due to contract expiry from a temporary job in interval $I_j$, or due to an idiosyncratic shock to a permanent job that results in $z \in I_j$. They flow out of $n_j^P(t)$ due to retirment or as idiosyncratic shocks arrive, or are all be fired suddenly when there is an aggregate shock $y'$ satisfying $R^P(y') \geq r_{j-1}$.

## 4.5 Solving for the wage

We can also solve for equilibrium wages by combining the bargaining rules with the relevant Bellman equations. For temporary workers,

$$w_T(z, y) = \eta\left[z + y + c\theta(y) - \delta F\right] + (1 - \eta)\, b,$$

[13] Note that if $y$ is already so low at time $t$ that $R^T(y(t)) > r_{j-1}$, then $n_j^T(t)$ will already be zero. Therefore the fourth term representing the mass of firing vanishes in this case; there is no mass left to fire.

| *Variable* | *Model* | | *Model: conditional SS* | |
|---|---|---|---|---|
| | *SS* | *Mean* | *Recession* | *Boom* |
| *Stocks* | | | | |
| $E(n^T)$ | 0.2895 | 0.2900 | 0.2940 | 0.2823 |
| $E(n^P)$ | 0.6096 | 0.5959 | 0.5818 | 0.6303 |
| $E(u)$ | 0.1009 | 0.1152 | 0.1242 | 0.0873 |
| *Probability rates* | | | | |
| $prob(T\|U)$ | 0.1525 | 0.1560 | 0.1280 | 0.1696 |
| $prob(P\|T)$ | 0.0176 | 0.0170 | 0.0176 | 0.0171 |
| $prob(U\|T)$ | 0.0335 | 0.0336 | 0.0344 | 0.0333 |
| $\rho + prob(U\|T)$ | 0.0356 | 0.0357 | 0.0365 | 0.0354 |
| $prob(U\|P)$ | 0.0063 | 0.0060 | 0.0068 | 0.0056 |
| $\rho + prob(U\|P)$ | 0.0084 | 0.0081 | 0.0089 | 0.0077 |

Table 2: First moments, $b/Ez = 0.8$ (monthly frequency)

if $z \geq R^C(y)$, and

$$w_T(z, y) = \eta\,[z + y + c\theta(y)] + (1 - \eta)\, b,$$

otherwise. Notice that the wage of temporary workers *decreases* by $\eta\delta F$ at the threshold $R^C(y)$, because a worker with $z \geq R^C(y)$ expects his/her job to last longer, and therefore obtains more surplus through expected future payments instead of payments now. For permanent jobs, the wage equation is

$$w_P(z, y) = \eta\,[z + y + c\theta(y) + rF] + (1 - \eta)\, b.$$

Notice that, conditional on the same level of productivity, the wage of permanent and temporary workers differ by the amount $\eta\left[r + \delta\mathbf{1}(z \geq R^C(y))\right] F$. Therefore, firing costs introduce a wedge between the wages of both types.

# 5 Dynamic results: $N = 2$ with large $F$

In this section, we study the business cycle dynamics of our model under the baseline parameterization, with two possible aggregate states. We begin by reporting some first moments in Table 2. Since the calibration is chosen for consistency with the average stocks of temporary and permanent workers in Spanish data ($n^T = 0.2895$ and $n^P = 0.6095$), these are reproduced precisely by the steady state of the model. However, given the model's extreme nonlinearity, its steady state differs from the mean of its dynamics in the presence of aggregate shocks. In particular, unemployment

| *Data* | *Model* | | | |
|---|---|---|---|---|
| | *Benchmark* | *Lower $F$* | *Lower $\delta$* | *Lower $b$* |
| *Stocks* | | | | |
| $sd(n^T) = 1.13\%$ | 0.74% | 0.77% | 0.80% | 0.46% |
| $sd(\ln(n^T)) = 3.86\%$ | 2.56% | 2.60% | 2.41% | 1.81% |
| $sd(n^P) = 0.58\%$ | 0.62% | 0.66% | 0.58% | 0.50% |
| $sd(\ln(n^P)) = 0.94\%$ | 1.05% | 1.09% | 1.06% | 0.75% |
| $sd(u) = 0.90\%$ | 1.06% | 1.08% | 1.09% | 0.66% |
| $sd(\ln(u)) = 9.22\%$ | 9.26% | 9.47% | 9.41% | 8.64% |
| *Flows* | | | | |
| $sd(JC)$ | 0.24% | 0.24% | 0.25% | 0.20% |
| $sd(\ln(JC))$ | 5.24% | 5.38% | 5.39% | 5.15% |
| $sd(JD)$ | 0.60% | 0.60% | 0.62% | 0.44% |
| $sd(\ln(JD))$ | 14.84% | 15.31% | 15.17% | 13.36% |

Table 3: Second moments, $b/Ez = 0.8$ (quarterly frequency, detrended HP-1600)

is almost one and a half percentage points higher in the mean of the economy with aggregate productivity shocks than it is in the steady state. This happens because recessions initially cause unemployment to dramatically overshoot the "conditional steady state" towards which it converges while the recessionary state lasts. The last two columns of Table 2 show conditional steady states: we see that in the limit of an arbitrarily long recession, the unemployment rate exceeds 12%, whereas in the limit of an arbitrarily long boom, it is less than 9%. The mean unemployment rate over time is closer to the conditional steady state for recessions than it is to that for booms, since the time average includes the initial spikes occurring in recessions.

Thus, looking only at the conditional steady states implied by long recessions and booms is insufficient to characterize employment volatility in this economy. Instead, Table 3 reports second moments under several parameterizations. For clarity, we report volatilities both in levels (as a percentage of the labor force), and in logs. Overall, the model does quite a good job of reproducing observed Spanish labor market fluctuations, which are calculated from quarterly EPA data, 2001:1-2008:3.[14] The coefficient of variation of unemployment in the model, 9.26%, comes remarkably close to the coefficient of variation in the data, 9.22%. Moreover, the relative volatility of the two labor market stocks in the model also fits the data quite well. In

[14] Both the data and the simulations from the model are HP-filtered with parameter 1600. Unfortunately our use of data classified by temporary/permanent status restricts us to a rather short sample.

Table 2, looking just at conditional steady states, permanent contracts seemed more volatile than temporary contracts (and moreover, at the conditional steady state, temporary employment is countercyclical). But in Table 3, we see that temporary employment is more volatile than permanent employment, both in the model and in the data. In relative terms, temporary jobs are more than twice as volatile as permanent jobs. Moreover, temporary jobs account for a larger share of employment fluctuations than permanent jobs in absolute terms too, even though on average temporary employment is less than one third of the total.

The alternative parameterizations consider several reforms that would the labor market more flexible. In the "lower $F$" parameterization, we decrease the firing cost by 20%. In the "lower $\delta$" parameterization, we decrease $\delta$ by 20% (that is, we increase the duration of eligibility for a temporary contract by 20%). These two policy changes have a surprisingly small effect. Finally, we also consider the effect of decreasing unemployment protection by 10% (i.e. by eight percentage points from 80% to 72% of average worker productivity), since the benefit level is a well-known factor to explain employment volatility in matching models. This reform causes a large decrease in unemployment (from 11.5% to 7.6%), as well as a large decrease in labor market volatility. Note that the decrease in volatility caused by a lower $b$ is especially pronounced in temporary jobs.

The effects of dual labor market policy on employment volatility are also illustrated in Figures 5-7. Figures 5 and 6 show the impulse responses of various labor market stocks and flows to an increase and a decrease in aggregate productivity (all variables are graphed as a percentage of the total labor force). As we have emphasized, the nonlinearity of the model makes these responses extremely asymmetric. At the transition from recession to boom, there is a hump-shaped response of temporary jobs, as new workers are hired, passing initially through temporary status and then eventually building up a higher stock of permanent matches. At the same time, job destruction of each type of worker briefly decreases by 0.2 percent of the workforce. In contrast, at the transition from boom to recession, there is a sudden burst of firing, with more than 3% of the workforce fired in each contract type (a total of almost 7% of the workforce is fired at this time). Both stocks of workers fall, with the stock of temps recovering quickly while the stock of permanent workers gradually decreases towards a new, lower conditional steady state.

The responses also depend on the starting point; the impulse responses shown here are calculated starting from the conditional steady state. In other words, Figure 5 is the effect of an increase in $y$ after an extremely long recession, and Figure 6 is the effect of a decrease in $y$ after an extremely long boom. Note that after an extremely long boom, a recession causes roughly equal levels of firing of temporary

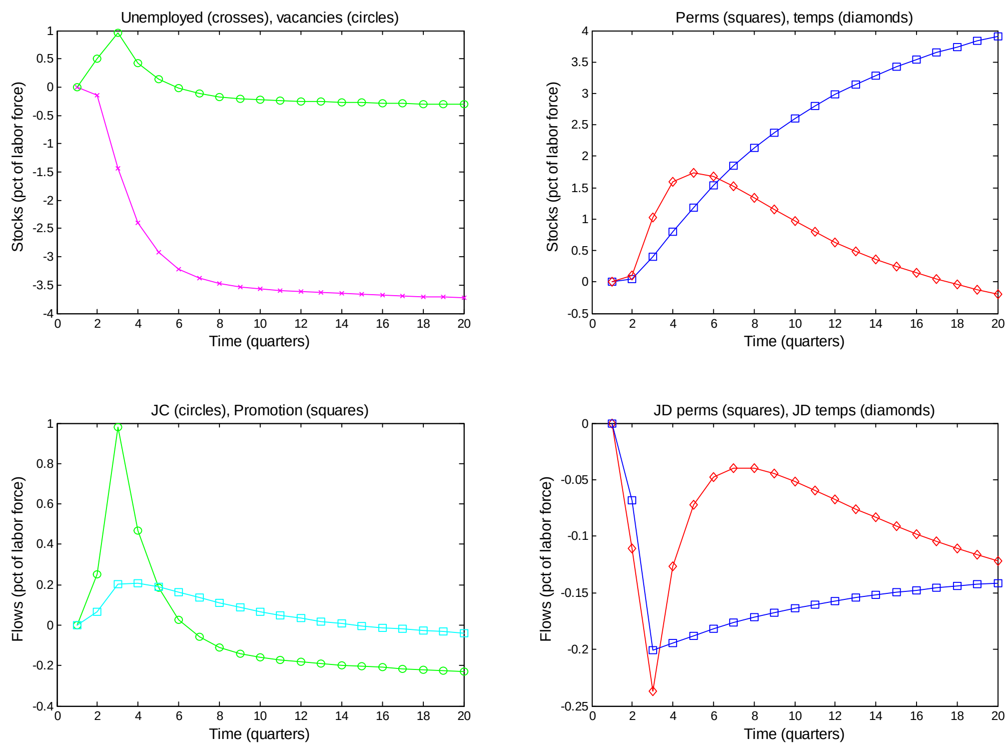


Figure 5: Impulse responses: recession to boom

and permanent jobs. This seems to suggest that both fluctuations in permanent jobs should be almost as important as fluctuations in temporary jobs to explain employment volatility overall.

However, such a conclusion would be mistaken, because the size of the burst in firing of temporary and permanent jobs at the beginning of a recession depends on the length of the preceding boom. The stock of permanent jobs builds up more slowly in a boom than the stock of temps, because workers must pass through temporary status before reaching permanent status, and because the productivity threshold for hiring is lower than the threshold for promotion. Therefore, mostly temporary jobs are fired after a short boom, whereas after a long boom a substantial number of permanent jobs separate too.

The differing ratios of temporary and permanent firing after expansions of dif-

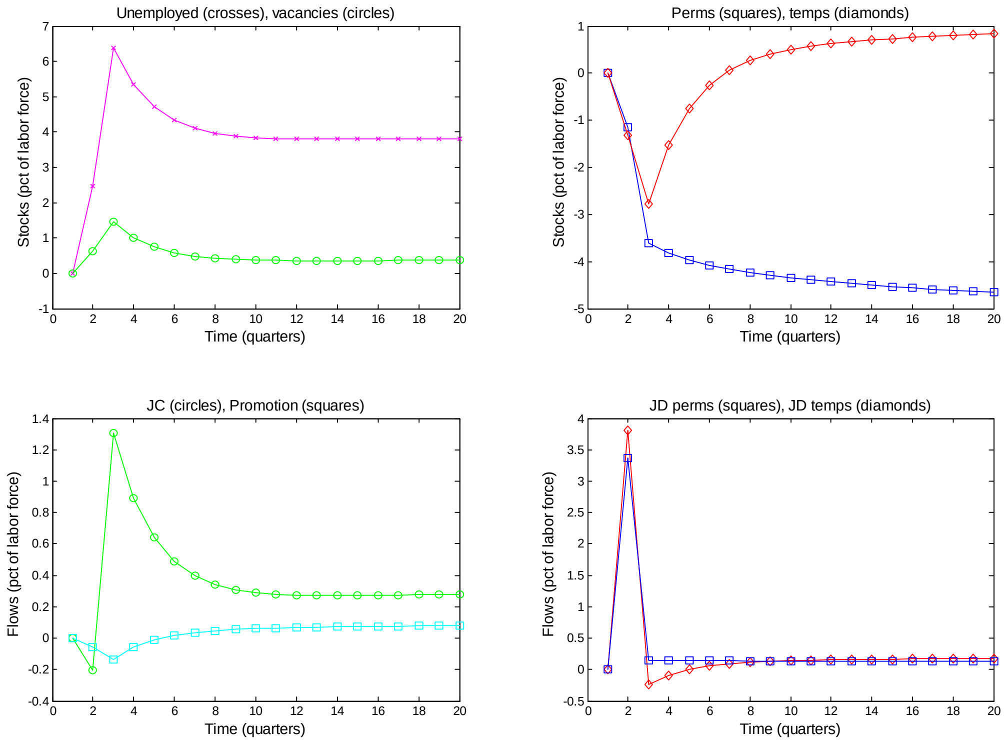


Figure 6: Impulse responses: boom to recession

ferent lengths can be seen quite clearly in the simulated example of Figure 7. Note that since promotion and firing of permanent jobs are very slow processes, a boom must be very long to get anywhere near its "conditional steady state". Instead, given cycles of realistic length, relatively few "fragile" permanent jobs are accumulated in booms. Thus, after the first few booms shown in Figure 7, the red spike representing temporary firing is much larger than the blue spike representing firing of permanent jobs. Only in the exceptionally long boom seen in the second half of the simulated sample do we observe a spike of permanent firing comparable to the spike in temporary firing.

Overall, then, as we already saw in Table 3, Figure 7 shows that temporary jobs play a much larger role for employment fluctuations than permanent jobs do. This is true both in relative terms and in absolute terms, in spite of the fact that the

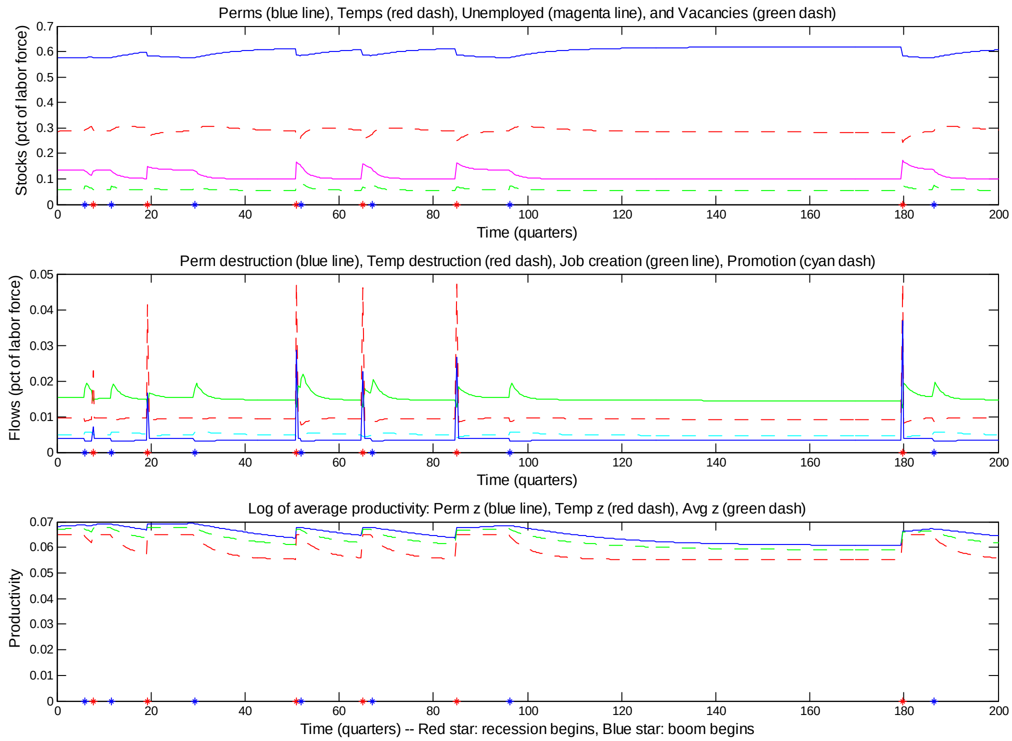


Figure 7: Example: dynamic simulation

average level of temporary jobs is lower than that of permanent jobs.

## 5.1 Eliminating duality

Next, we study what would happen if we replaced the dual contracting structure assumed in our benchmark model with a single type of contract. We maintain all the parameters of our benchmark specification, except for the policy parameters that drive duality. Thus, we now assume all ongoing jobs have exactly the same level of firing costs, $F^*$. We assume $0 < F^* < F$: the firing cost in the unified labor market exceeds the zero firing cost for temporary jobs under duality, but is lower than the cost $F$ of firing a permanent job under duality.

As for the timing of decisions, we assume that a matched pair observe their idiosyncratic productivity $z$ as soon as they meet. At this time, they must decide whether or not to form an employment relationship; if they do not, they can continue searching for other partners without paying the firing cost $F^*$. However, as soon as they begin working, they are legally considered employer and employee, and separation thereafter entails the cost $F^*$. The firm's surplus is therefore defined relative to the outside option $-F^*$, and thus total surplus includes $F^*$:

$$S(z, y) = W(z, y) - U(y) + J(z, y) - V(y) + F$$

where free entry, as before, implies $V(y) = 0$.[15]

Under these timing assumptions, there are two relevant reservation thresholds in any aggregate state $y$. There is a threshold $R^N(y)$ above which a pair will form a relationship upon meeting, which is determined by

$$J(R^N(y), y) = 0 \qquad \rightarrow \qquad S(R^N(y), y) = \frac{F}{1-\eta} > 0 \tag{25}$$

There is also a threshold $R(y)$ for continuation of any existing match, which is simply determined by the absence of any joint surplus from continuation:

$$S(R(y), y) = J(R(y), y) + F = W(R(y), y) - U(y) = 0 \tag{26}$$

The surplus function $S(z, y)$ is monotonically increasing for the same reasons we saw in the baselind model, and therefore we conclude that $R(y) < R^N(y)$ in each aggregate state $y$.

Calibrating the unified labor market only requires choosing an appropriate value of the firing cost $F^*$. Table 4 shows the effects of several different assumptions about the level of firing costs in the unified economy. First, we simply assume that the firing cost that applies to all contracts in the single contract model is half the cost $F$ necessary to fire a permanent job in the dual model. This level of firing costs makes hiring more costly overall, since firms no longer have the option of churning through temporary matches; therefore the unemployment rate rises by almost two percentage points. However, without the quick hiring and firing dynamics of temporary jobs, unifying the labor market causes a decrease in volatility. In particular, the standard deviation of the unemployment rate falls from 1.09% of the labor force under the dual structure, to 0.88% of the labor force under the unified structure.

However, this is not the only possible way to unify the labor market. Choosing $F^* = 0.5F$ sets the single firing cost to the mean of the two firing costs in the dual

[15] The value function notation in this section is the same as in our benchmark model except that, in the absence of duality, we can suppress the subscripts that indicate the two types of labor.

| *Variable* | *Model* | | | |
|---|---|---|---|---|
| | *Dual benchmark* | $F^* = 0.5F$ | *Fix total costs* | *Fix employment* |
| Firing cost | 2.0642 | 1.0321 | 1.1170 | 0.3645 |
| SS unemployment | 0.1010 | 0.1181 | 0.1194 | 0.1010 |
| SS firing costs paid | 0.0081 | 0.0078 | 0.0081 | 0.0034 |
| *Stocks* | | | | |
| $sd(n)$ | 1.09% | 0.88% | 0.89% | 0.81% |
| $sd(\ln(n))$ | | 1.03% | 1.03% | 0.93% |
| $sd(u)$ | 1.09% | 0.88% | 0.89% | 0.81% |
| $sd(\ln(u))$ | 9.50% | 6.55% | 6.61% | 6.97% |
| *Flows* | | | | |
| $sd(JC)$ | 0.25% | 0.13% | 0.13% | 0.15% |
| $sd(\ln(JC))$ | 5.38% | 4.93% | 4.99% | 4.56% |
| $sd(JD)$ | 0.61% | 0.34% | 0.33% | 0.37% |
| $sd(\ln(JD))$ | 15.11% | 16.37% | 16.72% | 13.91% |

Table 4: Second moments, $b/Ez = 0.8$ (quarterly frequency, detrended HP-1600)

model; but of course in the dual model, the two contract types represent different fractions of the labor force. Therefore an alternative is to choose $F^*$ so that the steady state flow of firing costs in the single contract model, which is $\lambda G(R)nF^*$, equals the steady-state flow of firing costs in the dual model, which is $\lambda G(R^P)n^P F$. By a numerical search, we find that this results in $F^* = 0.1170$, which is actually quite close to the value considered before. Therefore this policy change reduces the standard deviation of the unemployment rate from 1.09% to 0.88% of the labor force, which is a 22% decrease in variability.

Finally, we note again that imposing such a high $F^*$ from the beginning of any contract effectively makes hiring more costly for the firm. Therefore, another alternative is to choose the level of firing costs that lowers the steady state unemployment rate of the single contract model back down to the level associated with the steady state of the dual labor market. This requires a very substantial decrease in firing costs, to $F = 0.3645$, lowering the steady state flow of firing costs paid by almost 60%. This also implies a large decrease in the standard deviation of unemployment, from 1.09% to 0.88%, representing a decrease in variability of 35%.

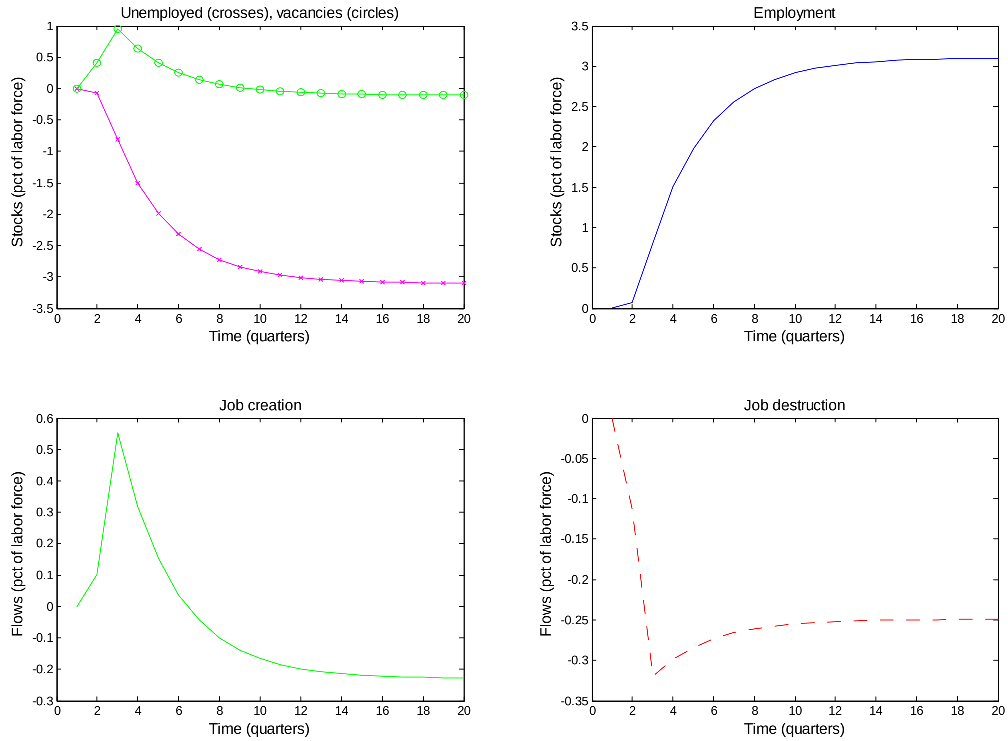


Figure 8: Recession to boom: single contract, fixing employment

# 6 Conclusions

**TO BE COMPLETED**

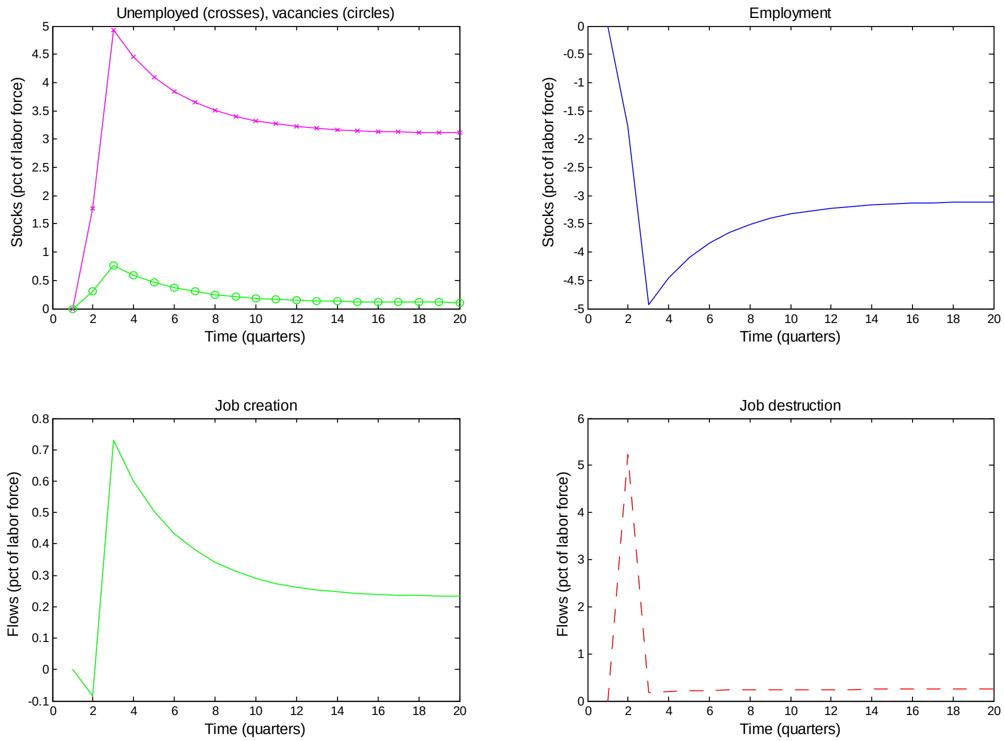


Figure 9: Boom to recession: single contract, fixing employment

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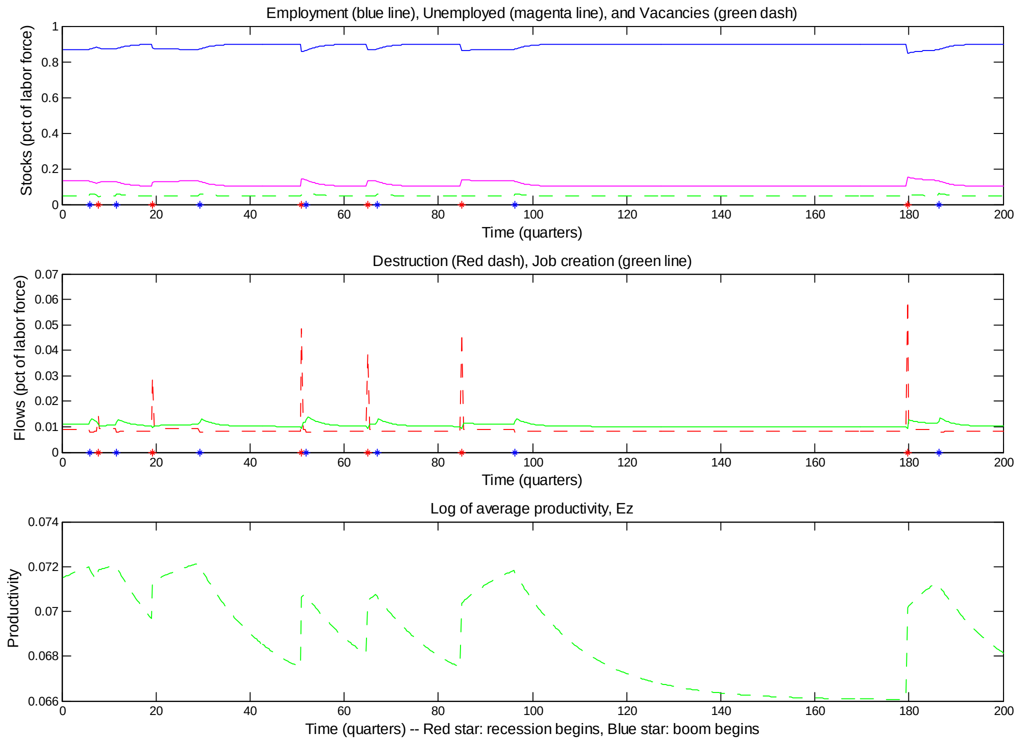


Figure 10: Dynamics: single contract, fixing employment

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# 1 Appendix: Dynamic equilibrium with $N$ aggregate states

Here we generalize the Bellman equations from Section 4 to allow for $N$ possible aggregate states $y_i$. Much of the analysis has already been given in Section 4, except that we can no longer calculate the surplus function explicitly on each productivity interval, since the number of intervals is large. Instead, following Costain and Jansen (2009), we describe a numerical method to calculate the slope on each interval and the jumps between intervals when the number and ordering of intervals is arbitrary.

## 1.1 Surplus functions

The value functions for workers and firms were already stated in Section 4.1. Following the same steps as in Sections 3 and 4, we can combine these equations to derive a Bellman equation for the match surplus. In analogy with equation (5), total match surplus for temporary jobs satisfies

$$(r+\rho+\lambda+\delta+\mu)\,S^T(z,y) = z+y-b-\frac{\eta c\theta(y)}{1-\eta}+\delta\mathbf{1}(z\geq R^C(y))\left(S^P(z,y)-F\right)$$
$$+\lambda\int_{R^T(y)} S^T(x,y)dG(x)+\mu\sum_{y':R^T(y')\leq z} M_{y'|y}S^T(z,y') \tag{27}$$

Note that $J^P(R^C(y),y)=0$ implies $S^P(R^C(y),y)-F=\frac{\eta}{1-\eta}F$. Therefore $S^T(z,y)$ is discontinuous at $z=R^C(y)$, with a jump equal to $(r+\rho+\lambda+\delta+\mu)^{-1}\frac{\delta\eta}{1-\eta}F$. This jump represents the discrete loss in the pair's joint value as $z$ decreases below $R^C(y)$, due to the inefficiency of promotion.

Furthermore, in the capital gains term due to the expected arrival of future aggregate shocks, there can be a discontinuity $z=R^C(y')$, for $y'\neq y$. Finally, note that there are *not* discontinuities at $z=R^T(y')$, since $S^T(R^T(y'),y')=0$.

The match surplus for permanent jobs includes the firing cost that must be paid to separate. It satisfies

$$(r+\rho+\lambda+\mu)\,S^P(z,y) = z+y-b+rF-\frac{\eta c\theta(y)}{1-\eta}+\lambda\int_{R^P(y)} S^P(x,y)dG(x)+\mu\sum_{y':R^P(y')\leq z} M_{y'|y}S^P(z,y') \tag{28}$$

Note that there are no discontinuities in this equation. In particular, there are *not* discontinuities at $z=R^P(y')$, since $S^P(R^P(y'),y')=0$.

### 1.1.1 Partitioning the productivity space

Equations (27)-(28) are continuous and differentiable at most but not all points. There are sudden changes in the form of equation (27) at points $z=R_i^C$ and $z=R_i^T$, and in equation (28) at the points $R_i^P$, for $i\in\{1,2,...,N\}$. Therefore, as in Prop. 5, it is convenient to analyze the surplus equations separately on each interval defined by two consecutive reservation thresholds. There are $N$ thresholds of each type, so the whole support of the productivity distribution can be broken into $3N+1$ relevant intervals bounded by reservation thresholds or by the lowest and highest possible values of $z$. Numbering backwards, we can list all the thresholds as

$$r_{3N}\leq r_{3N-1}\leq r_{3N-2}\leq ...\leq r_2\leq r_1$$

where for each $j \in \{1, 2, ..., 3N\}$, $r_j = R_k^a$, with $a \in \{T, C, P\}$ and $k \in \{1, 2, ..., N\}$. Then the typical interval takes the form

$$I_j = [r_j, r_{j-1})$$

where $r_j$ and $r_{j-1}$ are both reservation productivities.

If we then define $r_{3N+1} = 0$ and $r_0 = \infty$, then the full set of relevant intervals is

$$\begin{aligned} I_{3N+1} &= [r_{3N+1}, r_{3N}) = [0, R_N^P) \\ &\ldots \\ I_j &= [r_j, r_{j-1}) \\ &\ldots \\ I &= [r_1, r_0] = [R_1^C, \infty) \end{aligned}$$

Note that we have not ruled out the possibility that two or more reservation productivities might coincide, $r_j = r_{j-1}$; in this case interval $I_j$, by definition, is empty.

#### 1.1.2 Surplus slopes

On each of the intervals $I_j = [r_j, r_{j-1})$, the surplus functions are continuously differentiable. Differentiating both sides of (27), we obtain

$$(r + \rho + \lambda + \delta + \mu) \frac{\partial S^T}{\partial z}(z, y) = 1 + \delta \mathbf{1}(z \geq R^C(y)) \frac{\partial S^P}{\partial z}(z, y) + \mu \sum_{y': R^T(y') \leq z} M_{y'|y} \frac{\partial S^T}{\partial z}(z, y') \tag{29}$$

For concise notation, we write the slope of the surplus function for temporary jobs in state $y_i$ in interval $I_j$ as $\sigma_{ij}^T \equiv \frac{\partial}{\partial z} S^T(z, y_i),\ z \in I_j$. Likewise, for permanent jobs we define $\sigma_{ij}^P \equiv \frac{\partial}{\partial z} S^P(z, y_i),\ z \in I_j$.

Now consider the slope of the surplus function for temporary jobs in interval $I_j$. Note that for $z \in I_j = [r_j, r_{j-1})$, the condition $z \geq R^C(y)$ is equivalent to $r_j \geq R^C(y)$, and $R^T(y') \leq z$ is equivalent to $R^T(y') \leq r_j$. The surplus function for temporary jobs is zero in state $y_i$ if $R^T(y_i) \leq z$, so we only need to calculate the slopes $\sigma_{ij}^T$ in interval $I_j$ for states $i$ satisfying $R^T(y_i) \leq r_j$. For all these states $i$, the slope of the surplus function for temporary jobs at a point $z \in I_j = [r_j, r_{j-1})$ is characterized by

$$\sigma_{ij}^T = (r + \rho + \lambda + \delta + \mu)^{-1} \left[ 1 + \delta \mathbf{1}(r_j \geq R_i^C) \sigma_{ij}^P + \mu \sum_{k: R_k^T \leq r_j} M_{y_k|y_i} \sigma_{kj}^T \right] \tag{30}$$

For interval $I_j$, there is one equation like this for each $i$ satisfying $R^T(y_i) \leq r_j$, and the unknowns are all the $\sigma^T_{kj}$ satisfying $R^T(y_k) \leq r_j$. In other words, we have the same number of equations and unknowns to determine all the nonzero slopes $\sigma^T_{ij}$ associated with interval $I_j$.

Similarly, differentiating both sides of (28), we obtain

$$(r+\rho+\lambda+\mu)\,\frac{\partial S^P}{\partial z}(z,y) = 1 + \mu \sum_{y':R^P(y')\leq z} M_{y'|y}\frac{\partial S^P}{\partial z}(z,y') \tag{31}$$

which can be rewritten as

$$\sigma^P_{ij} \;=\; (r+\rho+\lambda+\mu)^{-1}\left[1+\mu \sum_{k:R^P_k \leq r_j} M_{y_k|y_i}\sigma^P_{ik}\right] \tag{32}$$

Here we have the right number of equations and unknowns to determine all the nonzero slopes $\sigma^P_{ij}$ associated with interval $I_j$.

### 1.1.3 Surplus jumps

The surplus equations also imply discontinuities in the surplus functions for temporary jobs at points $R^C(y)$, and no discontinuities elsewhere. To be precise, if we define $\Delta(z,y) \equiv \lim_{dz\to 0}\left[S^T(z+dz,y) - S^T(z-dz,y)\right]$, then $\Delta(z,y)$ is nonzero only at points $z = R^C(y')$ for $y' \geq y$. (This is assuming we define $S(z,y) = 0$ for $z < R^T(y)$, so that by definition jumps only occur at $z \geq R^T(y)$.)

To show this, we can use Bellman equation (27) to calculate the jump at any $z$. Since $S^P(z,y)$ is itself a continuous function, we have

$$(r+\rho+\lambda+\delta+\mu)\,\Delta(z,y) - \mu \sum_{y':R^T(y')\leq z} M_{y'|y}\Delta(z,y') =$$

$$= \delta \lim_{dz\to 0}\left\{\mathbf{1}(z+dz \geq R^C(y))\left(S^P(z+dz,y)-F\right) - \mathbf{1}(z-dz \geq R^C(y))\left(S^P(z-dz,y)-F\right)\right\}$$

$$= \delta \lim_{dz\to 0}\left\{\mathbf{1}(z+dz \geq R^C(y)) - \mathbf{1}(z-dz \geq R^C(y))\right\}\left(S^P(z,y)-F\right)$$

$$= \delta\left\{\mathbf{1}(z = R^C(y))\right\}\left(S^P(z,y)-F\right) = \delta\left\{\mathbf{1}(z = R^C(y))\right\}\frac{\eta}{1-\eta}F$$

This shows that there cannot be any discontinuities except at points $z = R^C_i$ for some $i$. To calculate all the jumps at point $R^C_i$, we can therefore calculate

$$\Delta(R^C_i, y_j) \;=\; (r+\rho+\lambda+\delta+\mu)^{-1}\left[\frac{\delta\eta}{1-\eta}F + \mu \sum_{y_k:R^T_k \leq R^C_i} M_{y_k|y_j}\Delta(R^C_i, y_k)\right] \tag{33}$$

which is a system of equations *only* involving the jumps at $z = R_i^C$. It is a system of equations involving the unknown jumps $\Delta(R_i^C, y_k)$ in the surplus functions of all states $k$ such that $R_k^T \leq R_i^C$. There is one equation for each of these unknowns, so there is a unique solution.

### 1.2 Equilibrium

The job creation and destruction conditions for the $N$-state model are virtually identical to those of the two-state model, (19)-(22), except that we sum over all $N$ states on the right-hand side. They can be written as

$$\frac{c}{q(\theta(y_i))} = (1-\eta)\int_{R_i^T} S^T(x, y_i)dG(x) \tag{34}$$

$$0 = R_i^T + y_i - b - \frac{\eta c\theta(y_i)}{1-\eta} + \lambda\int_{R_i^T} S^T(x,y)dG(x) + \mu\sum_{j:y_j\geq y_i} M_{y_j|y_i} S^T(R_i^T, y_j) \tag{35}$$

$$0 = R_i^P + y_i - b + (r+\rho)F - \frac{\eta c\theta(y_i)}{1-\eta} + \lambda\int_{R_i^P} S^P(x,y)dG(x) + \mu\sum_{j:y_j\geq y_i} M_{y_j|y_i} S^P(R_i^P, y_j) \tag{36}$$

$$(r+\rho+\lambda+\mu)\frac{F}{1-\eta} = R_i^C + y_i - b + (r+\rho)F - \frac{\eta c\theta(y_i)}{1-\eta} + \lambda\int_{R_i^P} S^P(x,y_i)dG(x) + \mu\sum_{j:y_j\geq y_i} M_{y_j|y_i} S^P(R_i^P, y_j). \tag{37}$$

Given hypothetical values of $R^P(y)$, $R^C(y)$, and $R^T(y)$, we can use the slope and jump formulas (30), (32), and (33) to numerically calculate the surplus functions and integrals of the surplus functions (looping over all intervals $I_j$). The surplus functions and integrals can then be plugged into (34)-(37), which are then $4N$ equations to jointly determine the equilibrium values of the $4N$ variables $R^P(y)$, $R^C(y)$, $R^T(y)$, and $\theta(y)$.

Having solved for the equilibrium reservation thresholds and tightness, we can then simulate employment and productivity dynamics as described in Section 4.4.

## 2 Appendix: Monotonicity of the surplus functions

**Lemma 6** *For each $y$, the surplus function for permanent contracts $S^P(z, y)$ is a continuous function, which equals 0 at $z = R^P(y)$. For $z > R^P(y)$ it is strictly* *increasing in $z$. At any $z$ that is not a permanent firing threshold ($z \neq R^P(y_j)$ for $j \in \{1, 2, ..., N\}$), the $z$-derivative of $S^P(z, y)$ is well-defined, satisfying*

$$\frac{1}{r+\rho+\lambda+\mu} \leq \frac{\partial}{\partial z} S^P(z,y) \leq \frac{1}{r+\rho+\lambda}. \tag{38}$$

**Proof.** $S^P(R^P(y), y) = 0$ is an equilibrium condition, and the value of $S^P(z, y)$ at $z < R^P(y)$ can be set to zero without loss of generality since it enters nowhere in the equations that define the model. $S^P(z, y)$ is also continuous at permanent firing thresholds $z = R^P(y')$, for $y' \neq y$, because $S^P(R^P(y'), y') = 0$.

At all points $z > R^P(y)$ that are not permanent firing thresholds, the surplus equation (28) is can be differentiated to give (31). Note that (31) can be regarded as a fixed-point problem for vectors of the form $\overrightarrow{\sigma}_i^P \equiv (\sigma_{i1}^P \ \sigma_{i2}^P \ \ldots\ldots \sigma_{iN}^P)'$. For each $i$, the Markov property of matrix $M$ implies that $\sum_{k:R^P(y_k)\leq z} M_{y_k|y_i} \leq 1$. Given this fact, is easy to verify that the mapping defined by (31) satisfies Blackwell's monotonicity and discounting conditions, with discount factor $\frac{\mu}{r+\rho+\lambda+\mu}$. Therefore the mapping is a contraction, and has a unique fixed point.

Moreover, if we apply mapping (31) to the vector $\overline{v} \equiv (r + \rho + \lambda)^{-1}(1\ 1\ \ldots 1)'$, the resulting vector is less than or equal to $\overline{v}$. Likewise, if we apply (31) to $\underline{v} \equiv (r + \rho + \lambda + \mu)^{-1}(1\ 1\ \ \ldots 1)'$, the resulting vector is greater than or equal to $\underline{v}$. Therefore the fixed point of (31) lies between $\overline{v}$ and $\underline{v}$. We therefore conclude that for any $z > R^P(y)$ which is not a reservation threshold, the slope of $S^P$ is exists and satisfies (38).

**QED.**

**Lemma 7** *For each $y$, the surplus function for temporary contracts $S^T(z, y)$ equals 0 at $z = R^T(y)$. For $z > R^T(y)$ it is strictly increasing in $z$. At any $z$ that is not a reservation threshold ($z \neq R^i(y_j)$ for $i \in \{T, C, P\}$ and $j \in \{1, 2, ..., N\}$), the $z$-derivative of $S^T(z, y)$ is well-defined, satisfying*

$$\frac{1}{r+\rho+\delta+\lambda+\mu} \leq \frac{\partial}{\partial z} S^T(z,y) \leq \frac{1}{r+\rho+\delta+\lambda}\left(1+\frac{\delta}{r+\rho+\lambda}\right). \tag{39}$$

*Furthermore, at any $z$ that is not a promotion threshold ($z \neq R^C(y_j)$ for $j \in \{1, 2, ..., N\}$), $S^T(z, y)$ is a continuous function. At the promotion thresholds, it jumps up by a nonnegative amount $\Delta(R^C(y_j), y_i)$, given by (33), bounded by*

$$\frac{1}{r+\rho+\delta+\lambda+\mu}\left(\frac{\delta\eta F}{1-\eta}\right) \leq \Delta(R^C(y_j),y_i) \leq \frac{1}{r+\rho+\delta+\lambda}\left(\frac{\delta\eta F}{1-\eta}\right). \tag{40}$$

**Proof.** To prove the first statements, we follow the same steps as in Lemma 6, noting that (32) defines a contraction mapping. By Lemma 6, the $\delta$ term in (32) lies between 0 and $\frac{\delta}{r+\rho+\lambda}$. Therefore, it is easy to show that the fixed point of (32) is bounded above by $\overline{v} \equiv (r+\rho+\delta+\lambda)^{-1}\left(1+\frac{\delta}{r+\rho+\lambda}\right)(1\ 1\ \ldots 1)'$, and below by $\overline{v} \equiv (r+\rho+\delta+\lambda+\mu)^{-1}(1\ 1\ \ldots 1)'$.

Since $S^P$ is continuous, and $S^T(R^T(y), y) = 0$, (27) shows that $S^T$ is continuous at thresholds $z = R^P(y_j)$ or $z = R^T(y_j)$. At the promotion thresholds $z = R^C(y_j)$, equation (27) implies jumps in $S^T$, which must satisfy (33). Equation (33) can also be seen as a contraction mapping. Using a bounding argument as before, the jumps $\Delta(z, y_i) \equiv \lim_{dz\to 0}\left[S^T(z+dz, y_i) - S^T(z-dz, y_i)\right]$ at $z = R^C(y_j)$ lie between $\frac{1}{r+\rho+\delta+\lambda+\mu}\left(\frac{\delta\eta F}{1-\eta}\right)$ and $\frac{1}{r+\rho+\delta+\lambda}\left(\frac{\delta\eta F}{1-\eta}\right)$.

**QED.**

**Proof of Lemma 2.** Together, Lemmas 6 and 7 imply that the surplus functions are increasing in $z$.

Monotonicity with respect to $y$: **TO BE COMPLETED.** For sufficient conditions, see Costain and Jansen (2009).